Adverse Selection and Liquidity Distortion*

Briana Chang†

First Draft: Nov, 2010
This version: Oct, 2013

Abstract

This paper develops a dynamic equilibrium model of market illiquidity driven by information frictions. The main contribution is to formalize liquidity as two dimensions, price and speed, and analyze how illiquidity manifests differently in varied informational settings: asymmetric information on the asset quality as well as on sellers’ trading motives. The model endogenously generates and separately identifies the effects of adverse selection on trading price and the trading volume. It shows how limited market participation, trading delays, and possibly fire-sales arise jointly as market outcomes.

Key words: Liquidity; Search frictions; Adverse selection; Fire Sales; Over-the-Counter.

JEL: D82, G1

*I am indebted to Andrea Eisfeldt, Kiminori Matsuyama, Dale Mortensen and Alessandro Pavan for their continuous support and encouragement. The paper has also benefited from the useful discussion with Matthias Doepke, Simone Galperti, Daniel Garrett, Veronica Guerrieri, Philipp Kircher, Arvind Krishnamurthy, Marzena Rostek, Robert Shimer, Martin Szydlowski, and Randall Wright. I also thank the seminar participants at CREI, Fuqua School of Business, Hass School of Business, Northwestern, Philadelphia Fed, Richmond Fed, Sauder School of Business, Toulouse, University of Pennsylvania, University of Toronto, UW-Madison, Wharton, the 2011 Scociety of Economic Dynamic Annual Meeting, the 2011 North American Summer Meeting of the Econometric Society, and the 2011 Chicago Federal Workshop on Money, Banking, Payments, and Finance.

†University of Wisconsin–Madison. Email: bchang@bus.wisc.edu
1 Introduction

One feature of the recent financial crisis is the sudden drop of trading volume in certain asset markets. For example, trading and issuance in mortgage-backed securities (MBS) has declined to negligible levels since the onset of the financial crisis in mid-2007.\footnote{\textsuperscript{1}See, for example, England (2008)} Such lack of transactions, leaving assets in the hands of financially constrained institutions despite gains from trade, cannot be explained by the standard paradigm, which predicts that the price should adjust downward to the level at which trade will occur. Meanwhile, a number of papers have shown evidence of adverse selection (as in Akerlof) in the market of MBS. Although market distortions are expected because of informational frictions, it is not clear whether adverse selection will lead to price undervaluation, drop in the volume, or both. In other words, in what circumstance will a seller accept a discounted price instead of holding on to their asset and waiting for a better price?

This paper develops a dynamic equilibrium model to study the effects of adverse selection on trading price and the trading volume. In the model, sellers’ preference over immediacy is determined by a combination of two forces: the quality of their assets (common value) and their distressed position (private value). The paper shows that the equilibrium outcome depends on uniformed investors’ inferences from sellers’ willingness to wait. Different information structures as well as different distress shocks therefore lead to different notions of market distortion. It therefore endogenously generates and separately identifies the effects of adverse selection on trading price and the trading volume.

Though the model is stark compared to the intricacies of financial markets, it provides an interpretation of the recent collapse of the AAA non agency MBS, where the drop in housing price makes some AAA-MBS, which were previously treated as safe assets, no longer risk free and potentially information sensitive. This view is consistent with the evidence in Agarwal et al. (2012), which shows that the evidence of adverse selection existed in the subprime mortgage market when the real estate bubble started to burst, but not before 2007. The model predicts that such information shock leads to a dry up in the volume and, furthermore, identifies the conditions under which distressed sellers will be willing to accept an undervalued price.

I study the environment where assets are traded in decentralized market. Contrary to the competitive paradigm with a centralized exchange and one market clearing price, trade is allowed to take place at different submarkets. Uniformed buyers create submarkets in which they commit to purchase the asset at the posting price, and sellers choose
the submarket optimally, taking into account price and the trading speed in each sub-
market. The trading speed is determined by the equilibrium buyer-seller ratio in each
submarket. The higher the ratio, (i.e., more buyers), the faster a seller can unload his
assets. Both price and speed - two dimensions of market liquidity - are determined jointly
by endogenous market participation in equilibrium.

Without informational frictions, each asset with a positive gain from trade will be sold
at the price that reflects the asset fundamental and at the optimal trading rate, (i.e. at
an optimal level of participation). In other words, a liquid market is one where sellers
can quickly find a buyer to purchase their asset at a fair price. I therefore define liquidity
distortion as the deviation from the first best outcome of the selling price and the trading
rate.

With adverse selection, the paper shows that, different notions of liquidity distortion
arise, depending on how traders sort themselves into different markets. For this, a com-
petitive search setup is useful as it explicitly allows traders to enter different markets. The
trading rate in different markets potentially can work as a screening device. The paper
provides new insight that the degree to which the market can screen agents depends on the
expected asset quality inferred from sellers’ waiting preferences. Through this channel,
different information structures lead to distinct notions of illiquidity.

