A Supply and Demand Model of the College Admissions Problem*

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Abstract

We develop a positive theory of decentralized college admissions problems with uncertainty, in which a continuum of heterogeneous students apply to two colleges. Student college application choices are nontrivial since they are costly and they are noisily evaluated. Colleges set admissions standards for signals of student caliber.

With a heterogeneous student population, we first characterize how the student acceptance chances at the colleges co-move in caliber. We then deduce how college application portfolios vary across students. In the spirit of supply and demand, admissions standards act like prices that allocate scarce slots to students. Noisy and costly applications prevent student-college sorting: Unless the colleges differ sufficiently in quality and the lesser one is not too small, better students need not apply most aggressively, nor need the better college set higher standards. We explore comparative statics, showing that shifts in capacity or application costs at one school affect both the standards and student body compositions at both.

We extend the model, to find that affirmative action at the better college may be met by a discriminatory admissions policy at the weaker school, and that the lesser college may poach students from its rival using early admissions.

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1 Introduction

The college admissions process has lately been the object of much scrutiny, both from academics and in the popular press. This interest owes in part to the strategic nature of college admissions, as schools use the tools at their disposal to attract the best students. Those students, in turn, respond most judiciously in making their application decisions. This paper examines the joint behavior of students and colleges in equilibrium.

We develop and flesh out an equilibrium model of the college admissions process, with decentralized matching of students and two colleges — one better and one worse, respectively, called 1 and 2. The model captures two previously unexplored aspects of the ‘real-world’ problem. First, college applications are costly, and second, colleges only observe a noisy signal of each student’s caliber. We assume that colleges seek to fill their capacity with the best students possible. Students meanwhile must solve a nontrivial portfolio choice problem. This tandem of noisy caliber and costly applications feeds the intriguing conflict at the heart of the student choice problem: Gamble on Harvard, settle for Michigan, or apply to Harvard while insuring with Michigan. By the same token, college standards are endogenous, reflecting student preferences and their capacities.

By analyzing how colleges and students interact in equilibrium, we make four main contributions. First, we provide a graphical framework that simplifies the equilibrium analysis. Second, we determine when students sort by caliber into colleges. Third, we show how costly and noisy applications induce some subtle interdependencies between the colleges. Finally, we show how an easy modification of our theory sheds some new light on two currently topical issues: affirmative action and early admissions.

Our first contribution is methodological: We produce an intuitive and yet rigorous graphical analysis of the student portfolio choice problem. With a heterogeneous student population, we characterize how the student acceptance chances at the colleges co-move. This explains how portfolio choices vary by student caliber. We can likewise parse how potential enrollment changes as colleges raise their admission standards: It is harder for any applicants to gain admission but also fewer students apply — our standards and portfolio effects. Treating admissions standards as prices, these two effects reinforce each other to establish in equilibrium a “law of demand”, in which college enrollment falls in its standard. We hope that this framework proves a tractable workhorse for future work. It embeds both the tradeoffs found in the search-theoretic problems analyzed by Chade and Smith (2006), and the colleges’ choice of capacity-filling admission standards.
Our second contribution is an analysis of college-student sorting. One might imagine that the best students are at least stochastically sorted into the best colleges. In fact, this need not hold. Intuitively, students can only be stochastically sorted if the better ones apply more aggressively. Precisely, this means: (i) the best students apply just to college 1; (ii) the middling/strong students insure by applying to both colleges 1 and 2; (iii) the middling/weak students apply just to college 2; and finally, (iv) the weakest students apply nowhere. We give a graphical proof that this need not occur in equilibrium unless colleges greatly differ in quality and the lesser is not too small.

Next, does the better college impose the tougher admission standard? In fact, this cannot be taken for granted, since college standards reflect not only the quality of the college but also their capacity. Roughly inverting our “law of demand”, college 1 must lower its admission standard to fill a larger capacity; at some point, its equilibrium admission standard conceivably might fall below that of college 2. In fact, we argue that this happens in equilibrium. Conversely, we prove that only sorting equilibria exist if the colleges differ sufficiently in quality and the higher ranked school is not too small.

In our third contribution, we uncover some new interdependencies between colleges. Absent noise, the better college does not care about decisions made by the lesser. But we show that admissions standards at both colleges fall if college 2 raises its capacity. UCLA is thus affected by admissions policies at USC. Moreover, the better ranked college profits from higher application costs charged by either school.

For our fourth contribution, we consider two extensions of our model that shed light on topical issues: affirmative action and early admissions. For these applications, we exploit an analogy with third degree price discrimination, and show that colleges equalize the “shadow values” of different groups. These groups depend on our context: for affirmative action, the groups are a target “minority” group and its complement, whereas for early admissions, the groups are those students applying early and regular.

First consider affirmative action. Here, we uncover an asymmetric result. When the lesser college introduces affirmative action policies in favor of the minority subgroup, the better college sees its worst minority students withdraw. Hence, it best responds by favoring those that do apply. Conversely, when the better college initiates the affirmative action policy, the weaker college will penalize minority students. This stems from an “acceptance curse” that it faces: Any student’s enrollment is bad news there since it implies that she was rejected by the better college. But this effect is stronger for minority students since they were advantaged by affirmative action at the better college.
Next, our framework affords some insights when one college offers an early admissions program. We show that this encourages more “aggressive” applications by students. As a consequence, we prove that college 1 would penalize early applicants. On the other hand, we show that college 2 might use early admissions to poach students from college 1.

The paper is related to several strands of literature. Gale and Shapley (1962) initiated the college admissions problem in their classic work in the economics of matching. As the prime example of many-to-one matching, it has been the province of cooperative game theory (Roth and Sotomayor 1990). Our model critically differs by the assumptions that matching is decentralized and subject to two frictions — the application cost and the noisy evaluation process. To analyze these features in a simple fashion, we confine our attention to homogeneous preferences and consider a model with two colleges. This yields a tractable benchmark analysis of what is otherwise a difficult equilibrium problem.

The informational friction and application costs pushes our paper closer to the large literature on decentralized frictional matching — for instance, Shimer and Smith (2000), Smith (2006), Anderson and Smith (2007), and Chade (2006). The focus of that two-sided matching literature has been on the difficulties of securing the sorting result found in Becker (1973). We find that in our many-to-one matching environment, the conditions for sorting require rather strong assumptions on payoffs and college capacities.

The student portfolio problem embedded in the model is a special case of the problem solved in Chade and Smith (2006). But the acceptance chances here are endogenous, since any one student’s acceptance chance depends on the equilibrium college admission threshold. In this respect, our paper also contributes to the literature on equilibrium models with non-sequential search (e.g. Burdett and Judd (1983), Burdett, Shi, and Wright (2001), and Albrecht, Gautier, and Vroman (2003)), as well as Kircher and Gale-nanos (2006)). Finally, Chade (2006) introduced the acceptance curse notion with type uncertainty. Lee (2007) shows that early admissions programs can mitigate such effects.

The paper is organized as follows. The model concept is found in Sections 2–4. The student problem is analyzed in Sections 5–7. Sections 8 and 9 shed light on the sorting character of equilibria. Section 10 contains some useful comparative statics. Section 11 and 12 slightly enrich the model to explore affirmative action and early admissions. Section 13 concludes. Proofs are found in the appendix.

1Niederle and Yariv (2007) explore a decentralized version of Gale and Shapley, while Chakraborty, Citanna, and Ostrovsky (2007) admit uncertain types in this world.
2Nagypál (2004) analyzes the problem facing a student, with perfectly correlated admissions.
2 An Overview of the Environment

We impose very little structure and concentrate on the essential features of the problem. We ignore the important consideration of heterogeneity in preferences of students over colleges or vice versa. Instead, we focus on two key frictions. First, student applications to colleges are costly. In practice, such costs can be quite high, as attested by the recent popularity of the “common application”, whose sole purpose is to lower the cost of multiple applications. Without application costs, there is no role for student choice.

Second, signals of student calibers are noisy. This informational friction creates uncertainty on the student side, and a filtering problem for colleges. It captures the difficulty faced by market participants, with students choosing “insurance schools” and “long shots”, and colleges trying to infer the best students from noisy signals. Without noise, sorting would be trivial: Better students would apply and be admitted to better colleges, for their caliber would be correctly inferred and they would be accepted. As we will see, sorting is less easily achieved with both application costs and evaluation noise. Indeed, there is a richer role for student choice in this environment.

We also make two other key modeling choices. First, we assume just two colleges. This is done for the sake of tractability. The $n$-college problem is very important, but will remain a challenging open problem in this literature for many years to come. We also fix their capacity. This is most defensible in the short run, and so it is best to interpret our model as focusing on the “short run” analysis of college admissions. We explore the simultaneous game in which students apply to college, and colleges decide whom to admit. However, we later briefly explore the possibility of “early admissions”.

A model with common and idiosyncratic components for the evaluation of a student is intractable. We can solve both extreme cases with perfectly correlated and conditionally i.i.d. signals. But we restrict to the analysis to the less trivial latter case. It is arguably more realistic too, since admission to a better college often does not guarantee admission to a lesser one. Conditionally i.i.d. signals exactly captures the case where students are apprised of all variables (such as the ACT/SAT or their GPA) common to their applications before applying to college. Students are uncertain as to how these idiosyncratic elements such as college-specific essays and interviews will be evaluated, but believe that the resulting signals are conditionally independent across colleges.

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3 The “common application” is a general application form that is used by over 150 colleges in an effort to simplify college applications.
3 The Model

There are two colleges 1 and 2 with capacities $\kappa_1$ and $\kappa_2$, and a unit mass of students with calibers $x$ whose distribution has a density $f(x)$ over $[0, \infty)$. We avoid trivialities, and assume that college capacity is insufficient for all students, as $\kappa_1 + \kappa_2 < 1$. Each college application costs a student $c > 0$. Preferences coincide, with all students preferring college 1. Everyone receives payoff 1 for attending college 1, $u \in (0, 1)$ for college 2, and no payoff for not attending college. To avoid trivialities, we later bound application costs above. Students maximize expected college payoff less application costs. College payoff equals the average enrolled student caliber times the measure of students enrolled.

Students know their caliber, and colleges do not. Appendix A.1 shows how our results can be at once re-interpreted if $x$ is a student’s signal of his own caliber. Colleges 1 and 2 each just observes a noisy conditionally independent signal of each applicant’s caliber. In particular, they do not know where else students have applied. Signals $\sigma$ are drawn from a conditional density function $g(\sigma|x)$ on a subinterval of $\mathbb{R}$, with cdf $G(\sigma|x)$. We assume that $g(\sigma|x)$ is continuous and obeys the strict monotone likelihood ratio property (MLRP). So $g(\tau|x)/g(\sigma|x)$ is increasing in the student’s type $x$ for all signals $\tau > \sigma$.

Students apply simultaneously to either, both, or neither college, choosing for each caliber $x$, a college application menu $S(x)$ in $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. Contemporaneously, colleges choose the set of acceptable students signals. They intuitively should use admission standards to achieve their objective functions — college $i$ admitting students above a threshold signal $\underline{\sigma}_i$. Appendix A.2 proves this given the MLRP property — despite an acceptance curse that college 2 faces (as it may accept a reject of college 1).

For a fixed admission standard, we want to ensure that very high quality students are almost never rejected, and very poor students are almost always rejected. For this, we assume that for a fixed signal $\sigma$, we have $G(\sigma|x) \to 0$ as $x \to \infty$ and $G(\sigma|x) \to 1$ as $x \to 0$. For instance, exponentially distributed signals have this property $G(\sigma|x) = 1 - e^{-\sigma/x}$. More generally, this obtains for signals drawn from any “location family”, in which the conditional cdf of signals $\sigma$ is given by $G((\sigma - x)/\mu)$, for any smooth cdf $G$ and $\mu > 0$ — eg. normal, logistic, Cauchy, or uniformly distributed signals. The strict MLRP then holds if the density is log-concave, so that $\log G'$ is strictly concave.

Alternatively, colleges could first commit to an admission standard. This yields the same equilibria until we study affirmative action and early admissions (proof omitted). In the interests of a unified treatment throughout the paper, we proceed in the simultaneous move world.
4 Equilibrium

A Nash equilibrium is a triple \((S^e(\cdot), \sigma^e_1, \sigma^e_2)\) such that

(a) Given \((\sigma^e_1, \sigma^e_2)\), \(S^e(x)\) is an optimal college application portfolio for each \(x\),

(b) Given \((S^e(\cdot), \sigma^e_j)\), college \(i\)'s payoff is maximized by admissions standard \(\sigma^e_i\).

In a sorting equilibrium, colleges’ and students’ strategies are monotone. This means that the better college is more selective \((\sigma^e_1 > \sigma^e_2)\) and higher caliber students are increasingly aggressive in their portfolio choice — namely, \(S^e(x)\) is increasing in \(x\) under the “strong set order” ranking \(\emptyset \prec \{2\} \prec \{1,2\} \prec \{1\}\). This order captures an intuitive increasing aggressiveness in student applications: The weakest apply nowhere; better students apply to the “easier” college 2; even better ones “gamble” by applying also to college 1; the next tier up shoot an “insurance” application to college 2; finally, the top students confidently just apply to college 1. Alternatively, monotone strategies ensure the intuitive result that the distribution of student calibers at college 1 first-order stochastically dominates that of college 2 (see Lemma 4 in Appendix A.8), so that all top student quantiles are larger at college 1. This is the most compelling notion of student sorting in our environment with noise (Chade 2006).

