Does Government Spending on Education Promote Growth and Schooling Returns?

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Abstract

Since human capital is a major driver of growth, the conventional wisdom suggests that the government should direct more resources to education. However, surprisingly the cross country data show little positive correlation between growth and public spending on education. In fact, the pattern is rather puzzling. Public spending on education tends to depress schooling return. The relationship between growth and education appears nonlinear. In this paper, we revisit this issue and try to understand these puzzling facts in terms of an endogenous growth model. We model return to schooling by adopting an asset pricing approach and show the explicit linkage between government intervention and growth via this schooling return.
1 Introduction


In this paper, we revisit this issue and focus specifically on the relationship between public expenditure on education, returns to schooling and growth. The cross country development facts summarized in Table 1 show some curious patterns.1 Countries with a higher share of public education spending in GDP (educ) have lower growth rates and lower returns to schooling (ROR) while growth and returns to schooling are positively correlated. Rich countries (measured in terms of per capita real GDP, GDPPC) spend more on education compared to poor countries.

<table>
<thead>
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<th></th>
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<th>ROR</th>
<th>Educ</th>
<th>GDPPC</th>
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<td>GDPPC</td>
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For the same sample, Figures 1 and 2 plot the relationship between edu-

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1 The data encompass a sample of 47 countries for which the returns to schooling data are available. Schooling return data are available from various world bank websites. For each of these variables, the time average is first computed for each country over the period 1960-2007. Countries with missing data have a shorter sample period.
cation spending share and growth by sorting the countries into low and high education spending groups taking median share of education spending as a cutoff point. The relationship between education spending and growth appears negative for low education share countries (correlation is -.23) while it is positive for high education share countries (correlation is .21). Although these correlations are not statistically significant at the 5% level, it suggests that the relationship between education spending and growth could be potentially nonlinear and U shaped.
The nonlinear relationship between growth and public spending has received attention in the growth literature. Blankeau and Simpson (2007) persuasively argue that the effect of public spending on growth is nonlinear. Their panel regression with bigger sample than ours reveals that public spending on education starts having a positive effect on growth for richer countries while for poor countries it potentially hurts growth. Our finding is consistent with Blankeau and Simpson because we find that rich countries also spend more on education (see, Fig 3).\(^2\)

\(^2\)Armellini (2008) also find a robustly positive relationship between per capita GDP and education spending ratio for bigger sample.
The objective of this paper is to understand these stylized facts in terms of an endogenous growth model with human capital. In our setting, growth, returns to schooling and the education spending ratio are endogenously driven by economic fundamentals. We focus on an important economic fundamental which is the degree of direct government intervention in the education sector. This intervention is modelled in terms of a schooling technology where the public spending on education appears as an infrastructural input together with private schooling efforts. Such a direct intervention induces technological complementarity between private and public inputs in the schooling technology. The marginal product of private schooling efforts depends positively on the level of government provided infrastructural input such as expenditure on teachers, school library, merit scholarships and other aids to promote learning.

As in Blankeau and Simpson (2008) we pay special attention to financing of this education spending. The government finances this education spending by taxing output. Given this framework, two scenarios lend themselves. First, government sets taxes suboptimally and countries differ only in terms of the tax distortion. Second, government sets taxes optimally and countries differ only in terms of deep parameters such as preferences and technology. The first scenario is unable to explain the observed U-shaped relation between
education and growth. The second scenario is potentially able to explain this cross-country relationship when countries differ in the degree of government intervention in education.

Curiously even though there is technological complementarity between private and public inputs in the human capital production, greater government intervention in schooling crowds out private schooling efforts. This crowding out of private schooling effort reflects private sector’s attempt to reallocate time between schooling and work to attain economic efficiency. This crowding out tends to lower growth while the technological complementarity induced by public spending on education promotes growth. In countries where historically the government plays a lesser role in the education sector, the former crowding out effect dominates the latter complementarity effect. The result is a reduction in growth when the government spends more on education. The complementarity effect dominates in countries where government infrastructural role in education is significant due to historically given circumstances. Given that richer countries spend more on public education, a testable hypothesis emerges from this model is that the degree of government complementarity in the education sector is higher in rich countries compared to poor.

The paper is organized as follows. The following section lays out the model. Section 3 characterizes the balanced growth properties of the model and characterizes the schooling returns. Section 4 performs quantitative analysis. Section 5 concludes.

