In the Wrong Hands:

Complementarities, Resource Allocation, and Aggregate TFP

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Abstract

I explore mismatch between firm quality and firm management as a mechanism for variations in total factor productivity (TFP) across countries. In my calibrated model, even minor deviations from efficient ( assortative) matching have sizeable effects on output and productivity. Underlying this result is the finding that the aggregate implications of matching frictions are highly sensitive to the degree of complementarity between firm and manager attributes. In addition, the relative dispersion of firm and managerial attributes is also key to quantifying the aggregate effects of matching frictions. The key model parameters are pinned down by calibrating the model to U.S. observations on the firm-size distribution and the level and distribution of managerial compensation. My results imply that “crony capitalism”, where key managerial positions are allocated on the basis of political connections rather than talent, imposes a substantial burden on economic welfare.

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1 Introduction

One of the most striking facts in macroeconomics is the variation of income per capita across countries. A factor of almost thirty separates GDP per capita in the most advanced OECD members from countries in sub-Saharan Africa.\(^1\) Recent work suggests that the lion’s share in these differences can be explained by cross-country variations in total factor productivity (TFP) rather than variations in human and physical capital accumulation.\(^2\) The aim of this paper is to explore one particular channel, namely mismatch between firm management and firm quality, to explain observed variations in aggregate productivity across countries.

I construct a model where production requires a project (firm) and a manager in addition to labor and capital. Managers of different abilities seek out projects of commensurate quality in a frictionless over-the-counter market. Together, ability and quality determine the firms’ span of control over capital and labor, both of which are traded in competitive factor markets. Decreasing returns to scale in capital and labor ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes. The equilibrium also features assortative matching between the managers’ ability and the firms’ quality.

Using a combination of firm-level and aggregate data, I calibrate the model to match key moments of the U.S. manufacturing sector. Most importantly, I identify the unobserved distributions of managerial talent and firm quality. I find that the parameters of the distributions are sensitive to the choice of the model’s technology parameters. In particular, imposing unit elasticity of substitution between talent and quality turns out to be a potentially restrictive assumption as far as the implied distribution of managerial ability is concerned.

The parameters of the calibrated model suggest that matching frictions in the market for firm and management attributes have considerable effects on output and productivity. I find that plausible degrees of mismatch can replicate observed variations in the distribution of manufacturing firms across countries, provided the elasticity of substitution between ability and

\(^1\) At purchasing power parity (ICP, 2008).
\(^2\) Recent contributions to this literature include, among many others: Klenow and Rodriguez-Clare (1997); Prescott (1998); Hall and Jones (1999); Howitt (2000); Restuccia et al. (2008); Jones (2008).
quality is low enough. Moreover, the distorted allocations generate economically large variations in measured TFP.

The paper is motivated by considerable evidence that mismatch of managers and projects can explain variations in the efficiency of production. A rich event study literature finds that managerial competence is a quantitatively important factor of efficiency at the firm level. Theoretical macro models, on the other hand, suggest that management quality plays a limited role in accounting for differences in aggregate productivity. My model reconciles these seemingly contradictory conclusions.

My research is related to recent work on the misallocation of resources across plants and firms. Restuccia and Rogerson (2008) and Hsieh and Klenow (2007a) find that the elimination of plant-specific taxes and subsidies can yield TFP gains of 30% or more. My paper formally models a complementary source of distortions, for which there is ample historical and contemporaneous support: crony capitalism. Indeed, there is abundant evidence to suggest that price distortions à la Restuccia and Rogerson (2008) are correlated with attributes of the firm or the manager (or a combination of both) in countries where cronies play an important economic and political role.

Finally, my work revisits the potential of complementarities to shed light on the vast income differences we observe across countries. The idea was first formulated in Kremer’s O-ring paper (Kremer, 1993). Jones (2008) explores the role of complementarities in a model with intermediate goods. I explore matching frictions and complementarities as a source of inefficiency in the production of a single good, rather than a chain of intermediate goods (or tasks). It is, however, natural to interpret my mechanism as a source of “bottlenecks” in a production process involving multiple steps (intermediate goods or tasks) that are complementary to one another, as in Kremer (1993) or Jones (2008).

The rest of this paper proceeds as follows. Section 2 reviews the empirical literature on the role of management in two broad classes of models. Section 3 sets up the benchmark model. I describe how latent firm and manager attributes are inferred from observable payment data and I define the undistorted competitive equilibrium. In section 4, I describe the aggregate and firm-level data and estimate relevant parameters. The model is calibrated to fit U.S. manufacturing data in section 5. Matching frictions
and the associated allocative distortions are discussed in section 6. Section 7 summarizes and concludes.

2 The Importance of Management: Micro vs. Macro Evidence

There is a vast literature in corporate finance – among others – on the contribution of management to efficiency at the firm level. Many of these studies conclude that managerial competence is an important ingredient for productivity and profitability.

One body of evidence suggests that the replacement of incompetent management is a key factor in productivity gains after privatizations in developing countries. La Porta and Lopez-De-Silanes (1999) find that bringing in fresh managers was key to productivity gains after the privatization of state-owned Mexican enterprises in the 1980s and 90s. Real sales per worker increased, on average, by 100% over this period. Similarly, Garcia et al. (2001) and Cole et al. (2005) report a fourfold increase in labor productivity growth in Chilean copper mines following the denationalization in 1990. Growth in real output per worker jumped from 3.5% (1970-1990) to 14% (1990-2000). Cole et al. (2005) argue that some, though not all, of the gains were realized thanks to improved expertise in both new and incumbent mines.

In a study of the Korean automotive industry, the McKinsey Global Institute found that total factor and labor productivity reached only 48% of U.S. levels in 1995.3 The study argued that two of the most important sources of inefficiency were (1) management’s inability to implement lean production and (2) the adoption of needlessly complex manufacturing processes. The latter reflected management’s inability to take manufacturing and assembly aspects into account at the vehicle design stage and was often mentioned as a source of low-quality output and costly rework.4

There is also a large corporate finance literature on the economic role of management. In an innovative empirical study, Bennedsen et al. (2007) doc-

3MGI (1998)

4A more comprehensive and detailed account of South Korea’s development experience – and the government’s heavy hand in the process – is available from the author upon request.
ument the performance effects of exogenous distractions of a firm’s CEO. Based on data for Danish limited liability firms, they find that the death of a CEO or a close family member is associated with significant declines in profitability as well as sales and investment growth. The death of a CEO causes, on average, a 1.7 percent point decline in operating returns on assets; the death of a family member triggers a 0.7 point decline. All events combined are associated with a 0.9 percentage point or 11% decline in operating returns. Moreover, the results suggest that managerial input is a particularly important aspect of performance in skill and capital-intensive firms.\textsuperscript{5}

Bebchuk and Cohen (2005) find that arrangements, which protect incumbent management from removal are associated with an economically significant reduction in firm value. Tobin’s Q is, on average, 17 points lower for firms with so-called staggered boards in a panel of U.S. public companies in 1995-2002. These results suggest that replacing “entrenched” management yields economically sizeable gains in firm value.

In contrast, recent macro studies on CEO compensation find a very limited role for management. Gabaix and Landier (2008) and Terviö (2008), for instance, find that managerial talent is very narrowly dispersed and argue that replacing CEOs of large US corporations by an arbitrarily chosen colleague hardly affects the market value of those firms. In this class of models, mismatch between firm quality and managerial ability is a trifling source of efficiency losses.

One contribution of this paper is to show how assumptions about the degree of complementarity drive the results found in Gabaix and Landier (2008) and Terviö (2008). A key assumption in these models is that the elasticity of substitution between the attributes of the firm (which I will call firm quality) and those of senior management (ability) is set to unity. Neither paper can identify the dispersion of firm and managerial attributes separately from technological parameters. Hence, the claim that CEOs are very similar to one another cannot be substantiated. In fact, a model with more complementarity between quality and ability undermines the finding

\textsuperscript{5}The effect of shocks is, in fact, concentrated in industries with high R&D expenditures, high investment rates and those exhibiting fast growth. This suggests that the degree of complementarity between the firm and senior management may vary across industries.
that variations in managerial input are economically unimportant.\textsuperscript{6} I show that with more complementarity, even small deviations from efficient allocations have a sizeable impact on output and aggregate TFP. Relaxing the unit elasticity assumption in my model is what allows me to reconcile the empirical results from the micro studies with those in Gabaix and Landier (2008) and Terviö (2008).

3 The Economic Environment

In recent work, Hsieh and Klenow (2007a) and Restuccia and Rogerson (2008) suggested that idiosyncratic price distortions – due to taxes or subsidies levied at the level of the firm – generate variations in the marginal product of variable inputs across firms. Here, I formalize the idea that two fixed factors, namely management ability and firm quality, need to be combined with capital and labor to produce output. In this model, mismatch of firm and management attributes is another type of allocative distortion.