Our concern with a sorting equilibrium may be motivated on efficiency grounds. If there are complementarities between student caliber and college quality, so that welfare is maximized by assigning the best students to the best colleges, any decentralized matching system must necessarily satisfy sorting to be (constrained) efficient. Since formalizing this idea would add notation and offer little additional insight, we have abstracted from these normative issues and focused on the positive analysis of the model.

5 The Student Optimization Problem

We begin by solving for the optimal college application set for a given pair of admission chances at the two colleges. This problem is in general hard, but an algorithm has recently been provided by Chade and Smith (2006). In our two college case, the solution is somewhat straightforward, and may be depicted graphically. From the graph, we can easily deduce sufficient conditions for monotone student behavior.

Consider the portfolio choice problem for a student facing the admission chances \(0 \leq \alpha_1, \alpha_2 \leq 1\). The expected payoff of applying to both colleges is \(\alpha_1 + (1 - \alpha_1)\alpha_2 u\).
Figure 1: **Optimal Decision Regions.** The left panel depicts (i) a dashed box, inside which applying anywhere is dominated; (ii) the indifference line for solo applications to colleges 1 and 2; and (iii) the marginal benefit curves $MB_{12} = c$ and $MB_{21} = c$ for adding colleges 1 or 2. The right panel shows the optimal application regions. A student in the blank region $\Phi$ does not apply to college. He applies to college 2 only in the vertical shaded region $C_2$; to both colleges in the hashed region $B$, and to college 1 only in the horizontal shaded region $C_1$.

The marginal benefit $MB_{ij}$ of adding college $i$ to a portfolio of college $j$ is then:

$$MB_{21} = \alpha_1 + (1 - \alpha_1)\alpha_2 u - \alpha_1 = (1 - \alpha_1)\alpha_2 u$$

(1)

$$MB_{12} = \alpha_1 + (1 - \alpha_1)\alpha_2 u - \alpha_2 u = \alpha_1 (1 - \alpha_2 u)$$

(2)

The optimal application strategy is then given by the following rule:

(a) **Apply nowhere if costs are prohibitive**: $c > \alpha_1$ and $c > \alpha_2 u$.

(b) **Apply just to college 1**, if it beats applying just to college 2 ($\alpha_1 \geq \alpha_2 u$), and nowhere ($\alpha_1 \geq c$), and to both colleges ($MB_{21} < c$, i.e. adding college 2 is worse).

(c) **Apply just to college 2**, if it beats applying just to college 1 ($\alpha_2 u \geq \alpha_1$), and nowhere ($\alpha_2 u \geq c$), and to both colleges ($MB_{12} < c$, i.e. adding college 1 is worse).

(d) **Apply to both colleges** if this beats applying just to college 1 ($MB_{21} \geq c$), and just to college 2 ($MB_{12} \geq c$), for then, these solo application options respectively beat applying to nowhere, as $\alpha_1 > MB_{12} \geq c$ and $\alpha_2 u > MB_{21} \geq c$ by (1)–(2).

This optimization problem admits an illuminating and rigorous graphical analysis. The left panel of Figure 1 depicts three critical curves: $MB_{21} = c$, $MB_{12} = c$, $\alpha_1 = u\alpha_2$. 

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From (1) and (2), we see that all three curves share a crossing point, since $MB_{21} = MB_{12}$, when $\alpha_1 = u\alpha_2$. Since $MB_{12} = c(1-c) < c$, this crossing point lies above and right of the point $\alpha_1 = u\alpha_2 = c$, below which applying anywhere is dominated.

Throughout the paper, we assume that $c < u(1 - u)$. For if not, then the curves $MB_{21} = c$ and $MB_{12} = c$ cross a second time inside the unit square. The analysis then trivializes because multiple college applications need not occur.

Cases (a)–(d) partition the unit square into regions of $(\alpha_1, \alpha_2)$ that correspond to each portfolio choice, suggestively denoted $\Phi, C_2, B, C_1$. These regions are shaded in the right panel of Figure. This picture summarizes the optimal portfolio choice of a student with arbitrary admissions chances $(\alpha_1, \alpha_2)$.

For an alternative insight into the student optimization, we could apply the marginal improvement algorithm of Chade and Smith (2006). There, a student first decides whether she should apply anywhere. If so, she asks which college is the best singleton. In Figure at the left, college 1 is best right of the line $\alpha_1 = u\alpha_2$, and college 2 is best left of it. Next, she asks whether she should apply anywhere else. Intuitively, there are two distinct reasons for applying to both colleges that we can now parse: Either college 1 is a “stretch” school (as a gamble) — namely, added second as a lower-chance higher payoff option — or college 2 is a “safety school”, added second for insurance. In Figure, these are the parts of region $B$ above and below the line $\alpha_1 = u\alpha_2$, respectively.

### 6 Admission Chances and Student Calibers

Let us now fix the thresholds $\sigma_1$ and $\sigma_2$ set by college 1 and college 2. Student $x$’s acceptance chance at college $i$ is now given by $\alpha_i(x) \equiv 1 - G(\sigma_i|x)$. Since a higher caliber student generates stochastically higher signals, $\alpha_i(x)$ is increasing in $x$. In fact, it is a smoothly monotone onto function — namely, it is strictly increasing and differentiable, with $0 < \alpha_1(x) < 1$, and the limit behavior $\lim_{x \to 0} \alpha_1(x) = 0$ and $\lim_{x \to \infty} \alpha_1(x) = 1$.

Taking the acceptance chances as given, each student of caliber $x$ faces the portfolio optimization problem of §5. She must choose a set $S(x)$ of colleges to apply to, and accept the offer of the best school that admits her. We now translate the sets $\Phi, C_2, B, C_1$ of acceptance chance vectors into corresponding sets of calibers. Let $C_1$ be the set of calibers that apply just to college 1. Likewise define $C_2$ and $B$.

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5For if $\alpha_2 = 1$, then $MB_{21} = c$ and $MB_{12} = c$ respectively force $\alpha_1 = 1 - (c/u)$ and $\alpha_1 = c/(1-u)$. Now, $1 - (c/u) > c/(1-u)$ exactly when $c < u(1-u)$. 

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Figure 2: The Acceptance Function with Exponential Signals. The figure depicts the acceptance function $\psi(\alpha_1)$ for the case of exponential signals. Students apply to nowhere ($\Phi$), college 2 only ($C_2$), both colleges ($B$) and college 1 only ($C_1$) as caliber $x$ increases. Student behavior is therefore monotone for the acceptance function depicted.

Key to our graphical analysis is a quantile-quantile function relating student admission chances at the colleges: Since $\alpha_i(x)$ strictly rises in the student’s type $x$, a student’s admission chance $\alpha_2$ to college 2 is strictly increasing in his admission chance $\alpha_1$ to college 1. Inverting the admission chance in the type $x$, the inverse function $\xi(\alpha, \sigma)$ is the student type accepted with chance $\alpha$ given the admission standard $\sigma$, namely $\alpha \equiv 1 - G(\sigma|\xi(\alpha, \sigma))$. This yields an implied differentiable acceptance function

$$\alpha_2 = \psi(\alpha_1 | \sigma_1, \sigma_2) = 1 - G(\sigma_2|\xi(\alpha_1, \sigma_1)) \quad (3)$$

Lemma 1 The acceptance function rises in college 1’s standard $\sigma_1$ and falls in college 2’s standard $\sigma_2$, and tends to 0 and 1 as the thresholds approach their extremes.

The proof of this and all results are in the appendix. To best characterize acceptance functions, we now define a function $h : [0, 1] \to [0, 1]$ as (weakly) regular if $h(\alpha)$ is a (weakly) increasing function on $[0, 1]$ with $h(0) = 0, h(1) = 1$, and the secant slopes $h(\alpha)/\alpha$ and $(1 - h(\alpha))/(1 - \alpha)$ (weakly) fall in $\alpha$. We find that this description fully captures how our acceptance chances relate to one another.

Theorem 1 (a) If $\sigma_1 > \sigma_2$, then the acceptance function $\alpha_2 = \psi(\alpha_1)$ is regular.
(b) For any smoothly monotone onto function $\alpha_1(x)$, and any regular function $h$, there exists a continuous signal density $g(\sigma|x)$ with the strict MLRP, and thresholds $\sigma_1 > \sigma_2$, for which admission chances of student $x$ to colleges 1 and 2 are $\alpha_1(x)$ and $h(\alpha_1(x))$. 

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This result gives a complete characterization of how student admission chances at two ranked universities should compare; the properties of this curve are testable implications. It says that if a student is so good that he is guaranteed to get into college 1, then he is likewise a sure bet at college 2; likewise, if he is so bad that college 2 surely rejects him, then college 1 follows suit. More subtly, as a student’s caliber rises, the ratio of his acceptance chances at college 1 to college 2 rises, as does the ratio of his rejection chances at college 2 to college 1. And conversely, these are the only restrictions placed on acceptance chances by an informative signalling structure.

For an application, suppose that caliber signals have the exponential density \( g(\sigma|x) = \frac{1}{x} e^{-\sigma/x} \). The acceptance function is then given by the geometric function \( \psi(\alpha_1) = \alpha_1^{\sigma_2/\sigma_1} \) (see Figure 2). Lemma 1 follows. This is increasing and concave — and so regular — when college 2 has a lower admission standard. In turn, the acceptance relation for the location family is easily seen to be \( \psi(\alpha_1) = 1 - G((\sigma_2 - \sigma_1)/\mu + G^{-1}(1 - \alpha_1)). \)

Omitted from Lemma 1 is another intuitive but loose property of the acceptance function. It is closer to the diagonal when signals are noisier, and farther from it with more accurate signals. For an extreme case, as we approach the noiseless case, a student is either acceptable to neither college, both colleges, or just college 2 (assuming that it has a lower admission standard). In other words, the \( \psi \) function tends to a function passing through \((0,0), (0,1), \) and \((1,1)\). For the location family, this notion is precise: The acceptance function \( \psi(\alpha_1) \) rises in the signal accuracy \( 1/\mu \) (see Persico (2000)).

Easily, the acceptance function tends to the top of the box \( \psi(\alpha_1) = 1 - G(-\infty) = 1 \) as \( \mu \to 0 \), and to the diagonal \( \psi(\alpha_1) = 1 - G(0 + G^{-1}(1 - \alpha_1)) = \alpha_1 \) as \( \mu \to \infty \). Near this extreme case, the student behavior is surely monotone, since as the student caliber rises, we proceed in sequence through the regions \( \Phi, C_2, B, \) and finally \( C_2 \). We now explore the intermediate cases with noise, and find when students use monotone strategies.

7 Portfolio Changes Across Student Calibers

As a student’s caliber rises, his admission chance at college 1 rises proportionately faster than at college 2. Indeed, the ratio \( \alpha_1(x)/\alpha_2(x) \) strictly rises in \( x \) by Lemma 1. This intuitively skews the optimal portfolio in \( \S 5 \) towards college 1 as the student caliber rises. This monotone behavior is depicted in Figure 2. We now explore this character.

In general, there are two reasons for non-sorting. First, the acceptance function may

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\( \text{6} \)The limit function is not well-defined: If a student’s type is known, just these three points remain.
Figure 3: Non-Monotone Behavior. In the left panel, the signal structure induces a piecewise linear acceptance function. Student behavior is non-monotone, since there are both low and high caliber students who apply to college 2 only ($C_2$), while intermediate ones insure by applying to both. In the right panel, equal thresholds at both colleges induce an acceptance function along the diagonal, $\alpha_1 = \alpha_2$. Student behavior is non-monotone, as both low and high caliber students apply to college 1 only ($C_1$), while middling caliber students apply to both. Such an acceptance function also arises when caliber signals are very noisy.

multiply cross the $MB_{12} = c$ curve, as in Figure 3. This defies sorting: some caliber $x$ gambles up by applying to both colleges, but a higher caliber $y > x$ plays it safe by applying to college 2 only. Figure 3 depicts a non-monotone sequence of application sets $\Phi, \{2\}, \{1,2\}, \{2\}, \{1,2\}, \{1\}$ as caliber rises. This can happen since the marginal benefit $MB_{12}$ in (2) rises in the expected payoff $\alpha_1$ of college 1 and falls in the expected payoff $\alpha_2u$ of college 2. So if $\alpha_2u$ rises faster than $\alpha_1$, then a better student may drop college 1 from her portfolio. We show that if $u \leq 0.5$, then this cannot happen. If a student includes college 1 in his portfolio, then any higher caliber student also does.

The next problematic case for sorting applies when college 1 is insufficiently more selective than college 2. For if both colleges impose the same standards, the acceptance function is the diagonal $\alpha_2 = \alpha_1$ (as in Figure 3). In this case, the worst students who apply anywhere will choose college 1, since $\alpha_1 > \alpha_2u$. It is impossible to preclude this behavior using primitives of the student optimization alone. Rather, the (endogenous) admission standard at college 1 must be sufficiently higher than at college 2.