2 The Model

The model is an adaptation of the Lucas-Uzawa (Lucas, 1988) model. There are two sectors, goods and education. A fixed time (normalized at unity) is allocated between schooling and goods production. Time $l_{Ht}$ allocated to schooling at date $t$ creates effective labour or human capital ($h_{t+1}$) in the following period. The productivity of schooling effort which is the same as the quality of schooling depends on pupil’s learning ability parameter ($A_H$) and the public spending on education ($g_t$):
The human capital thus evolves following the technology:

\[ h_{t+1} = (1 - \delta_h)h_t + A_H g_t^\eta (l_H t h_t)^{1-\eta} \]  

(1)

where \( 0 < \eta < 1 \). \( g_t \) is the government provided input for human capital acquisition. This input takes the form of public school infrastructure facilities such as expenditure on teachers, school library, merit scholarships and other aids to promote learning. Without this government support there is diminishing returns to human capital. There is a fixed rate of depreciation, \( \delta_h \) of human capital. This specification is similar to Glomm and Ravikumar (1997).

The parameter \( \eta \) represents the degree of government intensity or government intervention in the education sector. We model this government intensity as a schooling technology. This government intensity depends on the institutional features of a country which we do not model in this paper. Absent the government role in the education (\( \eta \) equals zero), the schooling technology reverts to the Lucas (1988) form.

Final goods \( (y_t) \) are produced with the help of human and physical capital via the Cobb-Douglas production technology:

\[ y_t = A_G k_t^\alpha (l_G h_t)^{1-\alpha} \]  

(2)

where \( l_{Gl} \) (that equals \( 1 - l_{Ht} \)) is the remaining time allocated to the production of goods and \( A_G \) is the total factor productivity (TFP) in the goods sector.\(^3\)

The investment goods technology is specified as follows:

\[ k_{t+1} = (1 - \delta_k)k_t + i_t^k \]  

(3)

where \( \delta_k \) is a fixed rate of depreciation of physical capital.

The government finances the education spending \( (g_t) \) by levying a proportional tax \( (\tau_t) \) on goods sector output, \( y_t \). In other words, the government budget constraint is:

\(^3\)We assume that leisure time is fixed.
\[ g_t = \tau_t y_t \]  

(4)

2.1 Private Sector Optimization

Private agents receive instantaneous utility \( U(c_t) \) from consumption \( (c_t) \) at date \( t \) and has an infinite horizon with a subjective utility discount factor \( \beta \). Given the tax sequence \( \{\tau_t\} \) the private agents choose the sequences \( \{c_t\}, \{i_t\}, \{l_Ht\} \), that maximize

\[
\text{Max} \sum_{t=0}^{\infty} \beta^t \ln(c_t)
\]

subject to the resource constraint:

\[ c_t + i_t = (1 - \tau_t)y_t \]  

(5)

and (1) through (3).

2.2 Government Optimization

The government is benevolent. It sets the tax rate, \( \tau_t \), optimally to maximize societal welfare assuming that the private sector behaves optimally. Once this optimal tax rate is chosen, the government is precommitted to this tax path.

Because of the government budget constraint (4), the ratio of public education spending to GDP is the same as the tax rate \( \tau_t \) levied on the private sector.

**Proposition 1** Along the balanced growth path, The optimal tax on the goods sector to finance education is given by:

\[
\tau = \frac{\frac{1 - \alpha}{1 - \eta} \frac{\eta H}{l_0}}{1 + \frac{\frac{1 - \alpha}{1 - \eta} \frac{\eta H}{l_0}}}
\]  

(6)

7
Proof. Appendix. ■

In economies where private schooling efforts \(l_{Ht}\) are higher, it is optimal to tax the goods sector more. However, whether agents will exert more effort in schooling depends on the schooling returns which we will analyze later.