I assume, for the time being, that there are no taxes and subsidies in the markets for labor and capital and hence rule out productivity losses from input price distortions. Instead, I focus on the assignment of managers with ability $a$ to firms with quality $q$ and I distinguish efficient from inefficient matches. Later, I will assess the aggregate productivity effect of mismatches with various degrees of severity to ascertain whether “crony capitalism” thusly defined can provide an explanation for observed per capita income and productivity differences.

3.1 Population and Firms

The model is populated by identical (family) households of measure one and each household has a measure $N$ of members.\textsuperscript{7} Each member is endowed with a single unit of labor and management ability $a$. Ability is distributed with c.d.f. $F_a(\cdot)$. Each household owns a measure $N$ of firms with quality $q$. Quality follows a distribution with c.d.f. $F_q(\cdot)$.

It is important to keep in mind that households are identical to one another in that every single one of them is endowed with a full support of

\textsuperscript{6}Moreover, the limiting Cobb-Douglas specification of my model exhibits considerably more heterogeneity in managerial talent compared to previous models.

\textsuperscript{7}For reasons that will become apparent shortly, I need at least four households.
management abilities and project qualities, in addition to a single unit of labor per period and household member.

Members of each household make an occupational choice between supplying a single unit of labor in their capacity as workers or to run a firm in their capacity as managers. If they choose the latter, a manager with ability $a$ is paired with a firm of quality $q$. The manager and firm characteristics are aggregated and jointly determine the span of control over capital and labor inputs, similar to Lucas (1978). I will describe the occupational choice in more detail in section 3.5.

All four inputs – capital, labor, quality, and ability – are traded in factor markets. As is standard in the literature, no transactions are carried out within the household, i.e. the factor inputs in each firm are supplied by (four) distinct households.

### 3.2 Benchmark Model

#### 3.2.1 Preferences

Households do not value leisure and order their preferences over consumption streams $\{C_t\}$ of the final good by:

$$\sum_{t=0}^{\infty} \beta^t N_t U \left( \frac{C_t}{N_t} \right)$$

where $U(\cdot)$ satisfies $U' > 0$, $U'' < 0$ and the Inada conditions $U'(0) = \infty$ and $U'(\infty) = 0$.\footnote{Since I am only interested in the steady state of the economy, I need not make any further assumptions about $U(\cdot)$ at this time.} Henceforth, let $c_t = \frac{C_t}{N_t}$.

At every point in time, each of the $N_t$ members of the household consumes an equal share of the household’s aggregate consumption bundle $C_t$. The household as a whole sums the valuations of each of its members.

#### 3.2.2 Technology

Firms produce the final good by combining the fixed factors managerial ability ($a$) and firm quality ($q$) with the variable inputs labor ($l$) and capital ($k$). The production technology exhibits decreasing returns to scale with respect to the variable inputs, both individually and jointly (i.e. $\alpha, \gamma \in (0, 1)$).
Firm-$(a,q)$ output is
\[ y = h(a, q)^{1-\gamma} (k^\alpha t^{1-\alpha})^\gamma \] (1)

where $h(a, q) = \psi[\nu_a a^\rho + \nu_q q^\rho]^{\frac{1}{\rho}}$ and $\psi$ is a normalizing constant (see section 3.4.2).

The elasticity of substitution between $a$ and $q$ is denoted by $\sigma = \frac{1}{1-\rho}$. The owners of the two fixed factors are joint residual claimants and factor payments always exhaust the (value of) output.\(^9\)

### 3.3 Household’s Problem

Members of a household are infinitely lived and maximize their lifetime utility subject to a budget constraint and the law of motion for capital. Assume that the members are rank-ordered by their management ability and indexed by $i$ (high $i$ is associated with high ability). Firm qualities are indexed analogously.

\[ \max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t N_t U(c_t) \] (2)

subject to

\[ rk_t + \int_{0}^{1} (\omega[i] + \pi_t[i]) di + \kappa w \geq c_t + x_t \] (3)

\[ x_t = k_{t+1}(1 + \eta) - (1 - \delta)k_t \] (4)

where $k_t = \frac{K_t}{N_t}$ is the capital stock per household member, $x_t = \frac{X_t}{N_t}$ denotes investment per capita, and $\eta = \frac{N_{t+1} - N_t}{N_t}$ is the growth rate of household size. $\kappa$ denotes the endogenous measure of household members who supply one unit of labor inelastically. For simplicity, I assume $\eta = 0$ unless noted otherwise.

Members who supply one unit of labor inelastically are paid the competitive wage rate $w$. Those who work as managers earn flow compensation $\omega[i]$ in equilibrium. Only a subset of household members will chose the latter occupation. In the budget constraint, this implies that $\omega[i] = 0$ for some

\(^9\)In this model, factor payments exhaust output because $a$ and $q$ are residual claimants. In fact, the aggregator $h(\cdot, \cdot)$ need not exhibit constant returns to scale for the production technology to be homogeneous of degree one.
i. Firm qualities that are matched with a manager are paid $\pi_i$; idle firms earn no quasi-rents (that is, $\pi_i = 0$ for some $i$). I describe the determination of the managers’ compensation in detail in section 3.4.2.

3.4 Firm’s Problem

The owners of a project face two distinct, yet simultaneous, problems in their production decision:

1. They must hire a manager and pay her the equilibrium wage, which, of course, depends on her type (ability).

2. They hire labor and capital in factor markets, with competitive prices $w$ and $r$, respectively.

Jointly, a project’s quality, say $q'$, and its manager’s ability, say $a'$, determine the span of control over variable inputs. In particular, the span of control is characterized by $h(a', q')$. A firm, in this context, consist of a project-manager pair.

For a given ability-quality match, the solution to an individual firm’s problem with respect to the variable inputs labor and capital depends not only on its own quality and that of its manager, but also on the distribution of all other ability-quality pairs, taking into account the occupational choices of households, and the prices of capital and labor.

Assume, for one moment, that all projects have been matched with a manager. I can then order the pairs by the match surplus $f(a, q)$ they generate and index them by $i$. Then $f(a[0], q[0])$ denotes the match with the smallest surplus of all possible pairs; $f(a[1], q[1])$, on the other hand, identifies the most productive match. In terms of notation, the reader may want to think of $a[i]$ as the ability of the manager matched with a project that generates the $i$-ranked surplus. Analogously, $q[i]$ is the project quality associated with the $i$-ranked pair. Since I restrict $F_a$ and $F_q$ to continuous distributions, the probability that $f(a[i], q[i]) = f(a[j], q[j])$, for $i \neq j$ equals zero. Hence, there is no need to consider the possibility of ties.

In what follows, I shall describe these two problems more formally.
3.4.1 Variable Inputs

In this basic model, we ignore the possibility of incentive problems between managers and owners of firm quality. Instead, they agree to maximize the joint quasi-rent of the firm. How these rents are apportioned to the manager and the owner is determined by the solution to the assignment problem, which I will describe shortly.

Firm-\([i]\) hires capital and labor inputs that satisfy:

\[
\Pi[i] = \max_{k,l} \left( \psi f(a[i], q[i]) \right)^{1-\gamma} g(k, l)^\gamma - rk - wl
\]

where

\[
f(a, q) = (\nu_a a^\rho + \nu_q q^\rho)^{\frac{1}{\rho}}
\]

\[
g(k, l) = k^\alpha l^{1-\alpha}
\]

Moreover, \(\alpha, \gamma \in (0, 1), k, l \geq 0, \) and \(\psi \in \mathbb{R}^{++}\). The technology need not exhibit constant returns to scale in all four inputs. In particular, since \(a\) and \(q\) are residual claimants, the payments to production factors always exhaust (the value of) output exactly, regardless of the values for \(\nu_a\) and \(\nu_q\).

The first-order conditions of this concave problem are:\(^{10}\)

\[
(\psi f(a[i], q[i]))^{1-\gamma} \gamma (k^\alpha l^{1-\alpha})^{\gamma-1} \alpha \left( \frac{k}{l} \right)^{\alpha-1} = r \quad (5)
\]

\[
(\psi f(a[i], q[i]))^{1-\gamma} \gamma (k^\alpha l^{1-\alpha})^{\gamma-1} (1-\alpha) \left( \frac{k}{l} \right)^{\alpha} = w \quad (6)
\]

Dividing (5) by (6) one can see immediately that the capital-labor ratio is equalized across firms. Firms run by different ability-quality pairs simply differ in the scale of operation, but not in the factor intensity of production.

Next, let me turn to the assignment problem of fixed inputs.