When the private information is on the common value only (as in Janssen and Roy
(2002), Inderst and Müller (2002) and Guerrieri et al. (2010)), the types who are willing
to wait longer are necessarily worth more to buyers, and the market can therefore fully
screen agents. In this case, there exists a unique equilibrium which is fully separating:
asset prices still reflect their fundamental values while sellers with high-quality assets suffer
a longer trading delay in finding a buyer. This equilibrium outcome then rationalizes the
phenomenon of "buyers’ strike," implies a low trading volume, and explains the claims
often heard in the popular media that financial sectors are clogged with illiquid assets.

However, liquidity effects might manifest themselves differently when there is multi-
dimensional private information: the asset quality (the common value component) and
sellers’ distress position (the private value component). Such a situation arises when, for
example, investors can neither observe the asset quality nor the financial health of banks.
In this environment, the seller who wants to unload his asset more quickly can be the type
who has a low-quality asset, or the one who simply has a relatively urgent need for cash.
Depending on the underlying distribution, if the types who are willing to wait longer do
not necessarily have more valuable assets on average, then buyers are no longer willing
to pay more to those who are willing to wait longer, which undermines the full screening mechanism and leads to semi-pooling equilibria.

In particular, when the impact of a distress position is large enough so that investors know the average asset quality among those distressed banks actually has a higher value, there exists a semi-pooling equilibrium which has a feature of fire sales. In such fire sale equilibrium, endogenously, few buyers are willing to offer a high price; and a distressed seller with a high quality asset finds it optimal to enter a pooling market, in which the equilibrium buyer-seller ratio is upward-distorted while the price is undervalued. Hence, the model predicts that distressed sellers will unload their assets quickly and accept a price discount in such environment.

The model therefore provides a micro-foundation for fire sales. Contrary to the standard explanation for fire sales developed in Shleifer and Vishny (1992) and Shleifer and Vishny (2010)—where, sellers are forced to sell at a dislocated price because high valuation high-valuation bidders are assumed to be sidelined—in this framework, all potential buyers are unconstrained, and sellers can choose to hold on to their asset for a better price. Since the model generates the fire sale endogenously, it therefore sheds lights on when fire sales would actually take place. Indeed, the evidence on fire sale of financial assets is mixed. The paper delivers a new insight that the existence of fire sales depends on the underlying information structure and the property of distress shocks, and further provides a testable implication for such phenomena.

Relation to the literature My work is closest to Guerrieri et al. (2010), who apply the notion of a competitive search equilibrium to a static environment with adverse selection and uninformed principals who are allowed to post contracts. As discussed in Guerrieri et al. (2010), this equilibrium concept is similar to the refined equilibrium concept developed in Gale (1992) and Gale (1996). This paper contributes to the literature by considering asymmetric information on both private and common values and is the first paper to show that it is the interplay of multidimensional private information that leads to semi-pooling equilibria. I also develop a new characterization method. I establish that the equilibrium can be solved directly as the problem of an imaginary market designer in two-sided matching markets. This approach not only simplifies the equilibrium characterization to solving a differential equation but further facilitates extending the analysis to a more general environment. In particular, the constructing algorithm in Guerrieri et al. (2010) is designed for the case when a fully separating equilibrium is obtained, while my approach can also be applied to characterizing semi-pooling equilibria.
These findings in regard to semi-pooling equilibria constitute a novel contribution to
the previous literature on competitive (direct) search (e.g., Moen (1997), Burdett et al.
(2001), Mortensen and Wright (2002) and Eeckhout and Kircher (2010)). From prior
research, we already know that, without asymmetric information on the common value,
a fully separating equilibrium is always obtained; moreover, this holds true whether the
information on the private value is complete or asymmetric. This is because the com-
petitive search framework separates agents into different submarkets according to their
different waiting preferences (Mortensen and Wright (2002) and Eeckhout and Kircher
(2010)). However, with adverse selection, different informational structures lead to differ-
ent market segmentations.

Building on Guerrieri et al. (2010), the contemporaneous work of Guerrieri and Shimer
(2011) also emphasizes the idea that liquidity works as a screening mechanism; in that
work, they consider private information on the common value only and therefore obtain
a fully separating equilibrium. They construct a model with rationing. As I will show in
the discussion section, an economy with rationing can be understood as a limit case of the
matching technology developed in my model. My framework is designed to handle a more
general trading environment in a decentralized market, which allows for a general payoff
function, two-sided heterogeneity,\textsuperscript{2} and, more importantly, heterogeneity in the sellers’
private values of the assets. The notion of a fire sale in Guerrieri and Shimer (2011)
refers to the drop in the trading price when the equilibrium resale value decreases. In
contrast, the term \textit{fire sale} in my paper refers to the case in which a relatively distressed
seller enters a \textit{pooling} market, in which he can sell his asset more quickly (because of an
upward distorted buyer-seller ratio) but takes an undervalued price (because of pooling
with worse assets). These features are unique to the constructing semi-pooling equilibria
and they also imply distinct market activities.