3 Balanced Growth Properties

Along the balanced growth path, the time allocations to goods and schooling sectors are stationary which we denote as \(l_H\) and \(l_G\) dropping the time subscripts. The ratios of output to capital \((y_t/k_t)\) and the physical to human capital \((k_t/h_t)\) are also constants. By virtue of proposition 1 it follows that the share of education in GDP is also constant. In other words, the steady state government spending share in GDP is given by:

\[
\frac{g_t}{y_t} = \tau \tag{7}
\]

Define the gross balanced growth rate as \(G\). There are three key balanced growth equations. Based on the first order condition for the physical capital stock we get:

\[
G = \beta \left[ (1 - \tau)(\alpha y_t/k_t) + 1 - \delta_k \right] \tag{8}
\]

Based on the first order condition for the human capital stock, one gets:

\[
G = \beta [1 - \delta_h + A_H(1 - \eta)\tau^n l_H^{1-\eta}(y_t/h_t)^\eta] \tag{9}
\]

Finally, using the human capital technology (1), we get a third balanced growth equation:

\[
G = 1 - \delta_h + A_H \tau^n l_H^{1-\eta} A_G^{(1-\alpha)\eta}(k_t/h_t)^{a\eta} \tag{10}
\]

These three equations solve for three unknowns, namely \(k/h, l_H\) and \(G\). The appendix provides the details of the derivation.
3.1 Return to Schooling

In this model, the human capital is an asset which yields a flow return. Think of human capital as a Lucas tree whose valuation is \( q_t h \) which is akin to Tobin’s \( q \) of physical capital. This valuation is driven by the return and opportunity cost of going to school.

It is easy to verify that this value of human capital is the same as the ratio of the shadow price of consumption to that of investment in schooling. In other words,

\[
q_t^h = \frac{\mu_t}{\lambda_t} \tag{11}
\]

where \( \mu_t \) and \( \lambda_t \) are the lagrange multipliers associated with the schooling technology (1) and the flow resource constraint (5). Using the Euler equation for human capital (see (F.6), one gets the following valuation equation for the human capital:

\[
q_t^h = m_{t+1} [\{q_{t+1}^h (1-\delta_h + A_H g_t^H (1-\eta)(1-l_{Gt+1})^{1-\eta} h_{t+1}^{-\eta})\} + \{A_G (1-\tau_{t+1}) (1-\alpha) k_{t+1}^\alpha h_{t+1}^{-\alpha} l_{Gt+1}^l \}]
\tag{12}
\]

where \( m_{t+1} \) is the intertemporal marginal rate of substitution in consumption given by \( \lambda_{t+1}/\lambda_t \).

Next verify from (F.4) in the appendix that

\[
q_t^h = \frac{(1-\tau_t) MPH_t^G}{MPH_t^E} \tag{13}
\]

where \( MPH_t^G \) and \( MPH_t^E \) are the marginal products of effective labour in the goods and education sectors respectively.

Rewrite (12) as

\[
q_t^h = m_{t+1} [q_{t+1}^h (1-\delta_h + l_{Ht+1} MPH_{t+1}^E) + l_{Gt+1} (1-\tau_{t+1}) MPH_{t+1}^G]
\tag{14}
\]

The valuation equation for human capital looks similar to a Lucas (1978)
tree valuation equation. The value of this tree at date $t$ is the discounted next period marginal product of human capital in the goods sector, $l_{t+1}MPH^G_{t+1}$ and the imputed next period value of unused portion of the tree $(1 - \delta_h)q^h_{t+1}$ plus the replenishment of it, $l_{t+1}MPH^E_{t+1}$ due to new education.

The return to schooling ($R^h_{t+1}$) is thus given by:

$$R^h_{t+1} = \frac{[q^h_{t+1}(1 - \delta_h + l_{t+1}MPH^E_{t+1}) + l_{t+1}(1 - \tau_{t+1})MPH^G_{t+1}]}{q^h_t}$$

(15)

Along the balanced growth path, $q^h_t$ and $\tau_t$ are stationary. Using (13) one obtains the following expression for the steady state return to human capital:

$$R^h = 1 - \delta_h + MPH^E$$

(16)

Using (16) one can rewrite the balanced growth equation as follows:

$$1 + g = \beta R^h$$

(17)

Comparison of (8) with (17) immediately reveals a familiar arbitrage condition that the return on human capital must balance the after tax return on physical capital. In other words,

$$R^h = (1 - \tau)(\alpha y/k) + 1 - \delta_k$$

(18)

### 4 Quantitative Analysis

It is important to be up-front about the goal of the quantitative analysis. Given the highly stylized nature of the model, a full blown calibration of the model economy to cross country stylized facts is beyond the scope of this present study. The central goal of this paper is rather to identify the economic fundamentals which could potentially give rise to a U shaped cross-country relationship between growth and public spending on education. Since the
education spending is tax financed, two working hypotheses lend themselves. First, all countries share the same deep parameters for preference and technology but only differ in terms of tax distortions. This basically means that the government in each country behaves suboptimally in the sense that it does not set the tax rate \((\tau)\) to maximize the societal welfare. This immediately means that \textit{ceteris paribus}, growth and education spending experiences differ across countries if different countries have different tax rates. The second hypothesis is that government in all countries are benevolent and set the tax optimally to maximize the welfare of citizens. Given this optimal tax rate, the only way growth and education spending could differ across countries if deep parameters differ.