3.4.2 Fixed Factors

If the scalar \(\psi\) satisfies the condition

\[
\psi = \left( \frac{w}{\gamma} \right)^{\frac{\gamma}{1-\gamma}} \left( \frac{1-\alpha}{\alpha} \right) \left( \frac{r}{w} \right)^{\frac{\alpha \gamma}{1-\gamma}} \left( \frac{1}{1-\alpha} \right)^{\gamma}\left( \frac{1}{1-\gamma} \right)\frac{1}{1-\gamma}
\]

\(^{10}\)For a given cutoff, say \(i\), the problem is indeed concave for all \(i \geq i\).
then the (net) surplus generated by $f(a, q)$ – and split between managers and owners of firm quality – is consistent with the gross surplus net of payments to the variable inputs labor and capital. That is,

$$f(a[i], q[i]) = (\psi f(a[i], q[i]))^{1-\gamma} g(k, l)^{\gamma} - rk - wl$$

It follows that I can indeed “partial out” the assignment problem from the firms’ hiring decisions for capital and labor.

Since firms hire managers at the extensive margin – and managers choose firms in the same way – this problem does not have necessary first-order conditions. Instead, the hiring decision satisfies feasibility, sorting, and participation constraints.\textsuperscript{11}

Feasibility requires that:

$$\Pi[i] \geq \pi[i] + \omega[i]$$  \hspace{1cm} (7)

The assignment problem’s sorting constraints are:

$$f(a[i], q[i]) - \omega[i] \geq f(a[j], q[i]) - \omega[j]$$

$$f(a[i], q[i]) - \pi[i] \geq f(a[i], q[j]) - \pi[j]$$  \hspace{1cm} (8)

The first line of (8) states that the owner of firm quality $a[i]$ prefers to be matched with the manager that maximizes his quasi-rent, taking into account her equilibrium compensation $\omega[i]$, rather than anyone else. Similarly, according to the second line, a manager prefers to be matched with the project that maximizes her compensation, taking into account the equilibrium profile of quasi-rents paid to the owners of firm quality.

Finally, the participation constraints are:

$$\omega[i] \geq w \quad \forall i \in [\kappa, 1]$$

$$\pi[i] \geq \pi \quad \forall i \in [\kappa, 1]$$  \hspace{1cm} (9)

where $\kappa$ denotes the value of the index $i$ that satisfies the participation constraint exactly.

For now, I assume that the outside options of firms and managers are identical for all types. As long as the outside option does not increase “too fast”

\textsuperscript{11}This assignment structure follows Terviö (2008).
with ability (quality), the constraint only binds for the lowest types.\footnote{The slope of the outside payment profile plays a more subtle role in a model with matching frictions. In particular, if the matching is not one-to-one, then the constraint is binding for some managers but not others of the same type. This further exacerbates the efficiency losses from non-assortative matching.}

Since \( f(a, q) \) is twice differentiable, \( \frac{\partial^2 f(a, q)}{\partial a \partial q} > 0 \) is a sufficient condition for positive assortative matching of \( a \) and \( q \) (see, among others, Becker, 1973). Under this condition, I can state the following lemma:

**Lemma 1** For given factor prices \( w \) and \( r \), if \( \frac{\partial^2 f(a, q)}{\partial a \partial q} > 0 \), and if \( a \) and \( q \) are distributed continuously, then the profiles of managerial compensation \( \omega[i] \) and payments to quality \( \pi[i] \) are generated by the following system of equations:

\[
\begin{align*}
\pi'[i] &= f_q(a[i], q[i]) q'[i] \\
\omega'[i] &= f_a(a[i], q[i]) a'[i]
\end{align*}
\]

with initial value condition

\[
f(a[\kappa], q[\kappa]) = w
\]

Equations (10) and (11) are derived from the two sorting constraints in (8). Substitute \( j = i - \epsilon \) in equation (8), rearrange, and divide both sides of the inequality by \( \epsilon \) to obtain:

\[
\frac{f(a[i], q[i]) - f(a[i - \epsilon], q[i])}{\epsilon} \geq \frac{\omega[i] - \omega[i - \epsilon]}{\epsilon}
\]

Taking the limit \( \epsilon \to 0 \) yields equation (11). Proceeding analogously for the manager’s sorting constraint, that is, the second line in (8), yields equation (10).

Integrating over active managers, the payment profiles for managers and firms are, respectively:

\[
\begin{align*}
\omega[i] &= w + \int_{\kappa}^{i} f_a(a[j], q[j]) a'[j] \, dj \\
\pi[i] &= \pi + \int_{\kappa}^{i} f_q(a[j], q[j]) q'[j] \, dj
\end{align*}
\]

The assumption of continuously distributed managerial ability and firm quality simplifies the analysis. Differential rent problems such as this one
satisfy the no-surplus condition spelled out in Ostroy (1980, 1984). This satisfies an alternative definition of a competitive equilibrium and eliminates the need to establish a bargaining protocol between firm owners and managers.\textsuperscript{13}

Taking the payment profiles $\omega[i]$ and $\pi[i]$ as given (i.e. observable), equations (10) and (11) form a system of two ordinary differential equations in $a[i]$ and $q[i]$, with initial value condition (12).

Positive assortative matching allows me to reinterpret $a[i]$ and $q[i]$ as rank-ordered managers and projects. Moreover,

\begin{align*}
a[i] &= F^{-1}_a(i) \\
q[i] &= F^{-1}_q(i)
\end{align*}

where $F_a$ and $F_q$ denote the cumulative distribution functions of $a$ and $q$.

### 3.5 Equilibrium

An equilibrium in this economy is defined as:

**Definition 1** An equilibrium consists of prices $\omega[i]$, $\pi[i]$, $r$, $w$, an occupational rank cutoff $\kappa$ (and hence a measure of active firms $1 - \kappa$), per capita income $c$, factor inputs $k[i]$ and $l[i]$ such that, for given prices:

1. Household members maximize utility subject to the budget constraint.
2. Firms maximize payments to fixed factors subject to the technology constraints.
3. The labor market clears:

$$\kappa = \int_{\kappa}^{1} l[j]dj$$

where the left hand side is the measure of labor supplied by households, namely all those who prefer not to take on management responsibility.
4. The capital market clears:

$$K = \int_{\kappa}^{1} k[j]dj,$$

\textsuperscript{13}See a similar discussion in Terviö (2008). The standard reference for differential rent models is Sattinger (1979, 1993).
where $K$ denotes the aggregate capital holdings of households.\textsuperscript{14}

5. Managers prefer their assigned firm to any other assignment at the equilibrium wage profile $\omega[\cdot]$.\textsuperscript{15}

6. Agents weakly prefer their occupational choice between being a worker and being a manager. The lowest ranked manager (of rank $\kappa$) is indifferent between the two occupations in equilibrium; higher-ranked managers strictly prefer their job to being a worker. Formally, the marginal manager is paid:

$$
\omega[\kappa] = h(a[\kappa], q[\kappa])^{1-\gamma} g(k(a[j], q[j]), l(a[j], q[j]))^\gamma \\
- rk(a[j], q[j]) - wl(a[j], q[j]) - \pi[\kappa] \\
= w
$$

7. Firms owners prefer their assigned manager to any other at the equilibrium profit profile $\pi[\cdot]$.

8. The owner of the marginal firm quality is paid the outside option:

$$
\pi[\kappa] = h(a[\kappa], q[\kappa])^{1-\gamma} g(k(a[j], q[j]), l(a[j], q[j]))^\gamma \\
- rk(a[j], q[j]) - wl(a[j], q[j]) - \omega[\kappa] \\
= \pi
$$

Having defined the decentralized equilibrium, I am now in a position to take the model to the data.

\section{Data and Estimation}

\subsection{Preliminaries}

In the model, firm size is determined endogenously by aggregating firm and management attributes through the function $f(\cdot, \cdot)$. The fundamental arguments of the function, however, are unobservable. The firm-level data in \textit{CompuStat North America} contains information on payments to senior managers and to the owners of firm quality and the assignment framework from section 3.4.2 can then be deployed to infer the underlying distribution(s) of attributes.

\textsuperscript{14}As long as we assume that there is a unit measure of households, the aggregate and average capital holdings are identical.

\textsuperscript{15}Recall that managers and owners are residual claimants. They split the surplus between $\omega[\cdot]$ and $\pi[\cdot]$ as specified in section 3.4.2.
In the special case of log-additive aggregation, the solution to the system of differential equations (11) and (10) is a pair of closed form expressions that characterize the profile of talent and quality exceedences, $\frac{a[i]}{a[\kappa]}$ and $\frac{q[i]}{q[\kappa]}$ respectively. For a more general CES aggregator with substitution elasticity different from one, however, I cannot characterize the distribution of $a$ and $q$ without identifying the cutoffs $a[\kappa]$ and $q[\kappa]$ at the same time. That is, I cannot solve the system of differential equations based on the payment profiles $\omega[i]$ and $\pi[i]$ alone. I need additional structure.