This paper is related to the literature focusing on the effect of search frictions in asset
markets, including the over-the-counter literature (such as Duffie et al. (2005), Duffie
et al. (2007), Weill (2008), and Lagos and Rocheteau (2009)) and the monetary search
literature (for example, Kiyotaki and Wright (1993), Trejos and Wright (1995), and Shi
(1995)).\textsuperscript{3} There are a few works which consider adverse selection in the random-matching
framework (such as, Williamson and Wright (1994), and recent works by Chiu and Koeppel
(2011), Lester and Camargo (2011)). Both Chiu and Koeppel (2011) and Lester and

\textsuperscript{2}The extension with heterogeneous buyers is in the online supplementary materials.

\textsuperscript{3}Williamson and Wright (2008) provides a detailed survey of this line of literature.
Camargo (2011) use their framework to study government intervention: Chiu and Koeppel (2011) focus on the timing of the intervention, and Lester and Camargo (2011) shows that the policy might slow down market recovery, depending on the fraction of lemons in the market. It is worth noting that the fact that agents are separated into different trading rate shows up in a similar fashion in a random search framework: for example, in equilibrium constructed in Lester and Camargo (2011), sellers with high-quality asset trade slower.

The rest of the paper is organized as follows. Section 2 introduces the basic model and establishes my approach to characterizing equilibria. Section 3 adds the additional dimension of private information on sellers’ trading motives. Section 4 and 5 discuss the implications of the model and its testable predictions. Section 6 discusses efficiency and policy implication.

2 Baseline Model

Players There is a continuum of sellers and each owns a single asset which is nondivisible; the assets vary in quality, which is indexed by \( s \in S \) and which is the sellers’ private information. Assume that \( S = [s_L, s_H] \subset \mathbb{R}_+ \) and let \( G^0(s) \) denote the measure of sellers with asset quality weakly below \( s \) at \( t = 0 \). The other side of the market consists of a large continuum of homogeneous buyers; that is, the measure of buyers is strictly larger than the measure of sellers. The measure of buyers who decide to enter the market is endogenously determined by the free-entry condition. I choose this structure to emphasize the idea that there is a large number of potential buyers out there; therefore, limited market participation, if it arises, is endogenous.

Payoffs While holding the asset \( s \), the seller enjoys a flow payoff \( s \) but must pay a holding cost \( c \). One can think of the holding cost as a simple way to model a seller’s need to "cash" the asset.\(^4\) For now, one should think of the holding cost \( c \) as an easy way to generate the gain from trades. As shown in the general model, the main result holds for a general payoff. A buyer, on the other hand, does not need to pay the holding cost and therefore simply enjoys the flow payoff of the purchased asset. In order to buy the asset,

\(^4\)As explained in Duffie et al. (2007), we could imagine this holding cost to be a shadow price for ownership due to, for example, (1) low liquidity, that is, a need for cash; (2) high financing cost; (3) an adverse correlation of asset returns with endowments; or (4) a relatively low personal value from using the asset, as in the case of certain durable consumption goods, such as, homes.
the buyer must search for a seller, incurring a flow search cost, $k > 0$, for the duration of his search.

Setup All agents are risk-neutral and infinitely lived, with a common discount rate $r$. Time is continuous. The setup employs a dynamic competitive search framework. Buyers (uninformed principals) post a trading price $p$ and sellers direct their search toward their preferred market. All traders have rational expectations about the equilibrium market tightness (i.e., the buyer-seller ratio) associated with each market; the market tightness in each market $p$ is denoted by $\theta(p)$ and will be endogenously determined in equilibrium. As is standard, in each market, traders match bilaterally subject to a random matching function. A seller who enters the market $(p, \theta(p))$ matches a buyer with the Poisson rate $m(\theta(p))$. The assumption that $m(\cdot)$ is a strictly increasing function of $\theta$ captures the idea that having relatively more buyers make it easier to sell. On the other hand, a buyer in market $(p, \theta(p))$ meets a seller at the rate $q(\theta(p))$, where $q(\cdot)$ is a strictly decreasing function of $\theta$. In other words, a higher buyer-seller ratio makes it harder for a buyer to meet a seller. Trading in pairs further requires that $m(\theta) = \theta \cdot q(\theta)$. For the baseline model, I further assume that traders leave the market once the trade takes place. It is, however, straightforward to introduce resale into the framework.$^5$ For simplicity, I assume throughout this paper that the matching function takes the Cobb-Douglas form so that $m(\theta) = \theta^{\rho}$ where $1 > \rho > 0$. The results, however, are robust to a different form of search technology with standard assumptions.$^6$

2.1 Benchmark: Complete Information

We first establish the benchmark with complete information, which is the canonical competitive search model put forth by Moen (1997). In our particular setup, buyers simply post a trading price and sellers direct their search toward their preferred market. Moreover, following the interpretation of Mortensen and Wright (2002), one can imagine the competitive search equilibrium as if there is a market maker who can costlessly set up a collection $\Theta$ of submarkets. Each market can be characterized by a pair $(\theta(p), p)$, which is known ex ante to participants. Given the posting price and the market tightness in each market, each trader then selects the most preferred submarket in which to participate (search). With the assumption that there is perfect competition among market makers, the market maker’s problem is then to maximizes traders’ utilities.