The first hypothesis is ruled out apriori because the tax rate that maximizes societal welfare also maximizes growth in this model (see proposition 2 in the appendix). This gives rise to an inverted U shaped relationship between education spending share, \(\tau\) (see the government budget constraint (4)) and growth as shown in Fig 4. Such an inverted U shaped relationship goes contrary to the U shaped relation between growth and education spending.

While examining the second hypothesis we encounter an immediate problem. There are seven deep parameters in the model, namely \(\alpha, \delta_h, \delta_k, \beta, A_H, A_G, \eta\). Which of these could be held responsible for the nonlinear relation between growth and education spending? To this end, we resort to a method of elimination. Pretend that everything else equal, countries only differ in terms of a single parameter. Does this variation help explain the U shaped relation?

To perform this quantitative exercise, we need to start off with some baseline parameter values. Using the US economy as the benchmark, \(\alpha\) is fixed at .36. The other parameters are fixed at \(\beta = .97, \delta_k = .1, \delta_h = .05, A_G = 3.2, A_H = .12\) to target about a 2.5% world growth rate. These parameter choices also give rise to an approximately 60:40 allocation of time between schooling and work as a benchmark given that leisure time is fixed. The parameter \(\eta\) that represents government intensity in education is chosen at .05 which gives rise to a steady state share of government spending around
6% through (6). This is close to the cross country average share of education in GDP.\textsuperscript{4}

We first examine the effect of cross country difference in time preference, $\beta$ on the steady state growth and the public spending ratio. Figure 5 plots the steady state growth rate and the public spending ratio ($\tau$) when countries only differ in the time preference $\beta$. More forward looking countries (larger $\beta$) invest more time in education (greater $l_H$). This means higher long run growth. The optimal share of education spending is also higher as $\tau$ is positively related to $l_H$ (see equation (6)). Growth and the public spending on education are thus positively correlated if countries differ in $\beta$ alone. The variation of the preference parameter $\beta$ is thus unable to explain the empirical U shaped relationship.

\textsuperscript{4}Comparative statics reported here are robust to other choices of parameters.
Figure 6 plots the same growth-spending relationship when countries differ only in terms of the learning ability $A_H$ alone. Higher learning ability induces agents to devote more time at school and less time at work. This promotes long run growth and the steady state public spending in GDP ($\tau$). The relationship between growth and public spending is thus positive when countries differ in learning ability only. This rules out the difference in learning ability as a potential candidate to explain the growth-spending nonlinearity.
Table 2 reports the growth and education spending experiences of countries which only differ in terms of the TFP. No perceptible relationship emerges. The same is true when one varies the capital share parameter $\alpha$ (see Table 3). Thus variation of these parameters cannot explain cross country differences in growth and education spending.$^5$

The variation of the standard preference and technology parameters are thus unable to explain the observed U shaped relationship between growth and public spending. We now turn our attention to the variation of the nonstandard parameter $\eta$ which represents the degree of government intensity

$^5$We also performed similar experiments by changing the depreciation parameter $\delta_k$ and found no relationship. Variation of $\delta_k$ mimics the variation in $A_k$. In other words a lower $\delta_k$ has similar effects as a larger $A_k$. These results are not reported for brevity.
Table 2: Growth, Education Spending and Schooling Efforts when Countries differ in TFP

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<th>Growth</th>
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<td>.66</td>
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<td>.06</td>
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Table 3: Growth, Education Spending and Schooling Efforts when Countries differ in Capital Share