The span-of-control model with endogenous occupational choice from section 3 provides such a framework. Namely, I can reverse the solution algorithm used in Terviö (2008). I use the fully specified general equilibrium model to inform the parameters of the talent and quality distributions. To carry out this exercise, I need to identify empirical targets with counterparts that the model can be calibrated to.

4.2 Data

I use a comprehensive data set containing the largest manufacturing firms by market capitalization in the United States. I start with the three thousand largest firms listed in the Russell 3000® index, year-by-year. I extract corporate data from CompuStat North America database for all those index members that are listed in the database. I then further restrict the sample to firms that are identified with two-digit NAICS codes 31-33, that is, corporations whose main line of business is identified as manufacturing.

I also collect data on executive compensation form the CompuStat ExecuComp database. Coverage of executive compensation is somewhat limited, especially in the early 1990s. For that reason, the sample of firms with sufficient corporate and compensation data shrinks yet more. ExecuComp reports compensation details for as many as 13 corporate officers. 89% of firms report on at least five officers and 96% report on their top four. I retain the five most highly compensated officers in all firms. In the calibration(s) I use the $J$ top-ranked executives, with $J \in \{1, 2, 3, 4, 5\}$.

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16 See Appendix A for the derivation and details.
17 I follow standard practice in excluding financial firms and utilities. The former are excluded as they are seen as fundamentally different from corporations in the real economy. The latter are – or used to be – regulated. Although manufacturing firms produce innumerable differentiated goods, they are sufficiently homogeneous for the present argument.
In the data set, I identify all relevant payment streams and asset transfers from the firm to its executives and its securities holders. From the firms’ perspective, payments to firm quality and management ability can be categorized into current or deferred payments. Cash dividends, for instance, are current payments to the firms’ securities holders. Salaries, bonuses, company match to 401(k) contributions, or private use of corporate vehicles are all examples of current executive compensation. Deferred payments to the firms’ owners are capital gains. Managers receive deferred compensation for current input in the form of stock options or stock grants. They are deferred in the sense that firms do not incur the full resource cost at the time the payments are granted.

In equation (1), as long as $\gamma < 1$ and since $a$ and $q$ are (joint) residual claimants, payments to the four factor inputs always exhaust the (value of) output. Compensation for current managerial input by means of stock grants or options, for example, must be recorded at the contemporaneous resource cost. Simply recording those transfers at their (fair) asset value would contaminate the true contribution of $a$ and $q$ to current output using the “attribution mechanism” in 3.4.2. The next section details the derivation of the periodic resource costs of observable current and deferred payment streams.

### 4.3 Computation of Flow Payments

Let $\Pi_i$ denote the firm’s profit after all factors of production have received their flow compensation, but prior to the distribution of financial assets.

$$\Pi[i]_t = y[i]_t - r_t k[i]_t - w_t l[i]_t - \omega[i]_{F,t} - \pi[i]_{F,t}$$

where it is understood that firm $i$ is of quality $q[i]$ and run by a manager of ability $a[i]$. $y[i]$ is firm $i$'s output net of investment and $d[i]$ denotes dividends paid to the owners of the physical capital stock. $\omega[i]_{F,t}$ and $\pi[i]_{F,t}$ denote current payments – $F$ for “flow” – to managers and owners of firm quality, respectively. They consist of dividends paid to owners of firm quality, on the one hand, and salaries, bonuses, etc. to managers, on the other.

---

18Terviö (2008) approaches the problem from a different angle and capitalizes all flow payments. His approach simplifies the calculation of payments to the owners of firm quality somewhat. On the other hand, I can avoid making assumptions about future flow payments to the firms’ top managers. My problem is thus essentially a static problem and I can focus on the manager’s contemporaneous contribution to output.
hand.

Let $V[.]^*$ denote the *ex dividend* market value of the firm and recall that

$$
V[i]_t^* = k[i]_t^* + \sum_{s=t}^{T} (1 + r_s)^{-s}\Pi[i]_s
$$

$$
= k[i]_t^* + V[i]_t,
$$

(15)

where $k[i]_t^*$ denotes the optimal size of firm $i$'s capital stock. Owners and managers claim shares of $V[i]_t$ and receive them as stock options, stock grants, or in the form of capital gains.\(^{19}\) The split reflects their respective contributions to current output and is determined by equations (10) and (11). That is,

$$
V[i]_t = \omega[i]_{V,t} + \pi[i]_{V,t},
$$

where $\omega[i]_{V,t}$ and $\pi[i]_{V,t}$ denote the value of these ownership claims. The flow cost to the firm of compensating $a[i]$ and $q[i]$ with such claims at the end of the period is simply the amortization payment $\omega[i]_{A,t}$ and $\pi[i]_{A,t}$ of an annuity with present value $\omega[i]_{V,t}$ and $\pi[i]_{V,t}$, respectively:

$$
\omega[i]_{V,t} = \sum_{s=t}^{\infty} (1 + r_s)^{-s}\omega[i]_{A,s}
$$

(16)

$$
\pi[i]_{V,t} = V[i]_t - \omega[i]_{V,t} = \sum_{s=t}^{\infty} (1 + r_s)^{-s}\pi[i]_{A,s}
$$

(17)

Assuming a constant net real interest rate known at time $t$, denoted by $r_t$,

it follows that:

$$
\omega[i]_{A,t} = r_t(1 - (1 + r_t)^{-T})^{-1}\omega[i]_{V,t}
$$

(18)

$$
\pi[i]_{A,t} = r_t(1 - (1 + r_t)^{-T})^{-1}\pi[i]_{V,t}
$$

(19)

The total flow payments to $a[i]$ and $q[i]$ can then be computed as:

$$
\omega[i]_t = \omega[i]_{F,t} + \omega[i]_{A,t}
$$

$$
\pi[i]_t = \pi[i]_{F,t} + \pi[i]_{A,t}
$$

\(^{19}\)Current profits are distributed to owners by means of dividends and to managers through bonuses. Since these are current payments, we need not worry about them here.
I add the flow compensation for the $J \in R^{++}$ most senior executives.\textsuperscript{20} The sum compensates the collective ability input provided by these corporate officers. However, I remain agnostic about the functional form of the aggregation of managerial talent and I do not attempt to identify the individual contributions. Formally,

$$a[i] = \Gamma(a[i]_{\text{rank 1}}, \ldots, a[i]_{\text{rank } J})$$

where $\Gamma$ is an unknown function. $\omega[i]$ is the compensation paid to $a[i]$.

Garicano and Rossi-Hansberg (2007) assume a hierarchical management structure where, in equilibrium, more able individuals are closer to the top of the management pyramid and different strata interact in well-defined ways. Moreover, higher quality firms will feature "higher" management pyramids with a "wider" base. Here I abstract from a richer interpretation of management teams to keep the model tractable. Moreover, in the special case with $J = 1$, it is irrelevant that smaller firms tend to report compensation on fewer of their top executives, on average.\textsuperscript{21}

As far as payments to the owners of firm quality are concerned, they can be defined as total payments to the firms’ securities holders net of payments to the owners of the physical capital stock. This is formally captured by equations (15), (17), and (19). For lack of a better alternative I use the book value in corporate reports as a proxy for the physical capital stock, $k[i]^*$. Appendix B contains a detailed description of the individual components in the managers’ compensation.

In the Black-Scholes value calculations ExecuComp assumes that stock options have an average maturity of seven years. We therefore assume $T = 7$ in equation (16). In equation (17), on the other hand, we assume $T \to \infty$ since shares of company stock do not have a well-defined maturity.

\textsuperscript{20}In the special case where I only consider CEOs, $J = 1$.

\textsuperscript{21}When $J \in \{2, 3, 4, 5\}$, my compensation measure captures the reduced managerial input both in terms of less competent executives at the very top of the executive pyramid and in terms of "flatter" executive hierarchies.
4.4 Empirical Evidence for Assortative Matching

Under the assumption that firms produce a single homogeneous final good and provided the technology exhibits sufficient complementarity between the firm and management attributes, the Pareto-efficient allocation exhibits positive assortative matching between ability and quality.\textsuperscript{22} In our model, equations (11) - (14) suggest this is equivalent to assortative matching between (flow) payments to ability and quality.

In US manufacturing data, the year-by-year Spearman rank correlation between total flow payments to the CEO and payments to firm quality fluctuates between 0.61 (in 1999) and 0.75 (in 2007) for those firms with positive payments to quality. If I include the five most senior executives (in terms of compensation), the rank correlation is between 0.67 (1999) and 0.80 (2007).