**Lemma 2 (Monotone Applications)** Student behavior is monotone in caliber if
(a) College 2 has payoff $u \leq 0.5$, so that if a student applies to college 1, then any better student will also apply to college 1, and
(b) College 2 imposes a low enough admissions standard relative to college 1 so that if a student applies to college 2, then any worse student applies to college 2 or nowhere.

The proof in the appendix of (a) argues that the marginal benefit locus \( MB_{12} = c \) has a rising secant slope if \( u \leq 0.5 \), and thus it can only cross the acceptance function \( \psi \) once — since \( \psi \) has the falling secant property. The proof of (b) shows that we need merely insist that \( \psi \) cross high enough above a known acceptance point \((\bar{\alpha}_1, \bar{\alpha}_2)\) in region \( B \). A sufficient condition for this awaits the analysis of the college behavior in §8.

8 Equilibrium via Supply and Demand Analysis

Each college \( i \) must choose an admission standard \( \sigma_i \), as a best response to its rival’s threshold \( \sigma_j \) and the student portfolios. With a continuum of students, the resulting enrollment \( E_i \) at colleges \( i = 1, 2 \) is a non-stochastic number:

\[
E_1(\sigma_1, \sigma_2) = \int_{B \cup C_1} \alpha_1(x)f(x) \, dx \tag{4}
\]

\[
E_2(\sigma_1, \sigma_2) = \int_{C_2} \alpha_2(x)f(x) \, dx + \int_{B} \alpha_2(x)(1 - \alpha_1(x))f(x) \, dx, \tag{5}
\]

suppressing the dependence of the sets \( B, C_1 \) and \( C_2 \) on the student application strategy.

To understand (4) and (5), observe that caliber \( x \) student is admitted to college 1 with chance \( \alpha_1(x) \), to college 2 with chance \( \alpha_2(x) \), and finally to college 2 but not college 1 with chance \( \alpha_2(x)(1 - \alpha_1(x)) \). Also, anyone that college 1 admits will enroll automatically, while college 2 only enrolls those who either did not apply or got rejected from college 1.

If we substitute optimal student portfolios into the enrollment equations (4)–(5), then they behave like demand curves where the admissions standards are the prices. Our framework affords analogues to the substitution and income effects in demand theory. The admission rate of any student obviously falls in its anticipated admission standard — the standards effect. Theorem 2 below exploits a compounding portfolio effect — that demand also falls due to an application portfolio shift, visible in Figures 2 and 3.

Each college’s applicant pool shrinks in its own admissions threshold, and expands in its rival’s. Together, we deduce the natural property of demand curves:

**Theorem 2 (The Falling Demand Curve)** If either college raises its admission standard, then its enrollment falls, and thus its rival’s enrollment rises.
Assume monotone student behavior. Careful inspection of Figures 2 and 3 reveals that college 1’s applicant pool not only shrinks but also improves when either admission standard rises: Indeed, it just loses its worst applicants. By contrast, college 2 loses both better and worse applicants — the top students who also applied to college 1 will deem their college 2 insurance no longer worth it, while the worst students who simply shot a solo application to college 2 will drop out altogether.

Since capacities imply vertical supply curves, we have now justified a supply and demand analysis, in which the colleges are selling differentiated products:

\[ \kappa_1 = \mathcal{E}_1(\sigma_1, \sigma_2) \quad \text{and} \quad \kappa_2 = \mathcal{E}_2(\sigma_1, \sigma_2) \]  

(6)

For now, let us ignore the possibility that some college might not fill its capacity. Then equilibrium without excess capacity requires that both markets clear \( \kappa_1 = \mathcal{E}_1(\sigma_1, \sigma_2) \) and \( \kappa_2 = \mathcal{E}_2(\sigma_1, \sigma_2) \). Since each enrollment (demand) function is falling in its own threshold, we may invert these equations. This yields for each school \( i \) the threshold that “best responds” to its rival’s admissions threshold \( \sigma_j \) so as to fill their capacity \( \kappa_i \):

\[ \sigma_1 = \Sigma_1(\sigma_2, \kappa_1) \quad \sigma_2 = \Sigma_2(\sigma_1, \kappa_2) \]  

(7)

Given the discussion of the enrollment functions, we can treat \( \Sigma_i \) as if it is a best response function of college \( i \). It rises in its rival’s admission standard and falls in its own capacity. In other words, the admissions standards at the two colleges are strategic complements. Figure 4 depicts an equilibrium as a crossing of these increasing best response functions.

By way of contrast, observe that without noise or without application costs, the better college is completely insulated from the actions of its lesser rival — \( \Sigma_1 \) is vertical. The equilibrium analysis is straightforward, and there is necessarily a unique equilibrium. In either case, the applicant pool of college 1 is independent of what college 2 does. For when the application signal is noiseless, just the top students apply to college 1. And when applications are free, all students apply to college 1, and will enroll if accepted.

With application costs and noise, \( \Sigma_1 \) is upward-sloping, and application pools depend on both college thresholds. When college 2 adjusts its admission standard, the student incentives to gamble on college 1 are affected. This feedback is critical in our paper. It leads to a richer interaction among the colleges, and perhaps to multiple equilibria.

In Figure 4, the best response function \( \Sigma_1 \) is steeper than \( \Sigma_2 \) at the crossing point. Let us call any such college equilibrium stable. It is robust in the following sense:
Figure 4: **College Responses and a Stable Equilibrium.** The functions $\Sigma_1$ (solid) and $\Sigma_2$ (dashed) give pairs of thresholds so that colleges 1 and 2 fill their capacities in equilibrium.

Suppose that whenever enrollment falls below capacity, the college relaxes its admission standards, and vice versa. Then this dynamic pushes us back to the equilibrium. So at this theoretical level, admission thresholds act as prices in a Walrasian tatonnement. Unstable equilibria should be rare: They require that a college’s enrollments respond more to the other school’s admission standard than its own.

**Theorem 3 (Existence)** A stable equilibrium exists. College 1 fills its capacity. Also, there exists $\bar{\kappa}_1(\kappa_2, c) < 1 - \kappa_2$ satisfying $\lim_{c \to 0} \bar{\kappa}_1(\kappa_2, c) = 1 - \kappa_2$ such that if $\kappa_1 \leq \bar{\kappa}_1(\kappa_2, c)$, then college 2 also fills its capacity in any equilibrium. If $\kappa_1 > \bar{\kappa}_1(\kappa_2, c)$, then college 2 has excess capacity in some equilibrium.

For some insight, we choose the capacity $\bar{\kappa}_1$ given $\kappa_2$ so that when college 2 has no standards, both colleges exactly fill their capacity. This borderline capacity is less than $1 - \kappa_2$ since a positive mass of students — perversely, those with the highest calibers — applies just to college 1, and some are rejected. (This happens whenever one’s admission chance at college 1 is at least $1 - c/u$, by (1).) It may be surprising that some college spaces can go unfilled in equilibrium despite insufficient capacity for the applicant pool. Essentially, if college 1 is “too big” relative to college 2, then college 2 is left with excess capacity. There is excess demand for college slots, and yet due to the informational frictions, there is also excess supply of slots at college 2, even at “zero price”.

When college 2 has excess capacity, it optimally accepts all applicants. Since college 1 maintains an admissions standard, college behavior is monotone. But this forces $\alpha_2 = 1$ for all students, and so the acceptance function traverses the top side of the unit square.
in Figure 2. In other words, as student caliber rises, they apply in order to colleges \( \{2\} \), then \( \{1,2\} \), and finally \( \{1\} \). Altogether, this is a sorting equilibrium.

9 Do Colleges and Students Sort in Equilibrium?

We now attack the harder problem of finding conditions for sorting equilibria without excess capacity. To wit, when is our portfolio effect monotone in the student caliber? When do “richer” students shift towards more expensive goods? Hearken back to the sufficient conditions in Lemma 2 where we argued that a sufficiently dominant college 1 (\( u \leq 0.5 \)) and a low enough admission standard at college 2 were sufficient for monotone student behavior. We now show that these conditions are also necessary for sorting to occur in equilibrium when the admission standards clear the market (7).

Theorem 4 (Non-Sorting in Equilibrium)

(a) [College 2 is Too Good] For any payoff \( u > 0.5 \) and any capacities \( \kappa_1, \kappa_2 > 0 \) with \( \kappa_1 + \kappa_2 < 1 \), a continuous signal density \( g(\sigma|x) \) with the MLRP exists for which an equilibrium exists having \( \sigma_1 > \sigma_2 \) but non-monotone student behavior.

(b) [College 2 is Too Small] There exists \( \bar{\kappa}_2(\kappa_1) > 0 \) so that college 2 sets a higher admissions standard than college 1 in a stable equilibrium, for any capacity \( \kappa_2 \leq \bar{\kappa}_2(\kappa_1) \).

We tackle part (a) constructively in the appendix, starting with the acceptance function depicted in the left panel of Figure 3 and then appealing to Theorem 1 to find some signal distribution that generates it. The message of part (b) is that even a bad college 2 may maintain higher standards if it is sufficiently small. In other words, admissions standards can be misleading measures of college quality. Because college 2 has higher admissions standards and lower payoff, it is no applicant’s first choice. But some students will still insure themselves with an application to college 2. If college 2 is sufficiently small, it may fill its capacity with these insurance applicants.

Thus far, we have found conditions under which sorting fails in some equilibrium. We next finish the picture and give sufficient conditions for the reverse conclusion that college matching entails sorting. Towards this conventional wisdom, we complete Lemma 2, providing a sufficient condition for its part (b). This is a partial converse of Theorem 4.

Theorem 5 (Sorting Equilibrium) There exists \( \kappa_1(\kappa_2) > 0 \) such that if \( \kappa_1 \leq \kappa_1(\kappa_2) \) and \( u \leq 0.5 \) — namely, college 1 is not too big and college 2 is not too good — then there are only sorting equilibria and neither college has excess capacity.
Changing College Sizes and Application Costs

We now continue to explore the supply and demand metaphor, and derive some basic comparative statics. The potential multiplicity of equilibria renders a comparative statics exercise difficult. The analysis of this section applies to all stable equilibria, and in particular to any unique sorting equilibrium. In Figure 5, we present the equilibrium effects of increases in the capacity of college 1 (left panel) and college 2 (right panel).

**Theorem 6 (College Capacity)** An increase in either college’s capacity lowers both college admissions thresholds.

Let us focus on a sorting equilibrium, and consider for example what happens if college 2 raises its capacity $\kappa_2$. Fixing the admission standard $\sigma_1$, this depresses $\sigma_2$. The marginal student that was indifferent between applying to college 2 only ($C_2$) and both colleges ($B$) now strictly prefers to apply to college 2 only. So fewer apply to college 1. Given this portfolio reallocation, college 1 will drop its admission standards.

Next, we turn to changes in application costs. Since we have assumed these are the same across colleges, we simply consider small unilateral increases.

**Theorem 7 (Application Costs)** If the application costs at any college $i$ slightly rise, then both admissions standards fall. If equilibrium is sorting, then the distribution of calibers of student enrolled at college 1 stochastically improves if students know their types.

---

7 It is not easy to ensure uniqueness of equilibrium. One case in which this holds is when $c$ is sufficiently small. This follows by continuity from the uniqueness of equilibrium in the costless case.
The logic for the first part is straightforward. When applications costs at a college rise, its applicant set shrinks, and it must in turn reduce its standards to compensate. Once again, this affects rival colleges. For example, if applications costs at college 2 rise, then it must lower its own standards. But now a student previously indifferent between applying to college 2 only and to both colleges will choose college 2 only, and so college 1’s set of applicants also shrinks. College 1 is forced to lower its standards.

For insight into the last part of Theorem 7, we show that higher costs help the top college in the special case of a sorting equilibrium. When the application cost at college 1 rises, its weakest applicants — for whom at college 1 was a stretch school — will now pass on this gamble, and apply to college 2 only. With this tougher student self-screening process, the quality of the applicant pool and of the enrolled students at college 1 rises.

More surprisingly, the better college also benefits from higher costs at the worse one. Since college 2 attracts fewer applicants, it must drop its admissions threshold to fill its capacity. It is now easier to gain admission into college 2, and the marginal benefit of a stretch application to college 1 falls. Its weakest applicants drop out, and thus the caliber distribution of its applicant pool and of the enrolled students at college 1 rises.

11 The Spillover Effects of Affirmative Action

We now illustrate the power and flexibility of our framework, and address the topical issue of affirmative action in college admissions. Specifically, we explore the equilibrium spillover effects of affirmative action preferences of one college on the other. Enriching the model, we assume that a fraction \( \mu \) of the applicant pool belongs to an under-represented minority; other students are in the majority. We assume a common caliber distribution, so that there is no other reason for differential treatment of the applicants.

Assume that students honestly report their minority status on their applications. Moreover, assume that students from both groups use monotone application strategies. Reflecting the colleges’ desire of a more diverse student body, let college \( i \) earn a bonus \( \pi_i \geq 0 \) for each enrolled minority student. Since race is observable, the colleges may set different thresholds for the two groups. Let the respective standards for majority and minority groups be \((\sigma_1, \sigma_2)\) and \((\sigma_1 - \Delta_1, \sigma_2 - \Delta_2)\). In other words, \( \Delta_i \) acts like
a “discount” accorded minority students. At each college, the expected payoff of the marginal admits from the two groups should coincide. This gives us two new equilibrium conditions, that account for the fact that ex post, colleges will behave rationally, and equate their expected values of under-represented and majority applicants.

\[
E[X + \pi_1 | \sigma = \sigma_1 - \Delta_1, \text{minority}] = E[X | \sigma_1, \text{majority}] \quad (8)
\]

\[
E[X + \pi_2 | \sigma = \sigma_2 - \Delta_2, \text{minority, accepts}] = E[X | \sigma_2, \text{majority, accepts}] \quad (9)
\]

where \(X\) is the random student caliber. So as with third degree price discrimination, colleges equate the shadow cost of capacity across groups. Along with market clearing (6) at each college, equilibrium requires solving four equations in four unknowns.