<table>
<thead>
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<th>Growth</th>
<th>$\tau$</th>
<th>$l_H$</th>
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in the education sector. Fig 7 plots the same relationship when countries differ only in terms of this education technology parameter. The relationship turns out to be nonlinear and U shaped reflecting the empirical regularity reported earlier.
To understand this nonlinearity, we plot the relationship between $\eta$ and the time allocation to schooling, $l_H$. The relationship is robustly negative (see Fig 7). Greater degree of complementarity between public spending on education and the effective labour in schooling actually crowds out private schooling effort. Although this crowding out effect is apparently counterintuitive, a closer examination reveals that a fundamental arbitrage condition (18) is at work in explaining this. Everything else equal, an increase in $\eta$ raises the optimal tax rate $\tau$ to finance education spending. Since the goods sector is more capital intensive, this increase in tax lowers the ratio of physical to human capital in the economy which raises the marginal product of physical capital. If agents can alter the time allocation, they will allocate more time to work and less to schooling to preserve the arbitrage condition.
This crowding out effect of government intervention in education is central to understand the U shaped relation between public spending and growth. Two opposing effects are at work when the government intervenes more in the education sector. First this crowding out effect lowers steady state growth. Second, greater steady state size of the government has a direct scale effect on growth by promoting the marginal product of human capital. The former crowding out effect dominates the latter when the size of the government (measured by $\tau$) is small. As $\tau$ crosses a threshold, the latter scale effect dominates which promotes growth. The net result is a U shaped relation between growth and education spending.
4.1 Welfare Implications

Does an increase in public spending on education necessarily make the society worse off in the long run? The answer depends on what drives the variation in public spending? Based on the expression of the steady state welfare (F.11) in the appendix certain observations follow. If the increase in public spending is the result of a greater learning ability \( A_H \) or forward looking nature of the society \( \beta \), it will necessarily promote welfare via the growth channel. On the other hand, if the increase in public spending is caused by a direct intervention by the government (larger \( \eta \)) it has a non-monotonic effect on welfare just as growth. If the distortionary effect of the increased government intervention is less compared to the complementarity effect, the society will be better off by direct intervention by the government in the education sector.

4.2 Is government intervention in education greater in rich countries?

The analysis suggests that the cross-country difference schooling technology parameter \( \eta \) is responsible for the nonlinearity in the growth and education spending relationship. Our hypothesis is that the complementarity effect of public spending is greater for rich countries. In other words, this means that the parameter \( \eta \) is higher for richer countries. This provides a potential explanation for the U shaped relationship between public spending on education and the per capita income found by Blanqueau et al. (2008). Recall that \( \eta \) represents the degree of direct government intervention in education. It is difficult to identify this parameter from the data directly. However, there are ample indirect evidence that the government role in education is historically more predominant in rich countries compared to poor countries. For example, in England the formal compulsory primary schooling was instituted by the government in 1880. Developing countries formal schooling system is relatively young compared to this. In addition, pupils in rich countries enjoy greater public sector externality in terms of infrastructural
facilities such as school library, internet facilities, education subsidy compared to the developing world. Although these are sketchy indicators of direct government involvement, it provides a working hypothesis to investigate in future research that public sector externality in education is stronger in rich countries.

5 Conclusion

References


Appendix

6.1 First order conditions

Let $\lambda_t, \mu_t$, be the lagrange multipliers associated with the flow budget constraint (5), human capital technology.

The lagrange is:

$$L = \sum_{t=0}^{\infty} \beta^t U(c_t) + \sum_{t=0}^{\infty} \lambda_t [AG(1-\tau_t)k_t^\alpha(l_{Gt}h_t)^{1-\alpha} + (1-\delta_k)k_t - c_t - h_{t+1}]$$

$$+ \sum_{t=0}^{\infty} \mu_t [(1-\delta_h)h_t + A_Hg_t^n(l_{Ht}h_t)^{1-\eta} - h_{t+1}]$$