$^5$See online Appendix for more details.

$^6$That is, $m(\cdot)$ is twice continuously differentiable and strictly concave.
Sellers’ and buyers’ expected utilities who enter the market with the pair \((\theta, p)\) can be expressed as follows, respectively:

\[
\begin{align*}
\rho V(s, \theta, p) &= s - c + m(\theta)(p - V(\theta, p, s)) \\
\rho U_b(s, \theta, p) &= -k + \frac{m(\theta)}{\theta}(\frac{s}{r} - p - U_b(\theta, p, s))
\end{align*}
\]

Free entry condition for buyers is assumed. That is, buyers’ entry and exit decisions are instantaneous and they will adjust until free entry condition holds. With perfect information, one can solve the equilibrium independently for each asset \(s\). The market maker’s optimization problems for each asset \(s\) is:

\[
\begin{align*}
V_{FB}(s) &= \max_{p, \theta} V(s, \theta, p) = \max_{p, \theta} \frac{s - c + pm(\theta)}{r + m(\theta)} \\
\text{st} : \quad U_b(s) &= \frac{m(\theta)(\frac{s}{r} - p) - \theta k}{r\theta + m(\theta)} = 0
\end{align*}
\]

One can easily see that \(\theta_{FB}\) solves following FOC:

\[
\frac{c}{k} = \frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)}
\]

Notice that \(\theta_{FB}\) is an increasing function of the cost ratio, \(\frac{c}{k}\). Namely, it is relatively easier for sellers to meet buyers, and it takes longer for the buyer to find the seller when the holding cost is higher. Also, the first best solution is independent of the asset quality. The intuition is clear since the gain from trade is simply the holding cost, which is independent of the asset quality. The price of each asset is then: \(p_{FB}(s) = \frac{s}{r} - \frac{k\theta_{FB}}{m(\theta_{FB})}\), the expected value of the asset minus the expected searching cost paid by buyers. One can easily check that IR constraint holds for all types of sellers. Obviously, first-best allocations can not be implemented in the environment with adverse selection. Facing the same market tightness, sellers always want to pretend a higher type so that they can get a higher payment.

### 2.2 Equilibrium with Adverse Selection

I now turn to an environment with adverse selection, in which sellers have private information about the asset quality. As in the complete information environment, buyers/sellers choose the price \(p\) they would like to offer/accept, and all traders have rational beliefs
about the ratio of buyers to sellers $\theta(p)$ in each market $p$. The key difference is that buyers now form rational beliefs about the distribution of sellers’ unobserved types in each market $p$, and that distribution determines the expected asset quality they receive in each market. Let $\mu(\cdot|p)$ denote the conditional distribution of sellers in each market $p$. That is, buyers expect that, conditional on a match in market $p$, $\mu(s|p)$ is the probability of a type-$s$ seller.

One convenient feature of this setup is that one can construct stationary equilibria, where the set of offered prices $P^*$, the market tightness in each market $\theta(p)$, and the composition of sellers in each market $\mu(\cdot|p)$ are time-invariant, while the aggregate distribution can evolve over time. This property makes our dynamic environment tractable. I now elaborate on how to construct stationary equilibria in this framework and focus on such equilibria throughout the paper.

Consider the set of time-invariant offer prices and the market tightness function. Facing the time invariant $(p, \theta(p))_{p \in P^*}$, sellers’ trading strategies are clearly stationary. That is, a seller simply stays in the same market until he trades. Given the stationary sellers’ trading strategies and the random matching in each market (i.e., all sellers in market $p$ trade with the same rate $\theta(p)$), the composition of sellers within each market is therefore stationary. As a result, the buyers’ expected matching value in each market $ar{J}(p) = \int J(\tilde{s}) \mu(\tilde{s}|p) d\tilde{s}$, where $J(s)$ denotes the value received upon buying the asset from a type-$s$ seller, is also time-invariant. From the free-entry condition $\left(p = \bar{J}(p) - \frac{k\theta(p)}{m(\theta(p))}\right)$, a time-invariant expected value $\bar{J}(p)$ then implies a stationary market tightness function $\theta(p)$. That is, at each point in time, buyers can enter and exit instantaneously to generate the correct ratio $\theta(p)$ in each market. Finally, one still needs to show that the set of offered prices $P^*$ is time-invariant. As will become clear later, the set of offer prices depends on the sellers’ equilibrium utilities and the range of underlying asset quality, both of which are time-invariant in the constructed environment. Hence, the above discussion suggests a possible stationary equilibrium. To characterize such an equilibrium, one needs to solve for only three objects: (1) the set of active markets $P^*$, (2) the equilibrium market tightness function $\theta(p)$, and (3) the composition $\mu(\cdot|p)$ in each market. Given $\{P^*, \theta(p), \mu(\cdot|p)\}$.