For any pair of discounts \((\Delta_1, \Delta_2)\) given to minority students at colleges 1 and 2, we let \((\sigma_1(\Delta_1, \Delta_2), \sigma_2(\Delta_1, \Delta_2))\) be admission standards for majority students that fill the capacity at both colleges. That is, the pair \((\sigma_1(\Delta_1, \Delta_2), \sigma_2(\Delta_1, \Delta_2))\) solves the capacity equations (6). Moreover, we assume that such a solution is stable. Then let \(V_i(\Delta_1, \Delta_2, \pi_i)\) be the shadow value difference in the LHS and RHS of (8)–(9), evaluated at capacity-filling standards \((\sigma_1, \sigma_2)\). An equilibrium is then a zero \(V_1(\Delta_1, \Delta_2, \pi_1) = V_2(\Delta_1, \Delta_2, \pi_2) = 0\). In particular, if \(\pi_1 = \pi_2 = 0\), then \(\Delta_1 = \Delta_2 = 0\) is an equilibrium.

Let us now define two new college best response functions. Let \(\Delta_i = \Upsilon_i(\Delta_j, \pi_i)\) when \(V_i(\Delta_1, \Delta_2, \pi_i) = 0\). Then an equilibrium is a crossing point of \(\Upsilon_1, \Upsilon_2\) in \((\Delta_1, \Delta_2)\)-space.

Our goal is to analyze the equilibrium comparative statics with respect to \(\pi_i, i = 1, 2\). The sign of these changes hinges upon the sign of the derivatives of \(V_i\) with respect to \(\Delta_j, i,j = 1, 2\), which are ambiguous in general. To see the difficulty, consider for example the effect on the shadow value difference \(V_i\) of an increment in the discount \(\Delta_1\) at college 1. The immediate effect is that \(V_i\) falls, as minorities meet a lower standard—fixing the majority standards. But there are two feedback sources as well, since the college standards adjust to satisfy the capacity constraint. In the appendix, we prove that the indirect effect on the shadow value difference \(V_i\) from \((i)\) changes in the standard \(\sigma_j\) owing to the discount \(\Delta_i\), or \((ii)\) changes in the standard \(\sigma_i\) attributed to \(\Delta_j\), are negligible locally around the no affirmative action point \(\Delta_1 = \Delta_2 = 0\).

---

8Here, \(\Delta_i\) can be interpreted as bonus points given to minority applicants, as in the old undergraduate admissions policy of the University of Michigan, struck down by the Supreme Court in \textit{Gratz v Bollinger}.

9The only exception is at corner solutions (e.g., when a college admits all students from a group).

10We know from Section 8 that there could be multiple solutions. The existence of a stable solution follows from a simple modification of the proof of Theorem 3.

11For this result alone, we also assume that the signal cdf derivative \(G_x\) is continuous.
Figure 6: Shadow Value Stable Equilibrium. The left panel illustrates a shadow value unstable equilibrium, which happens when the best response $Y_2$ slopes upward. A necessary condition for shadow value stability is that $\partial V_2 / \partial \Delta_2$ be negative, thus ensuring that $Y_2$ slopes downward. The right panel depicts a shadow value stable equilibrium.

we now ignore these two feedback effects in computing the total derivatives of in $\Delta_1, \Delta_2$.

First, $dV_1/d\Delta_1 < 0$ since lowering standards for minority students at college 1 not only depresses their average caliber via the standards effect, but also encourages worse minority applicants to apply — i.e. the portfolio effect reinforces this. Also, to fill capacity, the majority student standard must rise at college 1; their quality rises due to the portfolio and standards effects. Next, $dV_1/d\Delta_2 > 0$ solely via the portfolio effect: The worst minority applicants at college 1 now just apply to college 2, and the majority applicant pool at college 1 worsens (i.e., its marginal applicant’s caliber is lower) as the standard for majority applicants increase at college 2. Thus, $\Delta_1 = Y_1(\Delta_2, \pi_1)$ slopes up.

Next, $dV_2/d\Delta_1 < 0$ since the portfolio and standards effects reinforce: The best minority applicants at college 2 now only apply to college 1, and the remaining top tier of minority applicants gain admission to college 1 more often. The pool of majority applicants at college 2 improves since their admission standard rises to meet the capacity.

The sign of $dV_2/d\Delta_2$ is unclear. The standards effect is negative, but the portfolio effect is ambiguous: Its minority applicant pool clearly expands at the lower and upper ends. We now resolve this indeterminacy. Assume that if $V_i(\Delta_1, \Delta_2, \pi_i) > 0$, so that the shadow value of a minority student exceeds that of a majority student, then college $i$ responds by raising the minority advantage $\Delta_i$. If the equilibrium is stable under this adjustment process, then we call it shadow value stable. Suppose, for a contradiction,
Figure 7: **Affirmative Action Comparative Statics.** The left panel depicts a right shift of the best response discount curve $\Upsilon_1$ as the minority preference $\pi_1$ increases. The equilibrium shifts to $E_1$, with a higher discount $\Delta_1$ and a lower $\Delta_2$. This is justified for small minority preferences in Theorem 8. The right panel depicts how the best response $\Upsilon_2$ shifts up in $\pi_2$, increasing both equilibrium discounts $\Delta_1$ and $\Delta_2$.

that $dV_2/d\Delta_2 > 0$. Fix $\Delta_1$. Then whenever $\Delta_2 > \Upsilon_2(\Delta_1, \pi_2)$, we must have $V_2 > 0$, and thus the adjustment process leads to an even higher $\Delta_2$. So we must have $dV_2/d\Delta_2 < 0$ at a shadow value stable equilibrium, and the schedule $\Delta_2 = \Upsilon_2(\Delta_1, \pi_2)$ slopes down.

We are now ready to state the following result, whose proof is depicted graphically.\footnote{Such an equilibrium easily exists when $c = 0$, and by continuity when it is small enough.}

**Theorem 8 (Affirmative Action)** Fix $\pi_1 = \pi_2 = 0$. Assume that $\Delta_1 = \Delta_2 = 0$ is a shadow value stable equilibrium with monotone student behavior. As the preference for minority students at College 1 rises, it favors them and College 2 penalizes them. As the preference for minority students at College 2 rises, both colleges favor them more.

We should highlight here the surprising result that affirmative action at college 1 induces college 2 to penalize the minority. This is due to two effects. First, any students enrollment at college 2 is worse news for the minority, since it implies that she was rejected by the better college despite being favored. Second, there is a portfolio effect, since the best minority students that previously applied to college 2 now just apply to college 1, and so the pool of minority applicants at college 2 worsens.
12 Early Admissions

In a final illustration of the equilibrium model of the paper, we show how to derive some new insights into early admissions for students. We will uncover a close parallel with the analysis of affirmative action. We assume that students can apply early and acquire an option at a college, or regular. To stay focused, we analyze “early action” rather than early decision, so that early admission is non-binding. Moreover, we assume that rejection at a college is final, so that applicants are not “deferred”. Additionally, we assume that only one college among \( i = 1, 2 \) has an early admissions policy. In this way, we can shed light on some but not most competition between colleges. As in the analysis of affirmative action, college \( i \) may wish to discriminate between early and regular applicants. Specifically, we assume an early standard \( \sigma_i - \Delta_i \) and a regular standard \( \sigma_i \), where the early “discount” \( \Delta_i \) may well be negative.

For our first insight, we consider the student application problem, which is now more difficult. Despite the additional complexity, we offer an easy insight for optimal student behavior. Loosely, other things equal, applying early is better. Formally,

**Lemma 3** If college \( i \) offers early admissions, and does not penalize early applicants (so \( \Delta_i \geq 0 \)), then an early application to it is weakly preferable to a regular application.

Indeed, this is clear if just one regular application is planned. If one will apply to both colleges, then the cost of applying early or regular is the same; the benefits of an early application at college \( i \) are higher, since a regular application to college \( i \) is less beneficial in the event that one is also accepted at college \( j \). Intuitively, if a student is accepted early, he may not need to send an “insurance application” to the other school (accepted early at college 1); or may decline to “gamble up” (accepted early at college 2).

The equilibrium analysis requires an additional optimality condition upon the college offering early admission. If it admits students in both early and regular periods, then the expected caliber of the marginal enrollee in the two periods must coincide. In this way, we draw inspiration from the affirmative action section. By the logic of (8) and (9):

\[
E[X|\sigma = \sigma_i - \Delta_i, \text{applies early, accepts}] = E[X|\sigma = \sigma_i, \text{applies regular, accepts}] \tag{10}
\]

Namely, the shadow values of early and late applicants are equalized. To proceed, we first

\[13\] This slightly strengthens a result in Chade and Smith (2006) which implies that sequential dominates simultaneous applications when admissions chances are the same.
suppose that a representative fraction \( \mu \in (0, 1) \) of applicants has a “coupon” that allows them to apply early (though they may choose not to); thus, there always remain regular applicants. Since \( \mu \) can be near 1, this assumption is very weak. We assume monotone student behavior in the model with \( \mu = 0 \), i.e. without an early admission option.

**Theorem 9**  
*If college 1 has an early admissions program, it penalizes early applicants.*  
The argument is instructive. Suppose instead that it weakly favors them. By Lemma 3, the students that apply early consists of all those who would apply in the regular period, plus some lower caliber students that are induced to apply early. This is necessarily a lower set of students, with a lower shadow value, than in the regular period — namely, those without coupons. This violates the required optimality condition (10). So college 1 must penalize early applicants. Intuitively, if the early standards are weakly lower, then portfolio and standards effects both depress the shadow value of early applicants.

We next turn to the more interesting case when it is college 2 that offers the early admission program. We show that this enables it to “poach” students from college 1 — namely, some students who would have applied to both colleges regular, will only apply to college 2 early if they have a coupon, and will forego an extra application to college 1. To see this, suppose that college 2 favors early applicants, with a discount \( \Delta_2 \geq 0 \). Since any student then enjoys a weakly greater admission chance early than regular, he will apply early by Lemma 3. Also, of the students admitted at college 2, some will then drop their regular application to college 1. Recalling the threshold (2), this holds for students whose regular admission chances \( \alpha_{1R}, \alpha_{2R} \) satisfy

\[
\alpha_{1R}(x)(1 - u) < c < \alpha_{1R}(1 - \alpha_{2R}u)
\]

This suggested behavior — depicted in figure 8 — obtains in some equilibrium:

**Theorem 10**  
*There exists an equilibrium in which college 2 favors early applicants, and thus in which it poaches students from college 1.*

### 13 Concluding Remarks

We have reframed the college admissions problem in terms of supply and demand in which admissions standards act as prices. Our key assumptions have been costly applications and noisy evaluations. We have related student admission chances at two
Figure 8: Capturing Students with Early Action. The left and right panels depict the optimal student strategy regions under regular and early action programs, respectively. The key difference is the set of types on the solid (red) part of the acceptance function in the right panel: Students without coupons apply to both colleges and accept college 1 if accepted, but those with coupons apply early to college 2 and don’t bother to apply to college 1 if accepted. College 2 successfully pre-empts college 1, and poaches the students.

ranked colleges, and then built a graphical apparatus for analyzing student and college behavior. In this setting, college enrollment obeys the law of demand because a tougher admission standard not only cuts the fraction of students who meet it ("the standards effect"), but also reduces the number of students applying to it ("the portfolio effect").

We have shown that these realistic frictions may preclude college-student sorting in two ways: First, better students need not apply to better schools — if the worse college is either good enough or small enough, or the application process is noisy enough, a student may gamble on college 1 while more talented students do not. Second, college admissions standards needn’t reflect their quality — the worse college may set higher standards if it is small enough. In other words, we cannot reliably infer the quality of the school from its admissions standards without accounting for its size. This should be of concern to publications that use SAT scores of enrolled students in ranking schools.

We have also explored how the applications frictions induce interdependencies among the colleges: For instance, shifts in the capacity or application costs at one school affect the admission standard and student body composition at both. In particular, the better college should lower its admission standard when its lesser rival raises its capacity.

Finally, a natural modification of our model sheds new light on some topical issues. For affirmative action, we find that when the better college slightly favors a minority
group, the worse college penalizes it. Next, we explore early admission, finding that encourages aggressive applications by marginal students, and leads the better college to penalize early applicants. Conversely, the lesser college might use early admissions to poach students from its rival. Tractability has forced us to assume that just one college offers early admissions, which captures some cases, but not most in the USA. The richer problem with both colleges offering early admissions is an important but open problem.