First order conditions are:

$$c_t : \beta^t U'(c_t) = \lambda_t \quad (F.1)$$

$$k_{t+1} : -\lambda_t + \lambda_{t+1}[(1-\tau_{t+1})\alpha y_{t+1} + (1-\delta_k)] = 0 \quad (F.2)$$

$$h_{t+1} : \mu_t = \mu_{t+1}[1-\delta_h + A_Hg_{t+1}^n(1-\eta)h_{t+1}^{-\eta} h_{t+1}^{1-\eta}] + \lambda_{t+1}[A_G(1-\tau_{t+1})(1-\alpha)k_{t+1}^{\alpha} l_{Gt}^{\alpha} h_{t+1}^{-\alpha} h_{t+1}^{1-\alpha}] \quad (F.3)$$

$$l_{Gt} : \lambda_t (1-\alpha)(1-\tau_t)A_Gl_{Gt}^{\alpha} k_t^{\alpha} h_t^{1-\alpha} - \mu_t (1-\eta)g_t^n A_Hh_t^{1-\eta} h_{t+1}^{-\eta} = 0 \quad (F.4)$$

$$\tau_t : \lambda_t y_t = \mu_t A_Hy_t^{\eta-1}(h_{t+1}l_{Ht})^{1-\eta} g_t^n \quad (F.5)$$
6.2 Proof of Proposition 1

The expression for the optimal tax rate in proposition 1 immediately follows after substituting out $\lambda_t/\mu_t$ from (F.4) and (F.5). One gets the optimal tax rate:

$$\tau_t = \frac{\frac{1-\alpha}{1-\eta} \frac{\nu l_H}{l_G}}{1 + \frac{1-\alpha}{1-\eta} \frac{\nu l_H}{l_G}}$$

Next, we exploit the fact that along the balanced growth path, the time allocations to goods and schooling ($l_{Gt}$ and $l_{Ht}$) are constants. Unless the time allocations are constant, a constant balanced growth rate does not exist because the marginal product of capital will be time varying (see (F.2)). Since $l_{Gt}$ is a constant, this means that the optimal tax rate $\tau_t^*$ is also stationary.

6.3 Derivation of the Balanced Growth Equations

Hereafter we drop time subscripts for variables which are stationary along the balanced growth path. To prove (8), use (F.1) and (F.2).

To get (9), rewrite (F.3) as:

$$\frac{\mu_t}{\lambda_t} = \frac{\mu_{t+1}}{\lambda_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} [1 - \delta_h + A_H g_t^\eta (1 - \eta) (1 - l_{Gt+1})^{1-\eta} h_{t+1}^{-\eta}] \quad (F.6)$$

$$+ \frac{\lambda_{t+1}}{\lambda_t} \{ A_G (1 - \alpha) (1 - \tau_{t+1}) k_{t+1}^\alpha h_{t+1}^{-\alpha} l_{Gt+1}^{1-\alpha} \}$$

Using (F.1), check that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta c_t}{c_{t+1}}$. Use (F.5) to substitute out $\frac{\mu_t}{\lambda_t}$ and also use the balanced growth condition $\frac{\lambda_{t+1}}{\lambda_t} = \beta/(1 + g)$ which upon substitution in (F.6) yields:

$$G = \beta [1 - \delta_h + A_H (1 - \eta) \tau^\eta l_H^{-\eta} (y_t/h_t)^\eta] \quad (F.7)$$
To get (10) use (1), (2) and (4).

**Proposition 2** The tax rate that maximizes growth also maximizes the long run welfare.

**Proof.** The steady state welfare can be written as:

\[
W_t = \sum_{j=0}^{\infty} \beta^j \ln c_{t+j}
\]

\[
= \frac{\ln c_t}{1 - \beta} + \frac{\beta}{(1 - \beta)^2} \ln G
\]

\[
= \frac{\ln k_t}{1 - \beta} + \frac{\ln(c_t/k_t)}{1 - \beta} + \frac{\beta}{(1 - \beta)^2} \ln G
\]

Use the resource constraint (5) and the balanced growth condition to verify that

\[
\frac{c_t}{k_t} = \frac{(1 - \tau) y_t}{k_t} + (1 - \delta_k) - G
\]

(F.9)

Next plug (8) into (F.9) to find

\[
\frac{c_t}{k_t} = \frac{1 - \alpha \beta}{\alpha \beta} G - \frac{(1 - \delta)(1 - \alpha \beta)}{\alpha \beta}
\]

(F.10)

which upon substitution in (F.8) yields

\[
W_t = \frac{\ln k_t}{1 - \beta} + \ln(G - (1 - \delta)) + \frac{\beta}{(1 - \beta)^2} \ln G + \ln \left(\frac{1 - \alpha \beta}{\alpha \beta}\right)
\]

(F.11)

This shows that the steady state welfare is positively related to growth that.

Thus the growth maximizer tax rate is also a welfare maximizer. ■