7Note that in a setting of competitive search models with heterogeneous agents, it is well-known that type distribution does not play a role, as the standard result in the literature is a full separation (for example, Moen (1997)). In an environment with adverse selection, there are two main differences from the literature. First of all, the possibility of (semi-)pooling is allowed. In this case, the distribution of sellers’ types in other submarkets does not play a role, but the distribution within each market does matter and is governed by $\mu(\cdot|p)$. Second, as will become clear, the equilibrium market tightness of each
one can easily solve for the dynamics of the aggregate distribution, which does evolve over
time and affects aggregate statics (such as the aggregate price and the price dispersion).
I will discuss this at the end of this section.

In a stationary equilibrium, each market is then characterized by \((p, \theta(p), \mu(\cdot|p))\). The
expected utility of type-\(s\) sellers when entering the market \((p, \theta(p), c)\) yields:

\[
rU^s(p, \theta(p), s) = s - c + m(\theta(p))(p - U^s(p, \theta(p), s)).
\]

A seller cares about the trading price and the market tightness (but not about \(\mu(\cdot|p))\).
Given the active markets \(p \in P^*\), sellers directs their search toward their preferred mar-
kets. A seller can always choose the option of no trade. The equilibrium expected utilities
of seller \(s\) then must satisfy the following:

\[
V^*(s) = \max \left\{ \frac{s - c}{r}, \max_{p' \in P^*} U^s(p', \theta(p'), s) \right\}.
\]

Buyers’ payoffs in each market \(p\) can be expressed as:

\[
rU^b(p, \theta(p), \mu(\cdot|p)) = -k + \frac{m(\theta(p))}{\theta(p)} \left\{ \int \frac{s}{r} \mu(\bar{s}|p)d\bar{s} - p - U^b(p, \mu(\cdot|p), \theta(p)) \right\}.
\]

From a buyer’s viewpoint, he cares not only about the trade-off between the trading
price and the market tightness \((p, \theta(p))\) but also about the composition of sellers \(\mu(\cdot|p))\),
which determines the expected asset value \(\int \frac{s}{r} \mu(\bar{s}|p)d\bar{s}\) in that market. Buyers can choose
which market to enter given \(p \in P^*\); they also have the option to open a new market
\(p \notin P^*\). Hence, I now need to specify the belief off the equilibrium path in order to analyze
buyers’ decisions. This equilibrium concept is adopted from Guerrieri et al. (2010), and
resembles the refined Walrasian general-equilibrium approach developed in Gale (1992).8
When a buyer contemplates a deviation and offers a price \(p\) which has not been posted,
\(p \notin P^*\), he has to take the sellers’ equilibrium utilities \(V^*(s)\) as given and forms a belief
about the market tightness and the types he will attract.9 First of all, a buyer expects a
market depends on the range of the underlying distribution, which is the key consequence of adverse
selection.

8See Guerrieri et al. (2010) for detailed discussion regarding its relationship with different refinement
developed in the previous literature.
9Such a requirement is called the market utility property in the competitive search literature. Burdett,
Shi, and Wright (2001) prove that a competitive search equilibrium is the limit of a two-stage game with
finite numbers of homogeneous buyers and sellers, which can be understood as a micro-foundation for
the market utility property.
positive market tightness only if there is a type of seller who is willing to trade with him. Moreover, he expects to attract the type $s$ who is most likely to come to the market until it is no longer profitable for him to do so. Formally, define:

$$\theta(p, s) \equiv \inf\{\bar{\theta} > 0 : U^*(p, \bar{\theta}, s) \geq V^*(s)\}$$

$$\theta(p) \equiv \inf_{s \in S} \theta(p, s).$$

(2)

By convention, $\theta(p, s) = \infty$ when $U^*(p, \bar{\theta}, s) \geq V^*(s)$ has no solution; specifically, $\theta(p, s) = \infty$ for any $p < V^*(s)$. Intuitively, we can think of $\theta(p)$ as the lowest market tightness for which the buyer can find a seller. Now let $T(p)$ denote the set of types which are most likely to choose $p$:

$$T(p) = \arg \inf_{s \in S} \{\theta(p, s)\}.$$

Therefore, given $\theta(p)$, it is optimal for every type $s \in T(p)$ to enter the new market $p$ but not optimal for $s \notin T(p)$. Hence, the buyer’s assessment about $\mu(s|p)$ for any posted price $p$ needs to satisfy the following restriction:

For any price $p \notin P^*$ and type $s$, $\mu(s|p) = 0$ if $s \notin T(p)$

(3)

In the case in which $T(p)$ is a singleton, a buyer then expects that this deviation will attract only seller $T(p)$ and that therefore $\mu(s|p) = 1$ if $s = T(p)$, and $\mu(s|p) = 0$ for $s \notin T(p)$. After specifying the off-path equilibrium belief, I am now ready to define the equilibrium. Let $P$ denote the set of feasible prices: $P = [0, J(s_H)].$

**Definition 1** A stationary equilibrium consists of a set of offer prices $P^*$; a market tightness function in each market $p$, $\theta(\cdot) : P \rightarrow [0, \infty]$; and the conditional distribution of sellers in each submarket $\mu : S \times P^* \rightarrow [0, 1]$, such that the following conditions hold:

**E1** (optimality for sellers): let

$$V^*(s) = \max\left\{\frac{s - c}{r}, \max_{p' \in P^*} U^*(p', \theta(p'), s)\right\}$$

and for any $p \in P^*$ and $s \in S$, $\mu(s|p) > 0$ implies $p \in \arg \max_{p' \in P^*} U^*(p', \theta(p'), s)$.