These correlations are close to those reported in Terviö (2008). On average, I find a slightly tighter relationship and I thereby further corroborate the empirical evidence in favor of assortative matching in the US.\textsuperscript{23}

One possible reason for deviations from perfect assortative matching (PAM) can be measurement error. However, I am reluctant to attribute the full departure to measurement error alone. For one, manufacturing does not produce a single homogeneous good and variations in relative demand for these differentiated goods can be a source for less-than-perfect rank correlations. Moreover, the model is stylized and does not claim to capture the many factors on which top management’s impact on firm productivity depends. Bertrand and Schoar (2003), for instance, describe the effects of different management styles on firm performance. Similarly, Bloom and Van Reenen (2007); Bloom et al. (2007) document multiple dimensions of management ability that affect profitability, productivity, and survival rates.

\textsuperscript{22} As long as the substitution elasticity is less than zero (i.e. more complementarity than linear aggregation) the efficient allocation exhibits PAM.

\textsuperscript{23} In his paper, Terviö capitalizes payments to the firm and payments to the CEO are taken directly from the ExecuComp database (variable tdc1). This variable lumps together current (cash or in kind) compensation and asset transfers at fair value. Here, I attempt to construct a more consistent measure of executive compensation. In Terviö (2008), the reported rank correlations are between the market value of the firm and tdc1.
4.5 Small and Medium-Sized Firms: Inference Problem

The sample of firms described in previous sections does not, of course, include small or medium-sized firms. The US manufacturing sector, however, consists of a large number of such firms. To fix ideas about the relative importance of small, medium, and large firms in US manufacturing, it is worth looking at US Census data. To illustrate the point, I use data for 2000.

According to the US Census Bureau there were almost five million firms (and more than 6 million establishments) with at least one employee in the United States in 2000. Of those, 283,602 enterprises (and more than 330,000 establishments) were classified as manufacturing firms.

Unfortunately, there is no firm-level data that would allow me to identify payments to management ability and firm quality in smaller firms. Instead, one has to assume that the data generating process for $a$ and $q$ is the same for all manufacturing firms and that the “observed” $(a, q)$ pairs for large firms are the right tails of that unified process. The challenge at hand, then, is to characterize the distribution of large firms as sharply as possible and to identify a distribution whose tails fit that characterization tightly.

The model in this paper provides sufficient structure to make plausible inferences about the distribution of firms and managers across the full size spectrum of manufacturing firms. The task is disciplined (1) by the observable firm size distribution in the U.S. and (2) by the relationship between payments and unobserved characteristics in the right tail of that distribution. In section 4.6, I identify features of this relationship that can then be used as calibration targets in section 5.

---

24 Firms and establishments are reported as having no employees if they have no one on payroll during the mid-March pay period, but with employees on payroll for at least one other pay period during the entire 2000 calendar year. Firms in this category can be seasonally operational firms, firms that ceased to exist after January 1, 2000 but prior to the pay period including mid-March, 2000, or new firms that started operating after the pay period including mid-March, 2000. In 2000, 726,862 firms and 767,912 establishments were reported as having no employees.

25 I do not, of course, observe $a$ and $q$. Instead, they are inferred from the differential equations (11) and (10).
4.6 Estimation

One of the “best documented empirical regularit[ies] regarding levels of executive compensation” is what Gabaix and Landier (2008) call “Robert’s Law”, according to which executive compensation is proportional to (firm size)$^\tau$.\textsuperscript{26}

In Table 1, I report the coefficients of the following regression:

$$\log(\omega[i]) = \alpha + \tau \log(\text{firm size}) + \epsilon$$

(20)

I run the regression with two different proxies for firm size: (1) market capitalization and (2) the number of employees. Each regression is run twice, with and without industry fixed effects. For brevity, I only report the estimates for $\tau$. The coefficients confirm “Robert’s Law” and in the calibration I will target a value of 0.3.

For theoretical reasons that will become apparent in section 5, I need one additional target. The distribution of firm and management attributes completely determines the size distribution of firms in the model. The size distribution, together with the remaining primitives pins down the competitive equilibrium. To match all the targets, it is thus imperative that I pick distributional parameters for $a$ and $q$ that satisfy “Robert’s Law” as well as the size distribution of firms.

Figure 1 illustrates the distribution of U.S. manufacturing firms in 2000. In addition to the size bins (red stairs), the figure shows the maximum likelihood fit for the log-normal distribution, since this is the parametrization I will be working with in the calibration. Visual inspection reveals that it underestimates the measure of very large firms and I thus err on the side of caution.\textsuperscript{27} In the calibration, I choose the 98.5\textsuperscript{th} percentile of U.S. manufacturing firms as the empirical target. Below this cutoff all firms have 499 or fewer employees. This guarantees that I have a sufficient measure of large firms in the model.

Arguably, one of the key parameters in the model is the elasticity of substitution ($\sigma$) between firm quality and managerial competence. The model

\textsuperscript{26}The original quote is from Baker et al. (1988).

\textsuperscript{27}I err on the side of caution since in span-of-control style models all the “action” in terms of output and productivity is driven by the tails of the distribution.
<table>
<thead>
<tr>
<th>Panel A: CEOs</th>
<th>log(executive compensation)</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(market cap)</td>
<td></td>
<td>.345</td>
<td>.344</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.008)</td>
<td>(.010)</td>
<td>(.016)</td>
<td>(.018)</td>
</tr>
<tr>
<td>log(# of employees)</td>
<td></td>
<td>.313</td>
<td>.322</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.006)</td>
<td>(.006)</td>
<td>(.012)</td>
<td>(.013)</td>
</tr>
<tr>
<td>Industry Fixed Effects</td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>6619</td>
<td>6619</td>
<td>6594</td>
<td>6594</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>.31</td>
<td>.33</td>
<td>.25</td>
<td>.28</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Top-Five Executives</th>
<th>log(executive compensation)</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(market cap)</td>
<td></td>
<td>.340</td>
<td>.341</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.005)</td>
<td>(.006)</td>
<td>(.006)</td>
<td>(.007)</td>
</tr>
<tr>
<td>log(# of employees)</td>
<td></td>
<td>.295</td>
<td>.313</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.004)</td>
<td>(.002)</td>
<td>(.009)</td>
<td>(.009)</td>
</tr>
<tr>
<td>Industry Fixed Effects</td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>6640</td>
<td>6640</td>
<td>6615</td>
<td>6615</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>.59</td>
<td>.61</td>
<td>.44</td>
<td>.49</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Panel Regression: Executive Compensation and Firm Size

**Explanation:** The panel covers manufacturing firms listed in the Russell 3000 index between 1994 and 2007. Only firms with sufficient data on executive compensation are included. The first row of standard errors (in parentheses) is clustered by year. The second row of standard errors (in parentheses) is clustered at the firm level. The industry fixed effects are based on 3-digit NAICS codes.
itself does not provide enough structure to estimate the elasticity empirically. The reason is that any information about complementarity is contained in the cross-derivative of the aggregator $f(\cdot, \cdot)$, which plays no role in the determination of the competitive equilibrium. Appendix B outlines a promising estimation strategy based on a Simulated Method of Moments (SMM), which I invite the interested reader to take a look at.

5 Calibration

In this section, I calibrate the model to US manufacturing data. I treat the United States as an economy with no deviations from assortative matching. Some of the parameters have analogues in the growth model and I choose their values using standard procedures. Since the model features a non-degenerate firm-size distribution, I need to calibrate additional parameters for the model to match its targets.

I adopt standard values for the household’s discount factor $\beta$ and for the depreciation rate of capital $\delta$. Moreover, I assume that household size is
constant over time \((\eta = 0)\). Together, the three parameters determine the real interest rate in steady state. I assume that one model period corresponds to one year in the data. The share parameter \(\alpha\) is chosen to match the factor income shares in the US national income and product accounts. The parameter values are summarized in Table 2.

<table>
<thead>
<tr>
<th>Panel A: Standard Parameters</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.96</td>
<td>Real rate of return</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.15</td>
<td>Capital-output ratio</td>
<td></td>
</tr>
<tr>
<td>(\eta)</td>
<td>0</td>
<td>Constant household size</td>
<td></td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.85</td>
<td>Atkeson and Kehoe (2005)</td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.26</td>
<td>Income share of labor</td>
<td></td>
</tr>
<tr>
<td>(\pi[\kappa])</td>
<td>0</td>
<td>Equilibrium condition</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Distributional Parameters</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_q)</td>
<td>0</td>
<td>0</td>
<td>Normalization</td>
</tr>
<tr>
<td>(\mu_a)</td>
<td>0</td>
<td>–</td>
<td>Normalization</td>
</tr>
<tr>
<td>(\sigma_q)</td>
<td>6.986</td>
<td>3.5285</td>
<td>“Robert’s Law”</td>
</tr>
<tr>
<td>(\sigma_a)</td>
<td>0.054</td>
<td>3.5200</td>
<td>Compensation share</td>
</tr>
</tbody>
</table>

Table 2: Benchmark Calibration to US Data

The extent of decreasing returns is an important parameter in the model. Atkeson and Kehoe (2005) among others find a value for \(\gamma = 0.85\).