A Appendix: Proofs

A.1 Students’ Uncertainty about their Own Calibers

We have assumed that students know their caliber. We now prove that all the results except in \(\text{§12}\) obtain if they only see a noisy signal of their caliber.\footnote{In \(\text{§12}\) a new twist emerges, since students are afforded an opportunity to learn their true type. Rejection by college 1 may lead a student to downgrade his personal self-estimate and decline to apply to college 1; likewise, acceptance by college 2 might encourage a regular application to college 1.} We assume a density \(p(t)\) of types on \([t, \bar{t}]\). A student does not know \(t\) but only a signal \(X\) with density \(f(x|t)\). Similarly, a college sees a signal \(\sigma\) (conditionally independent from the student’s signal) from an applicant with density \(\gamma(\sigma|t)\). Both \(f(x|t)\) and \(\gamma(\sigma|t)\) satisfy MLRP — each is log-supermodular.\footnote{A positive function \(f(a, b)\) is strictly log-supermodular if \(f(a', b')f(a, b) > f(a, b')f(a', b)\) for all \(a' > a\) and \(b' > b\). If \(f\) is twice differentiable, then this is equivalent to \(f_{ab}f > f_a f_b\). This is the strict MLRP property for signal densities. It is well-known that this is preserved under partial integration.}

So the associated cdf \(\Gamma(\sigma|t)\) is also log-supermodular.

Let \(p(t|x)\) be the posterior belief of a student who observed \(x\), and suppose that each college \(i = 1, 2\) sets a threshold \(\sigma_i\). Then

\[
\alpha_i(x) = \int_t^{\bar{t}} [1 - \Gamma(\sigma_i|t)]p(t|x)dt = 1 - \int_t^{\bar{t}} \Gamma(\sigma_i|t)p(t|x)dt.
\]

Define \(G(\sigma|x) \equiv 1 - \int_t^{\bar{t}} \Gamma(\sigma|t)p(t|x)dt\), so that \(\alpha_i(x) = 1 - G(\sigma_i|x)\). Then (i) \(G\) is a cdf as a function of \(\sigma\); (ii) \(G\) is decreasing in \(x\); and (iii) \(1 - G(\sigma|x)\) is log-supermodular in \((\sigma, x)\) (as the integral of a product of log-supermodular functions). So results continue by reinterpreting all statements made about calibers as referring to signals of their caliber.

Lemma 4 now states that the cdf of accepted students’ signals at college 1 dominates that of college 2 in the sense of first-order stochastic dominance. Since (i) the set of applicants (based on their signals) at college 1 is higher than that at college 2 in the
strong set order, and (ii) the cdf \( P(t|x) = \int_t^1 p(s|x)ds \) is decreasing in \( x \), the cdf of accepted calibers at college 1 also dominates that of college 2.

A.2 Colleges Optimally Employ Admissions Thresholds

Let \( \chi_i(\sigma) \) be the expected value of the student’s caliber given that he applies to college \( i \), his signal is \( \sigma \), and he accepts. College \( i \) optimally employs a threshold rule if, and only if, \( \chi_i(\sigma) \) increases in \( \sigma \). For college 1 this is immediate, since \( g(\sigma|x) \) enjoys the MLRP property. We prove this for college 2, since it faces an acceptance curse. We assume that students of calibers in set \( C_i \) apply to college \( i \) only, and in \( B \) apply to both colleges.\(^{16}\)

\[
\chi_2(\sigma) = \frac{\int_{C_2} xg(\sigma|x)f(x)dx + \int_B xG(\sigma_1|x)g(\sigma|x)f(x)dx}{\int_{C_2} g(\sigma|x)f(x)dx + \int_B G(\sigma_1|x)g(\sigma|x)f(x)dx}
\]

(12)

It is easy to show that \( \chi_2(\sigma) \) is less that the expectation without the cdf’s \( G \) — because being accepted by a student reduces college 2’s estimate of his caliber, as there is a positive probability that the student was rejected by college 1; i.e., college 2 suffers an acceptance curse effect. Write (12) as \( \chi_2(\sigma) = \int_{B \cup C_2} xh_2(x|\sigma)dx \) using indicator function notation:

\[
h_2(x|\sigma) = \frac{(I_{C_2}(x) + I_B G(\sigma_1|x))g(\sigma|x)f(x)}{\int_{B \cup C_2} (I_{C_2}(t) + I_B G(\sigma_1|t))g(\sigma|t)f(t)dt},
\]

(13)

Then the ‘density’ \( h_2(x|\sigma) \) has the MLRP. Therefore, \( \chi_2(\sigma) \) increases in \( \sigma \).

A.3 Acceptance Function and Signals: Proof of Lemma \( \Box \)

Since \( G(\sigma_1|x) \) is continuously differentiable in \( x \), the acceptance function is continuously differentiable on \((0,1]\). Given \( \alpha \equiv 1 - G(\sigma|\xi(\alpha,\sigma)) \), partial derivatives have positive slopes \( \xi_\alpha, \xi_\sigma > 0 \). Differentiating (3),

\[
\frac{\partial \psi}{\partial \alpha_1} = -G_x(\sigma_2|\xi(\alpha_1,\sigma_1))\xi_\alpha(\alpha_1,\sigma_1) > 0 \\
\frac{\partial \psi}{\partial \sigma_1} = -G_x(\sigma_2|\xi(\alpha_1,\sigma_1))\xi_\sigma(\alpha_1,\sigma_1) > 0 \\
\frac{\partial \psi}{\partial \sigma_2} = -g(\sigma_2|\xi(\alpha_1,\sigma_1)) < 0
\]

\(^{16}\)We assume that students employ pure strategies, which follows from our analysis of the student optimization in \( \Box \). Measurability of sets \( B \) and \( C_2 \) owe to the continuity of our functions \( \alpha_i(x) \) in \( \Box \).
we obtain $x$ is strictly increasing in $h$ by the first secant property of $h$

Properties of the cdf $G$ imply $\psi(0, \underline{s}, \underline{z}_2) \geq 0$ and $\psi(1, \underline{s}, \underline{z}_2) = 1$. The limits of $\psi$ as thresholds approach the supremum and infimum owe to limit properties of $G$.

A.4 Acceptance Function Shape: Proof of Theorem 1

$(\Rightarrow)$ The acceptance function is regular. First, $G(\sigma|x)$ and $1 - G(\sigma|x)$ are strictly log-supermodular in $(\sigma, x)$ since the density $g(\sigma|x)$ obeys the strict MLRP. Since $x = \xi(\alpha_1, \underline{s})$ is strictly increasing, $G(s|\xi(\alpha_1, \underline{s}))$ and $1 - G(s|\xi(\alpha_1, \underline{s}))$ are then strictly log-supermodular in $(s, \alpha_1)$. So the secant slopes below strictly fall in $\alpha_1$, since $\underline{s}_1 > \underline{s}_2$:

$$\frac{\psi(\alpha_1)}{\alpha_1} = \frac{1 - G(\underline{s}_2|\xi(\alpha_1))}{1 - G(\underline{s}_1|\xi(\alpha_1))}$$

and

$$1 - \psi(\alpha_1) = \frac{G(\underline{s}_2|\xi(\alpha_1))}{G(\underline{s}_1|\xi(\alpha_1))}$$

$(\Leftarrow)$ Deriving a signal distribution. Conversely, fix a regular function $h$ and a smoothly monotone onto function $\alpha_1(x)$. Also, put $\alpha_2(x) = h(\alpha_1(x))$, so that $\alpha_2(x) > \alpha_1(x)$. We must find a continuous signal density $g(\sigma|x)$ with the strict MLRP and thresholds $\sigma_1 > \sigma_2$ that rationalizes the $h$ as the acceptance function consistent with these thresholds and signal distribution.

**Step 1: A discrete signal distribution.** Consider a discrete distribution with realizations in $\{-1, 0, 1\}$: $g_1(x) = \alpha_1(x)$, $g_0(x) = \alpha_2(x) - \alpha_1(x)$ and $g_{-1}(x) = 1 - \alpha_2(x)$. Indeed, for each caliber $x$, $g_i \geq 0$ and sum to 1. This obeys the strict MLRP because

$$\frac{g_0(x)}{g_1(x)} = \frac{\alpha_2(x) - \alpha_1(x)}{\alpha_1(x)} = \frac{h(\alpha_1(x))}{\alpha_1(x)} - 1$$

is strictly decreasing by the first secant property of $h$, and

$$\frac{g_0(x)}{g_{-1}(x)} = \frac{\alpha_2(x) - \alpha_1(x)}{1 - \alpha_2(x)} = -1 + \frac{1 - \alpha_1(x)}{1 - h(\alpha_1(x))}$$

is strictly increasing in $x$ by the second secant property of $h$.

Let the college thresholds be $(\underline{s}_1, \underline{s}_2) = (0.5, -0.5)$. Then $G(\underline{s}_1|x) = g_{-1}(x) + g_0(x) = 1 - \alpha_1(x)$ and $G(\underline{s}_2|x) = g_{-1}(x) = 1 - \alpha_2(x)$. Rearranging yields $\alpha_1(x) = 1 - G(\underline{s}_1|x)$ and $\alpha_2(x) = 1 - G(\underline{s}_2|x)$. Inverting $\alpha_1(x)$ and recalling that $\alpha_2 = h(\alpha_1)$, we obtain $\alpha_2 = h(\alpha_1) = 1 - G(\underline{s}_2|\xi(\underline{s}_1, \alpha_1))$, thereby showing that $h$ is the acceptance function consistent with this signal distribution and thresholds.

**Step 2: A continuous signal density.** To create an atomless signal distribution, we smooth this example using the triangular kernel $k(s) = \max\{1 - s, 0\}$. 

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Fix $\beta > 0$, and define $g(\sigma|x) = \beta \sum_{i=\{-1,0,1\}} g_i(x) k(\beta(\sigma - i))$. For any “bandwidth” $1/\beta \in (0,1/2)$, acceptance chances remain the same as with the discrete signals, since the masses at $\{-1,0,1\}$ is not transferred past the (respective) thresholds $\{0.5,-0.5\}$.

The strict MLRP implies that $g_i(x)$ is strictly log-supermodular in $(i,x)$. Also, the function $k(s)$ is concave in $s$, and thus log-concave in $s$ too. This implies that $k(\beta(\sigma - i))$ is log-supermodular in $(i,\sigma)$. This is in §1.5 in Karlin (1968), but for a self-contained treatment, let’s assume twice differentiability: Then $k_{\sigma i} k - k_\sigma k_i > 0$ iff $-k'' k + k' k' > 0$, which holds iff $k$ is log concave. Thus, $g_i(x) k(\beta(\sigma - i))$ is log-supermodular in $(i,x,\sigma)$. Finally, partially summing over $i = 1,2,3$ yields a log-supermodular function of $(x,\sigma)$, by Proposition 3.2 in Karlin and Rinott (1980) — the MRLP property.

\[ \Box \]

\section{A.5 Monotone Student Strategies: Proof of Lemma 2}

The proof proceeds as follows. First, we show that $u \leq 0.5$ implies that if a caliber applies to college 1, any higher caliber applies as well. Second, we produce a sufficient condition that ensures that the admissions threshold at college 2 is sufficiently lower than that of college 1, so that if a caliber applies to college 2, then any lower caliber who applies to college sends an application to college 2, and calibers at the lower tail apply nowhere. From these two results, monotone student behavior ensues.

\textbf{Proof of Part (a), Step 1.} We first show that the acceptance function $\alpha_2 = \psi(\alpha_1)$ crosses $\alpha_2 = 1/(1-c/\alpha_1)$ (i.e., $MB_{12} \equiv \alpha_1(1-\alpha_2 u) = c$) only once when $u \leq 0.5$. Since (i) the acceptance function starts at $\alpha_1 = 0$ and ends at $\alpha_1 = 1$, (ii) $MB_{12} = c$ starts at $\alpha_1 = c$ and ends at $\alpha_1 = c/(1-u)$, and (iii) both functions are continuous, there exists a crossing point. Clearly, they intersect when $\alpha_1(1 - \psi(\alpha_1) u) = c$. Now,

\[ [(1-\psi(\alpha_1) u)\alpha_1]' = 1 - u\psi(\alpha_1) - \alpha_1 u\psi'(\alpha_1) > 1 - u\psi(\alpha_1) - u\psi(\alpha_1) = 1 - 2u\psi(\alpha_1) \geq 1 - 2u \geq 0, \]

where the first inequality exploits $\psi(\alpha_1)/\alpha_1$ falling in $\alpha_1$ (Lemma 1), i.e. $\psi'(\alpha_1) < \psi(\alpha_1)/\alpha_1$; the next two inequalities use $\psi(\alpha_1) \leq 1$ and $u \leq 0.5$. Since $MB_{12}$ is rising in $\alpha_1$ when the acceptance relation hits $\alpha_2 = (1-c/\alpha_1)/u$, the intersection is unique.