**E2** (optimality for buyers):

**E2(a)** Free-entry, for any $p \in P^*$,

$$U^b(p, \theta(p), \mu(\cdot|p)) = 0.$$
$E2(b)$ Optimal price-posting: there does not exist any $p' \notin P^*$ such that
\[ U^b(p', \theta(p'), \mu(\cdot | p')) > 0, \]
where $\theta(p')$ and $\mu(s | p')$ satisfy (2) and (3), respectively.

That is, a stationary equilibrium \{\(P^*, \theta(p), \mu(\cdot | p)\)\} must satisfy both sellers’ and buyers’ optimal trading decisions. The first condition simply says that sellers must direct their search optimally, given the set of open markets $P^*$. The first part of the buyers’ optimality condition means that buyers must get zero in each market $p \in P^*$. Furthermore, the second part guarantees that buyers will not deviate by opening a new market $p' \notin P^*$, given the off-path belief we specified earlier.

### 2.3 Characterization

I now show that the equilibrium outcome can be characterized as the solution to a mechanism design problem which takes into account both sellers’ and buyers’ optimality conditions. Intuitively, one can think of a market designer who promises a price and a market tightness for each market in order to match traders from both sides. I solve the equilibrium with two steps. First, I characterize the set of feasible mechanisms $\alpha \in A$ that satisfy $E1$ (Lemma 1) and the free-entry condition $E2(a)$. Condition $E1$ guarantees that sellers truthfully report their type, and $E2(a)$ guarantees that buyer-seller ratio must be correct, which must equal the ratio of the measure of buyers who are willing to pay $p$ to the measure of type-$s$ sellers who are willing to accept $p$. Second, I use $E2(b)$ to identify a necessary condition for which a mechanism $\alpha \in A$ can be decentralized (Lemma 2). That is, given that buyers can post the price freely in decentralized markets, any price schedule recommended by the market designer has to be optimal for buyers. Otherwise, buyers will deviate by posting a price other than the ones recommended by the market designer.

To characterize the set of mechanisms that satisfy the sellers’ IC constraints, I set up the problem as a mechanism-design problem (of an imaginary market designer). By the revelation principle, one can focus direct revelation mechanisms without loss of generality. A direct mechanism is a pair $(\theta, p)$ where $\theta : S \to \mathbb{R}_+$ is the market tightness function and $p : S \to \mathbb{R}_+$ is the price function. The mechanism is interpreted as follows. A seller who reports his type $\hat{s} \in S$ will then enter the market with the pair $(\theta(\hat{s}), p(\hat{s})).$ Let $V(\hat{s}, s) = V(p(\hat{s}), \theta(\hat{s}), s)$ denote the payoff that type $s$ obtains when he reports $\hat{s},$ which can be expressed as:
\[
rV(\hat{s}, s) = s - c + m(\theta(\hat{s}))(p(\hat{s}) - V(\hat{s}, s)).
\]
The sellers’ IC condition then requires that $s \in \arg \max_s \frac{s - c + p(\hat{s})m(\theta(\hat{s}))}{r + m(\theta(\hat{s}))}$. A seller’s utility can be rearranged as:

$$V^*(s) = \max \left\{ \frac{s - c}{r}, \max_s \frac{s - c + p(\hat{s})m(\theta(\hat{s}))}{r + m(\theta(\hat{s}))} \right\}.$$

Notice that a seller can always choose not to participate and thereby get his autarky utility $\frac{s - c}{r}$. For convenience, we can think of not entering any market as choosing a market where the matching rate is zero. Since the mechanism has to satisfy the sellers’ IR constraint, we set $\theta(s) = 0$ whenever the IR constraint is binding. The following lemma then characterizes any mechanism $\alpha = \{p(\cdot), \theta(\cdot)\}$ which satisfies $E1$:

**Lemma 1** The pair of functions $\{\theta(\cdot), p(\cdot)\}$ satisfies the sellers’ optimality condition (E1) if and only if the following conditions are satisfied:

$$\frac{1}{r + m(\theta(s))} \text{ is non decreasing}; \quad (M)$$

$$V^*(s) = \frac{u(s) + p(s) \cdot m(\theta(s))}{r + m(\theta(s))} = V^*(s_i) + \int_{s_i}^{s} V_s(p(\hat{s}), \theta(\hat{s}), \hat{s})d\hat{s}; \quad (ICFOC)$$

$$V^*(s) \geq \frac{u(s)}{r}, \quad (IR)$$

where $u(s) = s - c$ in the baseline model.