Project qualities \(q\) that are not deployed in production are traded at a price of zero. It is thus intuitive to normalize payments to the marginal quality: \(\pi[\kappa] = 0\). It is worth emphasizing here that the marginal manager’s flow compensation \(\omega[\kappa] = w\) cannot be normalized. It is determined endogenously.

The depreciation rate \(\delta\) is calibrated to match the average capital-output
ratio for 1998-2005 in US manufacturing. Based on the National Income and Product Accounts, the average ratio for the period is 1.29. Gordon (1971) argues that the national accounts underestimate the price of capital and hence the capital-output ratio. I therefore follow Atkeson and Kehoe (2005) and assume a capital-output ratio of 1.46. The depreciation rate that matches the capital-output target is 11%.\textsuperscript{28}

The capital share $\alpha$ is calibrated to match labor’s average income share in manufacturing between 1998 and 2005 (63.7%).

In section 4.6 I identified two additional calibration targets: (1) the elasticity of management’s compensation relative to the elasticity of firm size, which equals 0.3 for large firms in U.S. manufacturing and (2) the 98.4\textsuperscript{th} percentile of U.S. manufacturing firms in 2000, which corresponds to a firm with 499 employees.\textsuperscript{29}

For the purposes of calibration, I hypothesize that both managerial ability $a$ and firm quality $q$ follow log-normal distributions with parameters $(\mu_a, \sigma_a)$ and $(\mu_q, \sigma_q)$, respectively. Since $a$ and $q$ are combined using a CES aggregator, I can normalize $\mu_q = 0$ without loss of generality. Note also that the share parameters $\nu_a$ and $\nu_q$ are not separately identified from the means of $\log a$ and $\log q$ (that is, $\mu_a$ and $\mu_q$) respectively.\textsuperscript{30} For given $\nu_a$ and $\nu_q$, this leaves three parameters ($\sigma_a, \sigma_q$ and $\mu_a - \mu_q = \mu_a$) to meet my targets.

In the special unit elasticity case we can further normalize $\mu_a = 0$. With Cobb-Douglas aggregation, the means of the logarithms of $a$ and $q$ are factored out as multiplicative constants. This, in turn, implies that we can match two targets – the firm size distribution and the relative elasticity – using the variances $\sigma_a$ and $\sigma_q$.\textsuperscript{31} In this calibration, management’s payment share in large firms is 0.71%. I did not target the share but the calibration is fairly close to the empirical counterpart of 1%. I keep the share target for

\textsuperscript{28}The depreciation rate may seem high compared to the values used elsewhere, e.g. Restuccia and Rogerson (2008). However, it is key to remember that I am targeting the capital-output ratio net of residential structures rather than the aggregate ratio.

\textsuperscript{29}In the model, I target the 98.4\textsuperscript{th} percentile of the firms that are operating in equilibrium, which corresponds to approximately the 99.96\textsuperscript{th} percentile of all potential projects.

\textsuperscript{30}The combination of log-normal distributions for both fixed factors and CES aggregation allows for this normalization. One cannot, in general, adopt the same normalization for more general classes of aggregators.

\textsuperscript{31}In the unit elasticity case, the variances $\sigma_a$ and $\sigma_q$ are not identified separately from the Cobb-Douglas exponents $\nu_a$ and $\nu_q$, respectively.
the cases with non-unit elasticity, where we have one additional parameter $-\mu_a$ to meet it.$^{32}$

Remarkably, the relative dispersion of $a$ and $q$ is highly sensitive to the substitution elasticity $\frac{1}{1-\rho}$. Panel B of Table 2 reports the log-normal parameters for a Cobb-Douglas aggregator ($\rho = 0$) and a CES aggregator with more complementarity ($\rho = -1$). The dispersion of managerial talent in the former case is very small and firm size is mostly driven by variations in $q$. The latter case, on the other hand, suggests that both firm and management attributes play a more important role in the determination of size. Importantly, for both elasticity parameters we match the firm size, elasticity, and payment share targets exactly. Figures 2 and 3 illustrate the distributions of managerial ability and firm quality as well as the calibrated payment profiles.

With unit elasticity of substitution, the model confirms the conclusions of Terviö (2008) and Gabaix and Landier (2008) that there is little variation in managerial talent. In fact, my model extends their finding from the sample of large publicly-traded firms to the universe of all (manufacturing) firms. This, truth be told, is not particularly surprising. For commonly used distributions, the partial derivative of the inverse distribution function (denoted here by $a'[i]$ and $q'[i]$) is highest in the tails. So if one was to find dispersion anywhere on the rank interval $[0, 1]$ it would indeed be in the neighborhood of the upper bound. The talent spacing in lower percentiles is typically tighter. My model differs from earlier work precisely in that it has a “complete” universe of firms, rather than just the tails of the distribution.

The change in relative dispersion of firm and management attributes as a function of the substitution elasticity suggests that mismatch of these two fixed factors may indeed be a source of productivity differences. In section 6, I analyze the effect of different types of mismatch on output, productivity, and the shape of the firm size distribution.

$^{32}$An alternative strategy would be to normalize output to unity by moving $\mu_q = \mu_a$ in the Cobb-Douglas case and $\mu_q - \mu_a$ in the non-unit elasticity cases. This would simply scale the model up or down but would not affect the competitive equilibrium.
Figure 2: Payments and Fundamental Attributes ($\frac{1}{1-\rho} = 1$)

Figure 3: Payments and Fundamental Attributes ($\frac{1}{1-\rho} = \frac{1}{2}$)
6 Quantitative Analysis of “Crony Capitalism”

In this section, I consider two broad classes of matching frictions. The first category of policies locks the occupational choice of household members in the competitive equilibrium into place. Firms and managers are then matched non-assortatively in a subset of the firm and management distributions. The second class of frictions affects occupational choices in that some individuals who would be supplying labor inelastically in the benchmark model run firms instead, and vice versa. Analogously, firm qualities that would idle in the competitive benchmark may be put into operation while others may be forced into “hibernation”. The mismatch is across the complete range of abilities and qualities, and I use evidence on variations in the firm size distribution across countries to discipline the extent of “crony capitalism”.

6.1 Matching Frictions: Occupational Choice Fixed

First, I consider matching frictions between firm qualities and managers confined to certain percentile ranges of the firm size distribution. Rather than being matched assortatively, \((a, q)\)-pairs are formed randomly, provided they are “located” between the same two critical percentiles, say \(i\) and \(i\), of their respective quality and ability distributions. For instance, if \(i = \kappa\) and \(i = 1 + \frac{1}{4}\), then firm qualities and managers in the first quartile of their respective distributions are matched randomly. However, no manager in the first quartile is ever paired with a firm quality in the second, third, or fourth quartile.

Clearly, the severity of the matching friction is a function of the number, say \(I\), of percentile ranges, spaced evenly between \(\kappa\) and 1. In the limit, as \(I \to +\infty\), the assignment is positive assortative. When \(I = 1\), matching between all firm qualities and managers above the occupational cutoff rank \(\kappa\) is completely random.

With a continuum of qualities and abilities, matching frictions of this nature affect the distribution of firm sizes. In particular, the distribution is less dispersed, since non-assortative matching tends to “eliminate” both very small and very large firms. Note, however, that the size support within each percentile – and hence in the full distribution of firms – remains unchanged compared to the competitive benchmark.
Recall that under this policy experiment, I fix the occupational cutoff at the competitive level $\kappa$. The change in the distribution of ability-quality pairs due to the matching frictions affects the employment level and aggregate capital stock in the distorted allocation for given factor prices $r$ and $w$. Remember also I restrict myself to comparing steady-state allocations, which implies that the interest rate $r$ is pinned down by $\beta$ and the depreciation rate $\delta$. Therefore, restoring full employment requires that the real wage $w$ adjust. This, in turn, affects the capital-labor ratio, which is proportional to $\frac{w}{r}$.

To evaluate the effect of such matching frictions, I randomly pair $N$ evenly rank-spaced firm qualities with equally many evenly rank-spaced managers.\textsuperscript{33} I then approximate the distribution of firms by a piece-wise linear function with $N$ grid points and compute aggregate output, employment and the economy’s capital stock.\textsuperscript{34} To compute aggregate TFP, I divide output by a composite input, where the latter is a Cobb-Douglas aggregate of the capital stock and employment using their respective income shares $\alpha$ and $1 - \alpha$.\textsuperscript{35}

Table 3 reports the effects of matching frictions on output and TFP for $I = \{10, 4, 2, 1\}$ (in four separate panels) and three values of the substitution elasticity: 1, $\frac{1}{2}$, and $\frac{1}{20}$. The results confirm earlier findings that under the assumption of unit elasticity aggregation, mismatch of fixed factors is a negligible source of efficiency and output losses. With higher degrees of complementarity, on the other hand, the losses increase dramatically. In Panel A ($I = 10$), the output loss widens from .02% under Cobb-Douglas aggregation to 11% when the elasticity of substitution is $\frac{1}{20}$; the decline in TFP jumps from .01% to 7.5%.