\textbf{Proof of Part (a), Step 2.} We now show that Step 1 implies the following single crossing property in terms of $x$: if caliber $x$ applies to college 1 (i.e., if $1 \in S(x)$), then any caliber $y > x$ also applies to college 1 (i.e., $1 \in S(y)$). Suppose not; i.e., assume that either $S(y) = \emptyset$ or $S(y) = \{2\}$. If $S(y) = \emptyset$, then $S(x) = \emptyset$ as well, as $\alpha_1(x) < \alpha_1(y)$ and $\alpha_2(x) < \alpha_2(y)$, contradicting the hypothesis that $1 \in S(x)$. If
\( S(y) = \{2\} \), then there are two cases: \( S(x) = \{1\} \) or \( S(x) = \{1, 2\} \). The first cannot occur, for by Lemma \( \alpha_2(x)/\alpha_1(x) > \alpha_2(y)/\alpha_1(y) \), and thus \( \alpha_2(y)u \geq \alpha_1(y) \) implies \( \alpha_2(x)u > \alpha_1(x) \), contradicting \( S(x) = \{1\} \). In turn, the second case is ruled out by the monotonicity of \( MB_{12} \) derived above, as caliber \( y \) has greater incentives than \( x \) to add college 1 to its portfolio, and thus \( S(y) = \{2\} \) cannot be optimal.

**Proof of Part (b), Step 1.** We first show that if the acceptance function passes above the point \((\bar{\alpha}_1, \bar{\alpha}_2) = \left( (1 - \sqrt{1 - 4c/\ell})/2, (1 - \sqrt{1 - 4c/\ell})/2 \right) \) — point \( P \) in the right panel of Figure 3 — then there is a unique crossing of the acceptance function and \( y < x \) for all \( \alpha_2 = c/u(1 - \alpha_1) \), i.e. \( MB_{21} = c \). Now, the acceptance function passes above \((\bar{\alpha}_1, \bar{\alpha}_2)\) if

\[
\psi(\bar{\alpha}_1, \underline{\alpha}_1, \underline{\alpha}_2) \geq \bar{\alpha}_2. \tag{14}
\]

This condition relates \( \underline{\alpha}_1 \) and \( \underline{\alpha}_2 \). Rewrite (14) using Lemma 1 as \( \underline{\alpha}_2 \leq \eta(\underline{\alpha}_1) < \underline{\alpha}_1 \), requiring a large enough “wedge” between the standards of the two colleges.

To show that (14) implies a unique crossing, consider the secant of \( \alpha_2 = c/u(1 - \alpha_1) \) (the curve \( MB_{21} = c \)). It has an increasing secant if and only if \( \alpha_1 \geq 1/2 \). To see this, differentiate \( \alpha_2/\alpha_1 = c/u\alpha_1(1 - \alpha_1) \) in \( \alpha_1 \). Notice also that \( MB_{12} = c \) intersects the diagonal \( \alpha_2 = \alpha_1 \) at the points \((\alpha_1^1, \alpha_2^1) = (1/2 - \sqrt{1 - c/4u}/2, 1/2u - \sqrt{1 - c/4u}/2u) \) and \((\alpha_1^2, \alpha_2^2) = (1/2 + \sqrt{1 - c/4u}/2, 1/2u + \sqrt{1 - c/4u}/2u) > (1/2, 1/2u) \).

Condition (14) gives \( \psi(\alpha_1^1, \underline{\alpha}_1, \underline{\alpha}_2) > \alpha_2^1 \). Since \( \underline{\alpha}_2 < \underline{\alpha}_1 \), we have \( \psi(\alpha_1, \underline{\alpha}_1, \underline{\alpha}_2) \geq \alpha_2 \) for all \( \alpha_1 \). Thus, the acceptance function crosses \( MB_{21} = c \) at or above \((\alpha_1^1, \alpha_2^1) \). And since \( \alpha_1^1 > 1/2 \), the secant of \( MB_{21} = c \) must be increasing at any intersection with the acceptance function. Hence, there must be a single crossing point.

**Proof of Part (b), Step 2.** We now show that this single crossing property in \( \alpha \) implies another in \( x \): If caliber \( x \) applies to college 2 (i.e., if \( 2 \in S(x) \)), then any caliber \( y < x \) that applies somewhere also applies to college 2 (i.e., \( 2 \in S(y) \) if \( S(y) \neq \emptyset \)). Suppose not; i.e., assume that \( S(y) = \{1\} \). Then there are two cases: \( S(x) = \{2\} \) or \( S(x) = \{1, 2\} \). The first cannot occur, for by Lemma \( \alpha_2(x)/\alpha_1(x) < \alpha_2(y)/\alpha_1(y) \), and thus \( \alpha_2(x)u \geq \alpha_1(x) \) implies \( \alpha_2(y)u > \alpha_1(y) \), contradicting \( S(x) = \{2\} \). The second case is ruled out by the monotonicity of \( MB_{21} \) given condition (14), as caliber \( y \) has greater incentives than \( x \) to apply to college 2, and thus \( S(y) = \{1\} \) cannot be optimal. Finally, \( S(y) = \emptyset \) if \( \alpha_2(y)u < c \) by (14), which happens for low calibers below a threshold. \( \square \)
Figure 9: Equilibrium Existence. In the left panel, since $\kappa_1 > \bar{\kappa}_1(\kappa_2)$, the best response functions $\Sigma_1$ and $\Sigma_2$ do not intersect, and equilibrium is at $E$ with $\sigma_2 = 0$. The right panel depicts the proof of Theorem 3 for the case $\kappa_1 < \bar{\kappa}_1(\kappa_2)$.

A.6 The Law of Demand: Proof of Theorem 2

We prove that the applicant pool shrinks at college 1 and grows at college 2 if $\sigma_1$ rises.

**Step 1:** The applicant pool at college 1 Shrinks. When $\sigma_1$ increases, the acceptance relation shifts up by Theorem 2, and thus the above type sets change as well. Fix a caliber $x \in C_2$ or $x \in \Phi$, so that $1 \notin S(x)$.\(^{17}\) We will show that $x$ continues to apply either to college 2 only or nowhere, and thus the pool of applicants at college 1 shrinks. If $x \in C_2$, then $\alpha_2(x)u - c \geq 0$ and $\alpha_2(x)u \geq \alpha_1(x)$, and this continues to hold after the increase in $\sigma_1$, since $\alpha(x)$ falls while $\alpha_2(x)$ is constant. And if $x \in \Phi$, then clearly caliber $x$ will continue to apply nowhere when $\sigma_1$ increases.

**Step 2:** The applicant pool at college 2 expands. Fix a caliber $x \in C_2$ or $x \in B$, so that $2 \in S(x)$. It suffices to show that caliber $x$ continues to apply to college 2 when the admission standard at college 1 increases. If $x \in C_2$, then $\alpha_2(x)u - c \geq 0$ and $\alpha_2(x)u \geq \alpha_1(x)$; these inequalities continue to hold after $\sigma_1$ rises, since $\alpha_1(x)$ falls while $\alpha_2(x)$ remains constant. And if $x \in B$, then $MB_{21} = (1 - \alpha_1(x))\alpha_2(x)u$ rises in $\sigma_1$, encouraging caliber $x$ to apply to college 2. Thus, $x \notin C_1 \cup \Phi$. Since $x$ was arbitrary, it follows that the applicant pool at college 2, $B \cup C_2$, expands when $\sigma_1$ increases. \(\square\)

A.7 Equilibrium Existence: Proof of Theorem 3

For definiteness, we now denote the infimum signal by $-\infty$, and the supremum signal by $\infty$. Fix any $\kappa_2 \in (0, 1)$, and let $\sigma_1^*(\kappa_2)$ be the unique solution to $\kappa_2 = \mathcal{E}_2(\sigma_1, -\infty)$.

\(^{17}\)With a slight abuse of notation, we let $\Phi$ denote the set of calibers that apply nowhere. The same symbol was previously used to denote the analogous set in $\alpha$-space.
i.e., when college 2 accepts everybody. (Existence and uniqueness of $\sigma^l_2(\kappa_2)$ follows from $E_2(-\infty,-\infty) = 0$, $E_2(\infty,-\infty) = 1$, and $E_2(\sigma_1,-\infty)$ increasing and continuous in $\sigma_1$.)

Define $\bar{\kappa}_1(\kappa_2) = E_1(\sigma^l_1(\kappa_2),-\infty)$. Let $\kappa_1 \geq \bar{\kappa}_1(\kappa_2)$. We claim that there exists an equilibrium in which college 2 accepts everybody, and college 1 sets a threshold $\sigma^l_1(\kappa_1)$, the unique solution to $\kappa_1 = E_1(\sigma_1,-\infty)$, which satisfies $\sigma^l_1(\kappa_1) \leq \sigma^l_1(\kappa_2)$. For since college 2 rejects no one, $\sigma^l_1(\kappa_1)$ fills college 1’s capacity exactly. The enrollment at college 2 is then $E_2(\sigma^l_1(\kappa_1),-\infty) \leq \kappa_2$ (as $\sigma^l_1(\kappa_1) \leq \sigma^l_1(\kappa_2)$ and $E_2(\sigma_1,\sigma_2)$ is increasing in $\sigma_1$), so by accepting everybody college 2 fills as much capacity as it can. This equilibrium is trivially stable, as $\Sigma$ is ‘flat’ at the crossing point (see Figure 9, left panel). Moreover, if $\kappa_1 > \bar{\kappa}_1(\kappa_2)$, then college 2 has excess capacity in this equilibrium.

Assume now $\kappa_1 < \bar{\kappa}_1(\kappa_2)$. We will show that the continuous functions $\Sigma_1$ and $\Sigma_2$ must cross at least once (i.e., an equilibrium exists), and that the slope condition is met (i.e., it is stable). First, in this case $\sigma^l_1(\kappa_2) < \sigma^l_1(\kappa_1)$ or, equivalently, $\Sigma^{-1}_2(-\infty,\kappa_2) < \Sigma_1(-\infty,\kappa_1)$. Second, as the standard of college 2 goes to infinity, college 1’s threshold converges to $\sigma^l_1(\kappa_1) < \infty$, the unique solution to $\kappa_1 = E_1(\sigma_1,\infty)$. This is the largest threshold that college 1 can set given $\kappa_1$. Similarly, as the standard of college 1 goes to infinity, college 2’s threshold converges to $\sigma^u_2(\kappa_2) < \infty$, the unique solution to $\kappa_2 = E_2(\infty,\sigma_2)$, i.e. the largest threshold that college 2 can set given $\kappa_2$. Third, for $\epsilon > 0$ small enough, the unique solution to $\kappa_1 = E_1(\sigma_1,\sigma^u_2(\kappa_2) - \epsilon)$ lies below the unique solution to $\kappa_2 = E_2(\sigma_1,\sigma^u_2(\kappa_2) - \epsilon)$. Equivalently, $\Sigma^{-1}_2(\sigma^u_2(\kappa_2) - \epsilon,\kappa_2) > \Sigma_1(\sigma^u_2(\kappa_2) - \epsilon,\kappa_1)$.

Since $\Sigma^{-1}_2(-\infty,\kappa_2) < \Sigma_1(-\infty,\kappa_1)$ and $\Sigma^{-1}_2(\sigma^u_2(\kappa_2) - \epsilon,\kappa_2) > \Sigma_1(\sigma^u_2(\kappa_2) - \epsilon,\kappa_1)$ (graphically, point A is to the left of point B in Figure 9), and $\Sigma_1$ and $\Sigma_2$ are continuous, by the Intermediate Value Theorem, they must cross at least once with the slope condition being satisfied (see Figure 9 right panel). Thus, a stable equilibrium exists when $\kappa_1 < \bar{\kappa}_1(\kappa_2)$. Moreover, in any equilibrium there is no excess capacity at either college, since $\Sigma^{-1}_2(-\infty,\kappa_2) < \Sigma_1(-\infty,\kappa_1)$.

Hence, a stable equilibrium exists for any $\kappa_2 \in (0,1)$. Capacities are exactly filled when $\kappa_1 \leq \bar{\kappa}_1(\kappa_2)$, while there can be excess capacity at college 2 whenever $\kappa_1 > \bar{\kappa}_1(\kappa_2)$.

Since $\kappa_2 = E_1(\sigma^l_1(\kappa_2),-\infty)$, $\bar{\kappa}_1(\kappa_2)$ equals $1 - \kappa_2$ plus the mass of students who only applied to, and were rejected by, college 1. This mass vanishes as $c$ vanishes, for then everybody applies to both colleges. So $\bar{\kappa}_1(\kappa_2)$ converges to $1 - \kappa_2$ as $c$ goes to zero. \hfill \Box
A.8 Sorting Equilibrium Implies Stochastic Dominance of Types

To justify our focus on this ex-ante notion, we observe that this induces ex-post sorting in enrollments: the best colleges stochastically get the best students.

**Lemma 4 (Sorting and the Caliber Distribution)** In a sorting equilibrium, the caliber distribution at college 1 first-order stochastically dominates that at college 2.

**Proof:** A monotone student strategy is represented by the partition of the set of types:

\[ \Phi = [0, \xi_2), C_2 = [\xi_2, \xi_B), B = [\xi_B, \xi_1), C_1 = [\xi_1, \infty) \] (15)

where \( \xi_2 < \xi_B < \xi_1 \) are defined by the intersection of the acceptance function with \( c/u \), \( \alpha_2 = (1-c/\alpha_1)/u \) (i.e., \( MB_{12} = c \)), and \( \alpha_2 = c/[u(1-\alpha_1)] \) (i.e., \( MB_{21} = c \)), respectively.