**Proof.** The proof follows from standard arguments in the mechanism-design literature (Milgrom and Segal (2002)). See Appendix.

Given a price schedule, one can define $H(p) \equiv E[s | p(s) = p]$ as buyers’ expected asset quality in market $p$. Any feasible mechanism $\alpha = \{p^\alpha(\cdot), \theta^\alpha(\cdot)\}$ must satisfy both Lemma 1 and the free-entry condition:

$$p = \frac{H(p)}{r} - \frac{k\theta(p)}{m(\theta(p))}. \quad (4)$$

Let $A$ be the set of feasible mechanisms; this set includes all possible pooling equilibria as well as separating ones. Nevertheless, $\alpha = \{p^\alpha(\cdot), \theta^\alpha(\cdot)\}$ is an equilibrium only if there is no profit for a buyer to deviate by opening a new market $p'$. That is, $\alpha$ must satisfy $E2(b)$. Let $V^*(s; \alpha)$ denote the expected payoff to a type-$s$ seller under mechanism $\alpha$. To facilitate the analysis, the following lemma identifies the type who is mostly likely to come $T(p')$, given any $\alpha \in A$:
Lemma 2 Given a mechanism \( \alpha \equiv \{p^\alpha(\cdot), \theta^\alpha(\cdot)\} \in A \) and \( \theta^\alpha(s) > 0 \ \forall s \), then for any \( p' \) outside the range of \( p^\alpha(\cdot) \), the unique type who will come to this market \( p' \) is given by:

\[
T(p') = s^+ \cup s^-
\]

where \( s^- = \inf\{s \in S | p' < p^\alpha(s)\} \) and \( s^+ = \sup\{s \in S | p' > p^\alpha(s)\} \).

With Lemma 2, I establish the necessary conditions for which \( 2A \) can be decentralized, which is summarized by Lemma 3 and Lemma 4. These two conditions pin down the unique equilibrium candidate.

Lemma 3 (Full Separation) There exists no submarket where seller types are pooled.

Intuitively, a buyer can post a new price \( p' \) which is only slightly higher than the original pooling price. In that case, he only pays a little bit more but definitely gets the best type in the original pooling (as implied by lemma 2), which therefore generates a profitable deviation. This result allows us to focus on a fully separating equilibrium. In each market, the price schedule and the market tightness function then have to satisfy the following equation, given by free-entry condition:

\[
p(s) = \frac{s}{r} - \frac{k\theta(s)}{m(\theta(s))}. \tag{5}
\]

Substituting this payment schedule into ((ICFOC)), we get:

\[
V^*(s) = \frac{s - c + \left(\frac{s}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))}\right)m(\theta^*(s))}{r + m(\theta^*(s))} = V^*(s_l) + \int_{s_l}^s V_s(p^*(s), \theta^*(s), \tilde{s}) d\tilde{s} ;
\]

Taking the derivative with respect to \( s \) on both sides, one then gets the following differential equation for \( \theta^*(s) \):

\[
\left[ c - k\left(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)}\right) \right] \frac{d\theta}{ds} = -\frac{\theta}{pr} (r + m(\theta)). \tag{6}
\]

In summary, in order to satisfy the incentive compatibility constraints and the free-entry condition, the market tightness function \( \theta^*(\cdot) \) has to satisfy the above differential equation, subject to the monotonic condition \((M)\).\(^{11}\) I use Lemma 4 to pin down the initial condition. Lemma 4 shows that the equilibrium solution must start from the first-best value \( \theta^*(s_L) = \theta^{FB} \). The intuition is clear: a downward-distorted market tightness occurs in order to prevent a lower-type seller from mimicking a higher-type seller. Therefore, there is no reason to distort the market tightness for the lowest type.

\(^{11}\) (6) is a separable, nonlinear, first-order differential equation with a family solution form: \( s = C + \int \left\{ \frac{c - k\left(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)}\right)}{-\frac{\theta}{pr} (r + m(\theta))} \right\} d\theta. \)
Lemma 4 In any fully separating equilibrium, the price must be continuous in $s$; in addition, the lowest type achieves his first-best utility and

$$\theta(s_L) = \theta^{FB}(s_L).$$  \hfill (7)

Hence, the unique candidate of the equilibrium market tightness $\theta^*(s)$ is the solution of (6) with the initial condition $\theta^{FB}$: $\theta^*(s) = \theta(s; \theta^{FB}(s_L))$,\footnote{One can see that the standard condition of uniqueness does not hold at $\theta_0(s_L) = \theta^{FB}$. In fact, there will be two solutions. However, the other solution increases with $s$ and therefore violates the monotonic condition.} which is illustrated in Figure 1. Given $\theta^*(s)$, the equilibrium price is also determined by (5). The equilibrium can be summarized as follows. Because of asymmetric information, sellers face a lower meeting rate but will get a higher payment: $\theta^*(s) < \theta^{FB}(s)$ and $p^*(s) = \frac{\xi}{r} - \frac{k\theta^*}{m(\theta^*)} > p^{FB}(s)$ $\forall s > s_L$. Endogenously, fewer buyers enter the market. Sellers’ equilibrium utilities are lower than in the first-best benchmark due to the distortion: $V^*(s) < V^{FB}(s)$ for $\forall s > s_L$.