Remarkably, even under more severe matching frictions, output and TFP decline minimally – less than .4% – if the elasticity of substitution is set to unity (first row in panels B, C, and D, respectively). On the other hand,

\textsuperscript{33}Consider an arbitrary percentile range with lower and upper rank bounds $i$ and $j$. Evenly rank-spaced firm qualities (managers) are separated by the rank distance $\frac{j - i}{N - 1}$ from their neighbors to the right and left.

\textsuperscript{34}The piece-wise linear approximation implies that quality and ability are distributed uniformly between the grid points. The points themselves lie exactly on their respective distribution functions. The approximation tends to overestimate the number of high-ability managers (high-quality firms). In the numerical counterfactuals, $N \geq 10^5$.

\textsuperscript{35}This procedure is similar to Restuccia and Rogerson (2008).
Table 3: Matching Frictions Within Various Ranges of Distribution

as the elasticity of substitution approaches the Leontief case (third row in
each panel), aggregate output falls by more than 25% and TFP by almost 19% under the assumption that $I = 1$ (Panel D).

It is, of course, not surprising that matching frictions are more harmful economically when ability and quality are complements. What has been overlooked thus far is that the underlying distributions of ability and talent are sensitive to the assumed degree of complementarity. Together, they imply that “crony capitalism” can depress output and TFP far more than previously thought. If, to the contrary, managers hardly differed in their abilities regardless of the value of $\frac{1}{\rho}$, then the size distribution would be driven almost exclusively by attributes of the firm and mismatch would indeed not be a meaningful source of inefficiency.

6.2 Matching Frictions: Endogenous Occupational Choice

If distortions in the market for managerial talent and firm quality are of a nature to generate mismatch between the full range of the two fixed factors, one would expect the enterprise size distribution to have a very different shape compared to the efficient benchmark. More specifically, the distribution in the distorted case must have thinner tails as high quality firms are not always matched with competent managers. In fact, some large firms may be run by managers who, in a frictionless economy, would be supplying labor to any one of the firms in the distribution inelastically. Very large enterprises account for a smaller share of all firms.

Figures 4 and 5 highlight the shape difference between US and Korean manufacturing. For ease of comparison, US manufacturing firms are sorted into the same size bins as those reported by the National Statistical Office of Korea and the $x$ and $y$ (log) scales are identical in the two figures.  

The parameter of interest in the Generalized Pareto Distribution (GPD) is $\hat{k}$, which characterizes the shape of the tail. A smaller number indicates a more concave plot in log-log space, and hence a more rapid decay in the probability mass as firms become larger. Both the mean and variance of the distribution are increasing in $\hat{k}$ and in the scale parameter $\hat{\sigma}$.  

---

36 The US firm size data is much less granular. The US Census tabulates data in as many as 44 size bins.
37 Note that the GPD slightly overestimates the probability of very large firms. For the current exercise this is of no consequence.
Since I assume the unobservable firm and talent attributes to be distributed log-normally, I can parametrize the degree of mismatch by the correlation coefficient between $a$ and $q$ (denoted by $\rho_{aq}$). In U.S. manufacturing, 96.50% of firms with at least five employees have 499 employees or less. Among Korean manufacturing firms, the corresponding share is 99.54%. A very small share of Korean firms has 500 or more employees.

Starting from the calibrated distributions of firm and management attributes in the U.S., I pick the $\rho_{aq}$ that lowers the variance by just enough to shift the 499 employee cutoff from the 96.50th to the 99.54th percentile of the firm size distribution. It is then a straightforward (though time-consuming) numerical exercise to compute the aggregate productivity decline relative to the efficient U.S. benchmark.

I do not mean to suggest that the variation in the shape of the firm size distribution is fully accounted for by frictions in the markets for fixed factors. Restuccia and Rogerson (2008) have demonstrated that idiosyncratic distortions in the market for variable inputs also affect the size distribution. In Korea, for instance, there is ample evidence for capital subsidies.
of a considerable magnitude. Krueger (2002) estimates capital subsidies in the order of ten percent of GDP in the late sixties and early seventies, and thus captures the effect of industrial policies put in place in the run-up to the so-called “heavy and chemical industries drive”.38

By calibrating to firm size distributions without taking into account distortions at the intensive margin for capital and labor, I characterize a lower bound on the importance of crony capitalism. Capital subsidies targeted at large enterprises – as was the case in Korea – tend to fatten the right tail of the firm size distribution, thus offsetting the effect non-assortative matching between fixed factors, which thins that same tail. Unfortunately, there is no micro data on payments to management and firm quality to disentangle the two effects.

38This episode is commonly referred to by the acronym HCI drive.
7 Conclusion

In this framework, mismatch of firms and managers generates productivity (output) losses of almost 20% (27%), provided the degree of complementarity between \( a \) and \( q \) is sufficiently high. The effect is driven by two – complementary – mechanisms.

First, the inferred dispersion of managerial competence is sensitive to the substitution elasticity with firm quality. In the Cobb-Douglas case, managers across the size distribution of firms are virtually identical. However, departures from unit elasticity in the Leontief direction imply much more variation in the managers’ ability. The second mechanism is mismatch itself. When managers are sufficiently homogeneous, mismatch has only a minor effect, since the size distribution is driven mostly by the distribution of fundamental attributes of the firm. On the other hand, when firm size is determined by heterogeneity in both firms’ and managers’ attributes, then mismatch is far more detrimental. Moreover, exactly how damaging mismatch is, depends on the substitution elasticity. This is, in essence, the “power mean” effect discussed in Jones (2008).

Here, both mechanisms move in the same direction. An elasticity of less than unity is associated with more dispersion in managerial talent (relative to the dispersion in firm attributes). The “power mean” property of the CES aggregator function then amplifies the effect of mismatch. Together, they explain how departures from Cobb-Douglas yield results that are very different from those reported in previous research and suggest that matching frictions can indeed impose substantial burdens on economic welfare.

Incidentally, a version of this model where the participation constraint depends on the manager’s ability offers insights into the effects of industry-specific salary caps: high-ability managers prefer their outside option, which enables less competent executives to take their place. The “flight” of managerial talent lowers, for each firm and in the aggregate, productivity and output. Such restrictions are currently discussed in the United States in the context of bail-out plans for the financial and automotive industries.

An important question for future research is the exact degree of complementarity between firm and management attributes. The evidence presented in Section 2 suggests a substitution elasticity significantly smaller than unity. Appendix C outlines a promising estimation procedure based
on Simulated Method of Moments (SMM). The procedure uses empirical deviations from assortative matching and assumptions about measurement error on payments to inform the elasticity of substitution.\textsuperscript{39}

In future theoretical work I plan to investigate some micro foundations of matching frictions. In ongoing research with Guillermo Ordoñez we ask under what conditions endogenously long-lived political coalitions are formed and how they affect the allocation of resources. This project is motivated by our belief that interactions between political and economic agents are key to understanding aggregate inefficiencies and we expect to break some new ground in the class of dynamic political economy models.

In a model similar to Shimer and Smith (2000) and Kiyotaki and Lagos (2007), search frictions generate equilibria with non-assortative matching between managers and firms. Variations in search frictions capture the stylized fact that markets for skilled individuals are shallow in developing countries compared to those in highly industrialized economies. A version with a richer participation structure also features “brain-drain” of high-ability individuals.