Fix \((\varrho_1, \varrho_2)\). Let \( f_1(x) \) and \( f_2(x) \) be the densities of calibers accepted at colleges 1 and 2, respectively. Formally,

\[
\begin{align*}
 f_1(x) &= \frac{\alpha_1(x)f(x)}{\int_{\xi_B}^{\infty} \alpha_1(t)f(t)dt} I_{[\xi_B, \infty)}(x) \quad (16) \\
 f_2(x) &= \frac{I_{[\xi_2, \xi_B]}(x)\alpha_2(x)f(x) + (1 - I_{[\xi_2, \xi_B]}(x))\alpha_2(x)(1 - \alpha_1(x))f(x)}{\int_{\xi_2}^{\xi_B} \alpha_2(s)f(s)ds + \int_{\xi_B}^{\xi_1} \alpha_2(s)(1 - \alpha_1(s))f(s)ds} I_{[\xi_2, \xi_1]}(x), \quad (17)
\end{align*}
\]

where \( I_A \) is the indicator function of the set \( A \).

We shall show that, if \( x_L, x_H \in [0, \infty) \), with \( x_H > x_L \), then \( f_1(x_H)f_2(x_L) \geq f_2(x_H)f_1(x_L) \); i.e., \( f_i(x) \) is log-supermodular in \((-i, x)\), or it satisfies MLRP. The result follows as MLRP implies that the cdfs are ordered by first-order stochastic dominance.

Using (16) and (17), \( f_1(x_H)f_2(x_L) \geq f_2(x_H)f_1(x_L) \) is equivalent to

\[
\begin{align*}
\alpha_1 I_{[\xi_B, \infty)}(x_H) \left( I_{[\xi_2, \xi_B]}(x_L)\alpha_2 + (1 - I_{[\xi_2, \xi_B]}(x_L))\alpha_2(1 - \alpha_1) \right) I_{[\xi_2, \xi_1]}(x_L) &\geq \\
\alpha_1 I_{[\xi_B, \infty)}(x_L) \left( I_{[\xi_2, \xi_B]}(x_H)\alpha_2 + (1 - I_{[\xi_2, \xi_B]}(x_H))\alpha_2(1 - \alpha_1) \right) I_{[\xi_2, \xi_1]}(x_H), \quad (18)
\end{align*}
\]

where \( \alpha_{ij} = \alpha_i(x_j) \), \( i = 1, 2 \), \( j = L, H \). It is easy to show that the only nontrivial case is when \( x_L, x_H \in [\xi_B, \xi_1] \) (in all the other cases, either both sides are zero, or only the right side is). If \( x_L, x_H \in [\xi_B, \xi_1] \), then (18) becomes \( \alpha_1 \alpha_2(1 - \alpha_1) \geq \alpha_1 \alpha_2(1 - \alpha_1) \), or

\[
(1 - G(\varrho_1 \mid x_H))(1 - G(\varrho_2 \mid x_L))G(\varrho_1 \mid x_L) \geq \\
(1 - G(\varrho_1 \mid x_L))(1 - G(\varrho_2 \mid x_H))G(\varrho_1 \mid x_H). \quad (19)
\]
Since \( g(\sigma \mid x) \) satisfies MLRP, it follows that \( G(\sigma \mid x) \) is decreasing in \( x \), and hence \( G(\sigma_1 \mid x_L) \geq G(\sigma_1 \mid x_H) \). Next, \( 1 - G(\sigma \mid x) \) is log-supermodular in \((x, \sigma)\), and hence
\[
(1 - G(\sigma_1 \mid x_H))(1 - G(\sigma_2 \mid x_L)) \geq (1 - G(\sigma_1 \mid x_L))(1 - G(\sigma_2 \mid x_H))
\]
as \( \sigma_1 > \sigma_2 \) in a sorting equilibrium. Thus, (19) is satisfied, thereby proving that \( f_i(x) \) is log-supermodular in \((-i, x)\), and so \( F_1 \) first-order stochastically dominates \( F_2 \). \( \square \)

A.9 Conditions for Sorting to Fail: Proof of Theorem 4

**Part (a): College 2 is Too Good.** Fix any \( \kappa_1, \kappa_2 \), such that \( \kappa_1 + \kappa_2 < 1 \). We shall proceed in two steps. First, we show that since \( u > 0.5 \), we can use Theorem 1 to construct a non-sorting equilibrium in which colleges’ behavior is monotone but students’ is not. Second, we show that all equilibria induce the same type of behavior.

**Step 1: Towards an Acceptance Function.** When \( u > 0.5 \), the secant from the origin to \( MB_{12} = c \) falls as \( \alpha_1 \) tends to \( c/(1-u) \) — as in the left panel of Figure 8. So for some \( \underline{c} < c/(1-u) \)s, a line from the origin to \((\underline{c}, 1)\) slices the \( MB_{12} \) curve twice. This would imply non-monotone student behavior if that line belonged to the acceptance function, such as: \( h: [0, 1] \rightarrow [0, 1] \) by \( h(\alpha) = \alpha/\underline{c} \) and on \([0, \underline{c}]\), and \( h(\alpha_1) = 1 \) for \( \alpha_1 \geq \underline{c} \).

**Step 2: A Piecewise-Linear Acceptance Chance \( \alpha_1 \).** Choose \( \xi \) and \( \bar{\xi} \) that uniquely solve \( \kappa_1 = \int_{\xi}^{\infty} f(x)dx \) and \( \kappa_2 = \int_{\xi}^{\bar{\xi}} f(x)dx \). Set \( \alpha_1(x) = 0 \) for \( x < \xi \). This function then jumps up to the rising line segment \( \alpha_1(x) = \omega(x)\underline{c} + (1 - \omega(x))c/(1-u) \) for \( x \in [\xi, \bar{\xi}] \), where \( \omega(x) \equiv (\bar{\xi} - x)/(\bar{\xi} - \xi) \). Lastly, \( \alpha_1 \) jumps up \( \alpha_1(x) = 1 \) for \( x > \bar{\xi} \).

**Step 3: Student Behavior.** Observe that \( h(0) = 0 \) and \( h(1) = 1 \), and that \( h \) is weakly increasing, with both \( h(\alpha)/\alpha \) and \( [1 - h(\alpha)]/[1 - \alpha] \) weakly decreasing. In this sense, \( h \) is a weakly regular function. This suggests that we set \( \alpha_2(x) \equiv h(\alpha_1(x)) \).

In this case, students \( x \in [0, \xi] \) are accepted with zero chance at either college, and so apply nowhere. Next, because \( h(\underline{c}) = 1 \), any calibers \( x \in [\xi, \bar{\xi}] \) are accepted with chance one at college 2, and with chance between \( \underline{c} \) and \( c/(1-u) \) at college 1. Further, any student \( \bar{\xi} \) strictly prefers just to apply to college 2 (as in Figure 8). To see this, observe that \( MB_{12} = (c/(1-u))(1 - \alpha_2u) > (c/(1-u))(1 - u) = c \) when \( \alpha_2 = c/(1-u) \) and \( \alpha_1 = 1 \). Lastly, calibers \( x > \bar{\xi} \) are always accepted at college 1 and only apply there.

**Step 4: Smoothing the Construction.** By smoothly bending the function \( h \) inside \((0, 1)\), an arbitrarily close function \( h^* \) is also regular. Next, we create a continuous
and smooth acceptance chance $\tilde{\alpha}$. Any four small enough numbers, $\varepsilon, \varepsilon, \bar{\varepsilon}, \bar{\varepsilon} > 0$, yield a unique Bezier approximation $\tilde{\alpha}$ tangent to $\alpha$ at the four points $\xi - \varepsilon, \xi + \varepsilon, \bar{\xi} - \bar{\varepsilon}, \bar{\xi} + \bar{\varepsilon}$. Then $\tilde{\alpha}_1$ — and so the enrollment at college 1 — falls in $\varepsilon$, and rises in $\bar{\varepsilon}$. Also, $\tilde{\alpha}_2 = h^*(\tilde{\alpha}_1)$ falls in $\varepsilon$, and rises in $\bar{\varepsilon}$, and it also falls in $\xi$, and rises in $\bar{\xi}$. Enrollment at college 2 shares this monotonicity, but the enrollment at college 1 is unaffected by $\varepsilon$ and $\varepsilon$.

Fix a small $\bar{\varepsilon} > 0$. Choose $\varepsilon > 0$ so that college 1 still fills its capacity. WLOG, enrollment at college 2 has fallen. Then choose $\varepsilon > 0$ large enough so that college 2 is over its capacity, then for some $\varepsilon > 0$, the former enrollment at college 2 is restored.

Theorem \[\text{4}\] now yields a signal density $g(\sigma|x)$ and thresholds $\sigma_1 > \sigma_2$ such that $h^*$ is the acceptance function. We have thus constructed a nonsorting equilibrium. \[\square\]

**Part (b): College 2 is Too Small.** The proof is constructive, exploiting our graphical analysis. To begin, consider the point $(\alpha_1, \alpha_2) = (c, c/u)$ on the line $\alpha_2 = \alpha_1/u$. Then the acceptance function evaluated at $\alpha_1 = c$ is below $c/u$ if and only if

$$\psi(c, \sigma_1, \sigma_2) < c/u. \quad (20)$$

We will restrict attention to pairs $(\sigma_1, \sigma_2)$ such that (20) holds. In this case, any student who applies to college starts by adding college 1 to his portfolio, and this happens as soon as $\alpha_1(x) \geq c$, or when $x \geq \xi(c, \sigma_1)$. Then enrollment at college 1 is given by

$$E_1(\sigma_1, \sigma_2) = \int_{\xi(c, \sigma_1)}^{\infty} (1 - G(\sigma_1|x)) f(x) dx,$$
which is independent of \( \sigma_2 \). Thus, for any capacity \( \kappa_1 \in (0, 1) \), a unique threshold
\( \sigma_1(\kappa_1) \) solves \( \kappa_1 = E_1(\sigma_1, \sigma_2) \). (The \( \Sigma_1^{-1} \) function is “vertical” when (20) holds, since
the applicant pool at college 1 does not depend on college 2’s admissions threshold.)

The analysis above allows us to restrict attention to finding equilibria within the set
of thresholds \( (\sigma_1, \sigma_2) \) such that \( \sigma_1 = \sigma_1(\kappa_1) \) and \( \sigma_2 \) satisfies \( \psi(c, \sigma_1(\kappa_1), \sigma_2) < c/u \).

Enrollment at college 2 is given by
\[
E_2(\sigma_1(\kappa_1), \sigma_2) = \int_B G(\sigma_1(\kappa_1)|x|(1 - G(\sigma_2|x))f(x)dx,
\]
which is continuous, decreasing in \( \sigma_2 \), and increasing in \( \sigma_1 \) (see Theorem 2). Thus,
\( \kappa_2 = E_2(\sigma_1(\kappa_1), \sigma_2) \) yields \( \sigma_2 = \Sigma_2(\sigma_1(\kappa_1), \kappa_2) \), which is strictly decreasing in \( \kappa_2 \).

Given \( \kappa_1 \), let \( \bar{\kappa}_2(\kappa_1) \) be the level of college 2 capacity so that
equilibrium ensues if both colleges set the same threshold.\(^{18}\) Since \( \Sigma_2 \) strictly falls in \( \kappa_2 \),
for any \( \kappa_2 < \bar{\kappa}_2(\kappa_1) \), an equilibrium exists with \( \sigma_2 > \sigma_1(\kappa_1) \). Then (a) for any \( \kappa_1 \in (0, 1) \)
and \( \kappa_2 \in (0, \bar{\kappa}_2(\kappa_1)] \), there is a unique equilibrium with \( \sigma_1 = \sigma_1(\kappa_1) \) and \( \sigma_2 \geq \sigma_1(\kappa_1) \),
having (b) non-monotone college and student behavior (Figure 10, left).\(^{19}\) \( \square \)

A.10 Conditions for Equilibrium Sorting: Proof of Theorem 5

Fix \( \kappa_2 \in (0, 1) \). We first show that the stable equilibrium with no excess capacity
derived in Theorem 3 is also sorting when the capacity of college 1 is small enough.
More precisely, there is a threshold \( \kappa_1(\kappa_2) \), smaller than the bound \( \bar{\kappa}_1(\kappa_2) \) defined in
the proof of Theorem 3, such that for all \( \kappa_1 \in (0, \kappa_1(\kappa_2)) \), there is a pair of admissions
thresholds \( (\sigma_1, \sigma_2) \) that satisfies \( \kappa_1 = E_1(\sigma_1, \sigma_2) \), \( \kappa_2 = E_2(\sigma_1, \sigma_2) \), and \( \sigma_2 < \eta(\sigma_1) \)
(i.e., a sorting equilibrium), and \( \partial \Sigma_1/\partial \sigma_2 \partial \Sigma_2/\partial \sigma_1 < 1 \) (i.e., the equilibrium is stable).

The proof uses three easily-verified properties of the function \( \eta \): (a) \( \eta \) is strictly
increasing; (b) \( \sigma_2 = \eta(\sigma_1) \to \infty \) as \( \sigma_1 \to \infty \); (c) \( \sigma_1 = \eta^{-1}(\sigma_2) \to -\infty \) as \( \sigma_2 \to -\infty \).

For any \( \kappa_1 \in (0, \bar{\kappa}_1(\kappa_2)) \), we know from Theorem 3 that there exists a pair \( (\sigma_1, \sigma_2) \)
that satisfies \( \kappa_1 = E_1(\sigma_1, \sigma_2) \) and \( \kappa_2 = E_2(\sigma_1, \sigma_2) \), with \( (\partial \Sigma_1/\partial \sigma_2)(\partial \Sigma_2/\partial \sigma_1) < 1 \).

Claim 1 The pair \( (\sigma_1, \sigma_2) \) is a sorting equilibrium when \( \kappa_1 \) is sufficiently small.

Proof: Let \( M(\kappa_2) = \{ (\sigma_1, \sigma_2) | \kappa_2 = E_2(\sigma_1, \sigma_2) \text{ and } \sigma_2 = \eta(\sigma_1) \} \). Graphically, this is
the set of all pairs at which \( \sigma_2 = \Sigma_2(\sigma_1, \kappa_2) \) crosses \( \sigma_2 = \eta(\sigma_1) \).

\(^{18}\)It is not difficult to show that \( \psi(c, \sigma_1, \sigma_2) < c/u \) is satisfied if \( \sigma_2 \geq \sigma_1(\kappa_1) \).

\(^{19}\)We are not ruling out the existence of another equilibrium that does not satisfy (20).
If $M(\kappa_2) = \emptyset$ we are done, for then $\mathfrak{g}_2 = \Sigma_2(\mathfrak{g}_1, \kappa_2) < \eta(\mathfrak{g}_1)$ for all $\mathfrak{g}_1$, including those at which $\kappa_1 = \mathcal{E}_1(\mathfrak{g}_1, \mathfrak{g}_2)$ and $\kappa_2 = E_2(\mathfrak{g}_1, \mathfrak{g}_2)$. To see this, note that (i) $\mathfrak{g}_1 = \eta^{-1}(\mathfrak{g}_2) \rightarrow -\infty$ as $\mathfrak{g}_2 \rightarrow -\infty$, while we proved in Theorem 3 that $\mathfrak{g}_1 = \Sigma_2^{-1}(\mathfrak{g}_2, \kappa_2)$ converges to $\mathfrak{g}_1^l(\kappa_2) > -\infty$. Also, (ii) $\mathfrak{g}_2 = \eta(\mathfrak{g}_1) \rightarrow \infty$ as $\mathfrak{g}_1 \rightarrow \infty$, while we proved in Theorem 3 that $\mathfrak{g}_2 = \Sigma_2(\mathfrak{g}_1, \kappa_2)$ converges to $\mathfrak{g}_2^u(\kappa_2) < \infty$. Properties (i) and (ii) reveal that if $\Sigma_2$ and $\eta$ do not intersect, then $\Sigma_2$ is everywhere below $\eta$.

If $M(\kappa_2) \neq \emptyset$, let $(\mathfrak{g}_1^s(\kappa_2), \mathfrak{g}_2^s(\kappa_2)) = \text{sup } M(\kappa_2)$, which is finite by property (b) of $\eta(\mathfrak{g}_1)$ and since $\mathfrak{g}_2 = \Sigma_2(\mathfrak{g}_1, \kappa_2)$ converges to $\mathfrak{g}_2^u(\kappa_2) < \infty$ as $\mathfrak{g}_1$ goes to infinity (see the proof of Theorem 3). Now, as $\kappa_1$ goes to zero, $\mathfrak{g}_1 = \Sigma_1(\mathfrak{g}_2, \kappa_1)$ goes to infinity for any value of $\mathfrak{g}_2$, for college 1 becomes increasingly more selective to fill its dwindling capacity. Since $\mathfrak{g}_2$ is bounded above by $\mathfrak{g}_2^u(\kappa_2)$, there exists a threshold $\kappa_1(\kappa_2) \leq \bar{\kappa}_1(\kappa_2)$ such that, for all $\kappa_1 \in (0, \bar{\kappa}_1(\kappa_2))$, the aforementioned pair $(\mathfrak{g}_1, \mathfrak{g}_2)$ that satisfies $\kappa_1 = \mathcal{E}_1(\mathfrak{g}_1, \mathfrak{g}_2)$ and $\kappa_2 = E_2(\mathfrak{g}_1, \mathfrak{g}_2)$ is strictly bigger than $(\mathfrak{g}_1^s(\kappa_2), \mathfrak{g}_2^s(\kappa_2))$, thereby showing that it also satisfies $\mathfrak{g}_2 < \eta(\mathfrak{g}_1)$. Hence, a sorting stable equilibrium exists for any $\kappa_2$ and $\kappa_1 \in (0, \bar{\kappa}_1(\kappa_2))$, with both colleges filling their capacities (see Figure 10 right panel).

To finish the proof, notice that, if there are multiple equilibria, both colleges fill their capacity in all of them (graphically, the conditions on capacities ensure that $\Sigma_2$ starts above $\Sigma_1$ for low values of $\mathfrak{g}_1$ and eventually ends below it). Moreover, adjusting the bound $\bar{\kappa}_1(\kappa_2)$ downward if needed, all equilibria are sorting (graphically, for $\kappa_1$ sufficiently small, the set of pairs at which $\Sigma_1$ and $\Sigma_2$ intersect are all below $\eta$). □

A.11 Standards and Application Costs: Proof of Theorem 7

Let $c = c_1 = c_2$ be the initial equal applications costs. If $c_i$ increases, the applicant pool at college $i$ shrinks, and so the $\Sigma_i$ curve shifts down, while $\Sigma_j$, $j \neq i$, remains unchanged. Thus, the functions now cross at a lower threshold pair, and so $\mathfrak{g}_1, \mathfrak{g}_2$ both fall.

In a sorting equilibrium, the applicant pool at college 1 consists of calibers $x \in [\xi_B, \infty)$. From the last part, any cost increase depresses $\mathfrak{g}_1$ in equilibrium. It follows that $\xi_B$ rises in equilibrium — since college 1 has the same capacity as before, if it is to have lower standards, it must also have fewer applicants. Let $(\xi_B^0, \mathfrak{g}_1^0)$ be the old equilibrium pair and $(\xi_B^1, \mathfrak{g}_1^1)$ the new one, with $\xi_B^0 < \xi_B^1$ and $\mathfrak{g}_1^0 > \mathfrak{g}_1^1$. Then the distribution function of enrolled students at college 1 under equilibrium $i = 0, 1$ is:

$$F_i^1(x) = \frac{\int_{\xi_B^i}^x (1 - G(\mathfrak{g}_1^i | t)) f(t) dt}{\int_{\xi_B^i}^\infty (1 - G(\mathfrak{g}_1^i | t)) f(t) dt}$$

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We must show $F_1^i(x) \leq F_0^i(x)$ for all $x \in [\xi_B, \infty)$. For any $x$, the denominators on both sides equal $k_1$, so cancel them. Now notice that $0 = F_1^i(\xi_B) < F_0^i(\xi_B)$ and $\lim_{x \to \infty} F_1^i(x) = \lim_{x \to \infty} F_0^i(x) = 1$. Since both functions are continuous in $x$, if $\partial F_1^i/\partial x > \partial F_0^i/\partial x$ for all $x \in [\xi_B, \infty)$, then $F_1^i(x) < F_0^i(x)$. But this requires $(1 - G(\sigma_1^0|x)) f(x) > (1 - G(\sigma_0^0|x)) f(x)$, which follows from $\sigma_1^0 < \sigma_0^0$. □

A.12 Affirmative Action: Negligible Feedback Effects

We show that in a neighborhood of $\Delta_1 = \Delta_2 = 0$, changes in $\Delta_i$ will have a negligible impact on $\sigma_j$, $i,j = 1,2$ at any stable solution solution to the capacity equations (6). Given any discount pair $(\Delta_1, \Delta_2)$, the capacity equations with two groups are:

$$
\begin{align*}
\kappa_1 &= \mu \xi_1^m(\sigma_1 - \Delta_1, \sigma_2 - \Delta_2) + (1 - \mu) \xi_1^M(\sigma_1, \sigma_2) \\
\kappa_2 &= \mu \xi_2^m(\sigma_1 - \Delta_1, \sigma_2 - \Delta_2) + (1 - \mu) \xi_2^M(\sigma_1, \sigma_2),
\end{align*}
$$

(21)

(22)

where $\xi_1^m, \xi_1^M$ are the respective fractions of minority and majority groups enrolled at college $i$, defined just as in (4) and (5), for the sets of signals (15). Since the signal density $g = G_\sigma$ and its derivative $G_x$ are both continuous, all derivatives of the enrollment function (using Leibnitz rule) are continuous too.

Differentiating equations (21) and (22) with respect to $\Delta_1$:

$$
\begin{align*}
\frac{\partial \sigma_1}{\partial \Delta_1} &= \mu \frac{\partial \xi_1^m}{\partial (\sigma_1 - \Delta_1)} \left( \frac{\mu}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_1^M}{\partial (\sigma_2 - \Delta_2)} \right) - \frac{\partial \xi_2^m}{\partial (\sigma_1 - \Delta_1)} \left( \frac{\mu}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_2^M}{\partial (\sigma_2 - \Delta_2)} \right) \\
\frac{\partial \sigma_2}{\partial \Delta_1} &= \mu \frac{\partial \xi_2^m}{\partial (\sigma_1 - \Delta_1)} \left( \frac{\mu}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_2^M}{\partial (\sigma_2 - \Delta_2)} \right) - \frac{\partial \xi_1^m}{\partial (\sigma_1 - \Delta_1)} \left( \frac{\mu}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_1^M}{\partial (\sigma_2 - \Delta_2)} \right),
\end{align*}
$$

where the denominator, from Cramer’s Rule, equals

$$
J = \left( \frac{\partial \xi_1^m}{\partial (\sigma_1 - \Delta_1)} + (1 - \mu) \frac{\partial \xi_1^M}{\partial (\sigma_1 - \Delta_1)} \right) \left( \frac{\partial \xi_2^m}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_2^M}{\partial (\sigma_2 - \Delta_2)} \right) - \left( \frac{\partial \xi_2^m}{\partial (\sigma_2 - \Delta_2)} + (1 - \mu) \frac{\partial \xi_2^M}{\partial (\sigma_2 - \Delta_2)} \right) \left( \frac{\partial \xi_1^m}{\partial (\sigma_1 - \Delta_1)} + (1 - \mu) \frac{\partial \xi_1^M}{\partial (\sigma_1 - \Delta_1)} \right)
$$

is positive in any stable equilibrium — i.e. the two group version of the condition that the slope of $\Sigma_1$ exceed the slope of $\Sigma_2$ in [8]. Now, $\partial \sigma_1/\partial \Delta_1 = \mu > 0$ and $\partial \sigma_2/\partial \Delta_1 = 0$ when $\Delta_1 = \Delta_2 = 0$, because the derivatives of the function $\xi_1^m, \xi_1^M$ at colleges $i = 1,2$

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coincide. Thus, the feedback effects vanish when $\Delta_1 = \Delta_2 = 0$, and are negligible in a neighborhood of it, by continuity of the enrollment derivatives. The analysis of the derivatives of $\bar{\sigma}_i$, $i = 1, 2$, with respect to $\Delta_2$ is analogous. □

A.13 Student Poaching: Proof of Theorem 10

For simplicity, assuming that the infimum signal is $\sigma = 0$ — which is WLOG by a simple monotone transformation of the signal. Let $(\Delta_2^e, \sigma_1^e, \sigma_2^e)$ solve the capacity equations as well as $\sigma_2^e = \Delta_2^e > 0$. In other words, college 2 admits all early applicants.

Fix $(\sigma_1^e, \sigma_2^e)$. Given $\Delta_2$, let $V_2(\Delta_2)$ be the difference between early and regular shadow values, namely, the LHS minus the RHS of equation (10).

If $V_2(\Delta_2^e) \geq 0$, then $(\Delta_2^e, \sigma_1^e, \sigma_2^e)$ is an equilibrium where early applicants are favored, since $\Delta_2^e > 0$. For each college fills its capacity, and either $V_2(\Delta_2^e) = 0$, or $V_2(\Delta_2^e) > 0$ which is a maximum for college 2, since it admits all early applicants.

Assume instead that $V_2(\Delta_2^e) < 0$. We will show that for some intermediate discount $\hat{\Delta}_2 \in (0, \Delta_2^e)$, college 2 favors its early applicants in some equilibrium.

Claim 2 The shadow value difference is positive when $\Delta_2 = 0$, or $V_2(0) > 0$.

Proof: The expected payoff for the lowest caliber that applies early or regular to college 2 equals the application cost. Also, $MB_{21} = c$ for the highest caliber who applies early or regular to college 2 (see Figure 8). Altogether, the same set of students apply early and regular to college 2 coincide when it sets the same standards for both groups.

Next, recalling the discussion around (11), a larger fraction of the early than the regular group accept college 2 if admitted. Thus, college 2 suffers from less of an acceptance curse in the early than in the regular period. Thus, the shadow value of early students exceeds that of regular students, proving the claim. □

Now, since the caliber and signal densities are continuous, $V_2(\Delta_2)$ is also continuous in $\Delta_2$. So by the Intermediate Value Theorem, there exists a $\hat{\Delta}_2 \in (0, \Delta_2^e)$ with $V_2(\hat{\Delta}_2) = 0$. Thus, $(\Delta_2^e, \sigma_1(\Delta_2^e), \sigma_2(\Delta_2^e))$ is an equilibrium where early applicants are favored. □
References


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