\textsuperscript{39}I wish to thank Jesus Fernandez-Villaverde, Jin Hahn, Dan Ackerberg, and Lee Ohanian for suggesting this approach.
References


A Closed Form Solution to System of Ordinary Differential Equations

Under the assumption that \( a[i] \) and \( q[i] \) are combined by a Cobb-Douglas aggregator, the system of differential equations formed by 11 and 10 has a closed form solution. The two equations can, in fact be rewritten as:

\[
\begin{align*}
\frac{a'[i]}{a[i]} & = \omega'[i]\nu_a^{-1}a[i]^{-\nu_a}q[i]^{-\nu_q} \\
\frac{q'[i]}{q[i]} & = \pi'[i]\nu_q^{-1}a[i]^{-\nu_q}q[i]^{-\nu_q}
\end{align*}
\]

(21)  (22)

Let

\[
z[i] = a[i]^{-\nu_a}q[i]^{-\nu_q} = (\omega[i] + \pi[i])^{-1}
\]

since payments exhaust surplus

Moreover, let

\[
\begin{align*}
\tilde{z}[i] & = \ln z[i] \\
-\tilde{z}[i] & = \ln (\omega[i] + \pi[i])
\end{align*}
\]

Since \( \omega[i] + \pi[i] = \exp (-z[i]) \), if follows that

\[
\begin{align*}
\tilde{z}'[i] & = \frac{1}{\exp (-z[i])}(\omega'[i] + \pi'[i]) \\
a'[i] & = \frac{d\ln a[i]}{di} = \omega'[i]\nu_a^{-1}z[i] \\
q'[i] & = \frac{d\ln q[i]}{di} = \pi'[i]\nu_q^{-1}z[i]
\end{align*}
\]

I can then solve for

\[
\ln \left( \frac{a[i]}{a[\kappa]} \right) = \tilde{a}[i] - \tilde{a}[\kappa]
\]

\[
= \int_\kappa^i \tilde{a}'[j]dj
\]

\[
= \int_\kappa^i \omega'[j]\nu_a^{-1}z[j]dj
\]

\[
= \int_\kappa^i \omega'[j]\nu_a^{-1}(\omega[j] + \pi[j])^{-1}dj
\]

42
Exponentiating both sides of the equation, I get

$$\frac{a[i]}{a[\kappa]} = \exp \left( \int_{\kappa}^{i} \frac{\omega'[j]}{\nu_a(\omega[j] + \pi[j])} dj \right)$$

(23)

Similarly, I can characterize the exceedences of $q[i]$ over a threshold as

$$\frac{q[i]}{q[\kappa]} = \exp \left( \int_{\kappa}^{i} \frac{\pi'[j]}{\nu_q(\omega[j] + \pi[j])} dj \right)$$

(24)

For substitution elasticities different from unity, the system of ordinary differential equations does not have an analogous reduced form solution, which relies on multiplicative separability of the aggregator. For general CES aggregators, additional structure is needed to characterize the cutoffs $a[\kappa]$ and $q[\kappa]$ and the distribution of ability and talent above the rank cutoff $\kappa$. 

This appendix describes the individual components of executive compensation in a panel of U.S. manufacturing firms. I rely on ExecuComp data, published by Standard & Poor’s. The database reports several components of executive compensation in some detail, which allows me to classify them as current or deferred payments. In particular, I can identify the following components for up to thirteen executives per firm and year:

1. Salary;
2. Bonus;
3. Value of stock options at grant date;
4. Stock awards and restricted stock grants;
5. Long term incentive payouts; and
6. Other miscellaneous payments such as:
   - severance payments;
   - debt forgiveness;
   - imputed interest on preferential loans to executive officers;
   - tax reimbursements;
   - signing bonuses;
   - 401(k) contributions; and
   - life insurance premiums.

In each case, I’m interested in the contemporaneous resource cost of compensation for current input. Salary, bonus, and most miscellaneous items are cash or in-kind payments for current managerial services.

Severance payments are conceptually problematic since they are not only paid for executive services in the pay period. Similarly, long-term incentive pay is, by definition, not meant to compensate contemporaneous managerial input. Since I don’t have sufficient detail on the “input period” that long-term incentive pay covers, I cannot calculate the firm’s resource cost of future long-term payouts.
ExecuComp reports Black-Scholes values at grant date for options and fair values for stocks so I need not worry about pricing the assets correctly for now. I do, however, need to convert the asset prices to flow payments using the amortization method in section 4.3.

For each executive, I compute the contemporaneous resource cost of his / her annual compensation. I include as many as five executives per year and firm. 89% of firms that report on at least one executive also report on at least five executives. Since I use $J \in \{1, 5\}$ executives in the quantitative exercise my results may be affected by the variability in coverage in my sample. Fortunately, robustness checks confirmed this was not much of a concern.

As mentioned before, including long-term incentive payouts (LTIPs) in managerial pay is potentially problematic, as managers are compensated for inputs that are not contemporaneous. However, as a share of current compensation, LTIPs account for only 4.7% among top-ranked executives (standard deviation: 0.132), 4.1% among second-ranked executives (standard deviation: 0.120), and less than 4% for lower-ranked managers. Omitting LTIPs does not affect the estimation results in section 4 significantly.

The mix of financial assets and current compensation in executive pay varies by rank. Stocks and stock options account, on average, for 42.2% of total compensation (consisting of current compensation plus the value of financial assets) among top-ranked executives (standard deviation: 0.298). The corresponding shares for lower-ranked executives are 39.8% (second ranked), 38.8% (third ranked), 38.0% (fourth-ranked), 36.7% (fifth-ranked).

\[40\] In general, smaller firms report on fewer managers. This pattern holds regardless of the measure of firm size (number of employees, sales, total assets).
C Future Empirical Work: Simulated Method of Moments

This appendix contains a brief outline of future empirical work. While certain elements of the procedure are already in place, additional time and effort is required to implement the algorithm.

I hypothesize that the tails of managerial talent and firm quality follow Generalized Pareto Distributions (GPD) with CDF:

\[ F(x) = 1 - \left(1 + \frac{kx}{\sigma}\right)^{-\frac{1}{k}} \]  

(25)

where \( k \) is the shape parameter and \( \sigma \) is the scale parameter. Note that the two-parameter GPD assumes the data consist of exceedences. If they weren’t one would simply subtract a threshold \( \mu \) (i.e. the smallest value in the sample) from every observation to obtain exceedences from the raw data. \( \mu \) is the so-called location parameter in a three-parameter GPD. Here I assume \( \mu = 0 \).

Following are step-by-step descriptions of the simulation procedure:

1. Draw a \( 2 \times N \) matrix of uniform random variables. Denote the two columns by \( I_1 \) and \( I_2 \). To guarantee stochastic equicontinuity, I draw the matrix only once (McFadden, 1989).

2. Draw a \( 2 \times N \) vector of standard normal random variables. Denote the columns by \( \epsilon \) and \( u \).

3. Fix seven parameters: \( \theta = (\rho, k_A, \sigma_A, k_Q, \sigma_Q, \sigma_\epsilon, \sigma_u) \), where \( \sigma_\epsilon \) and \( \sigma_u \) are measurement errors associated with management compensation and payments to firm quality, respectively.

4. Compute \( \hat{\omega}[i_1] \) and \( \hat{\pi}[i_2] \) using \( \theta \) as well as equations (13) and (14). Each \( i_1 \) corresponds to an element in \( I_1 \) and analogously for \( i_2 \). Denote the vectors by \( \hat{\Omega} \) and \( \hat{\Pi} \), respectively.

5. Compute

\[
\begin{align*}
X &= \hat{\Omega} + \sigma_\epsilon \epsilon \\
Y &= \hat{\Pi} + \sigma_u u
\end{align*}
\]

41I would like to thank Jesus Fernandez-Villaverde, Dan Ackerberg, Jin Hahn, as well as my advisor Lee Ohanian for suggesting this line of inquiry.
6. Compute the following seven sample moments:

\[
\begin{align*}
\frac{1}{N} \sum_{i=1}^{N} x[i] & \quad \frac{1}{N} \sum_{i=1}^{N} y[i] \\
\frac{1}{N} \sum_{i=1}^{N} x^2[i] & \quad \frac{1}{N} \sum_{i=1}^{N} y^2[i] \\
\frac{1}{N} \sum_{i=1}^{N} x^3[i] & \quad \frac{1}{N} \sum_{i=1}^{N} y^3[i] \\
\frac{1}{N} \sum_{i=1}^{N} x[i]y[i] & \\
\end{align*}
\]

7. Pick \( \theta \) that minimizes the weighted “distance” between the simulated and empirical moments:

\[
\frac{1}{N} \sum_{i=1}^{N} x[i] - \frac{1}{N} \sum_{i=1}^{N} \omega[i] \quad \frac{1}{N} \sum_{i=1}^{N} \omega^2[i] \\
\frac{1}{N} \sum_{i=1}^{N} x^2[i] - \frac{1}{N} \sum_{i=1}^{N} \omega^2[i] \\
\frac{1}{N} \sum_{i=1}^{N} x^3[i] - \frac{1}{N} \sum_{i=1}^{N} \omega^3[i] \\
\frac{1}{N} \sum_{i=1}^{N} y[i] - \frac{1}{N} \sum_{i=1}^{N} \pi[i] \\
\frac{1}{N} \sum_{i=1}^{N} y^2[i] - \frac{1}{N} \sum_{i=1}^{N} \pi^2[i] \\
\frac{1}{N} \sum_{i=1}^{N} y^3[i] - \frac{1}{N} \sum_{i=1}^{N} \pi^3[i] \\
\frac{1}{N} \sum_{i=1}^{N} x[i]y[i] - \frac{1}{N} \sum_{i=1}^{N} \omega[i] \pi[i]
\]

The weighting matrix \( \Sigma \) is determined iteratively. The initial matrix is the \( I \)-matrix. The initial \( \hat{\theta}_0 \) is then used to construct \( \hat{\Sigma}_0 \) and so forth until the estimates \( \theta \) and \( \Sigma \) converge.