Lastly, one can easily verify that the IR constraint holds for all sellers since there is no cost for entering the market and the trading price is higher than the outside option: $p^*(s) > p^{FB}(s) > \frac{\xi - c}{r}$. Furthermore, buyers do not find it profitable to open markets other than those that are already open. The argument is as follows. Denote by $(p_L, p_H)$ the lower bound and the upper bound, respectively, of the support of the function $p(s)$ constructed as above. From Lemma 2, if buyers post a price $p' > p^H$, they will attract only the highest type. The corresponding $(p', \theta')$ has to provide the highest type with the same utility; however, such a pair $(p', \theta')$ leads to a further distortion.\footnote{See Appendix for the formal proof.} As a result, a

![Figure 1: Equilibrium market tightness $\theta^*(s)$](image-url)
buyer’s utility is lower due to the additional distortion. Such a deviation is therefore not profitable. Similarly, if posting \( p' < p^L \), a buyer will attract the lowest type with a pair of \((p', \theta')\). Conditional on the lowest type obtaining his first best utility, any pair of \((p', \theta')\) other than \((p^{FB}, \theta^{FB})\) implies that a buyer’s utility will be negative. The above argument thus confirms that no profitable deviation exists for buyers. Hence, we have the following proposition:

**Proposition 1** The unique solution to the mechanism design problem subject to conditions \( E1 \) and \( E2 \) is given by the market tightness function \( \theta^* : S \rightarrow \mathbb{R}_+ \), and the price function \( p^* : S \rightarrow \mathbb{R}_+ \), where \( \theta^*(s) \) is the unique solution to (6) with the initial condition \( \theta(s_L) = \theta^{FB}(s_L) \), and where \( p^*(s) \) is given by (5).

**Corollary 1** The unique decentralized equilibrium outcome is characterized by:

1) a set of offered prices (active submarkets) \( P^* = \{ p \in \mathbb{R}_+ | p = p^*(s) \text{ for } s \in S \} \), where the price function \( p^*(\cdot) \) is as given in Proposition 1;

2) the market tightness for each submarket \( \Theta^* : P^* \rightarrow \mathbb{R}_+ : \Theta^*(p) \equiv \theta(p^{-1}(p)) \), where \( p^{-1} : P^* \rightarrow S \) denotes the inverse of \( p^* \);\(^{14}\)

3) the share of type \( s \) in each submarket: \( \mu(s|p) = I\{p^*(s) = p\} \).

### 2.4 General Payoff

This section shows that the main result is robust to a traders’ payoff function that is more general. This generalization is important as it further facilitates the analysis in the later section. As before, there is a mass of heterogeneous sellers. Each seller has one asset and the asset quality is indexed by \( s \in S \), which is sellers’ private information. The flow payoff of the asset \( s \) to the seller is now given by \( u(s) \), where \( u \) is a continuously differentiable function, \( u : S \rightarrow \mathbb{R}_+ \). The indices \( s \) are ordered so that \( u(s) \) is increasing in \( s \), that is, \( u'(s) > 0 \). On the other side of the market, there is a large mass of buyers. The flow payoff of an asset \( s \) is given by \( h(s) \), which is a strictly positive function.

**Assumption 1** The function \( h(s) \) is (1a) a continuously differentiable function and (1b) strictly increasing in \( s \), \( h_s(\cdot) > 0 \).

**Assumption 2** For all \( s \in S \), the gain from trade is positive: \( g(s) = h(s) - u(s) > 0 \) \( \forall s \in S \).

\(^{14}\)Note that the price function \( p \) is strictly increasing and therefore invertible.
**Proposition 2** Under Assumptions 1 and 2, a unique fully separating equilibrium \( \{p^*(\cdot), \theta^*(\cdot)\} \) exists. The market tightness function \( \theta^*(s) \) solves the following differential equation:

\[
\left[ h(s) - u(s) - k\left(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)}\right) \right] \frac{d\theta}{ds} = -(r + m(\theta)) \cdot \frac{\theta h_s(s)}{\rho} r.
\]

The initial condition is given as \( \theta^*(s_L) = \theta^{FB}(s_L) \), where the first-best market tightness \( \theta^{FB}(s) \equiv \max_{\theta \in \mathbb{R}_+} U^*(s, \theta) \). The corresponding price schedule \( p^*(s) \) satisfies the following equation:

\[
p^*(s) = \frac{h(s)}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))}.
\]

**Lemma 5** Under Assumptions 1 and 2, (1) the first best solution \( \{\theta^{FB}(s), p^{FB}(s)\} \) is not implementable; and (2) the equilibrium market tightness \( \theta^*(s) \) is downward-distorted compared to the first-best, that is,

\[
\theta^*(s) < \theta^{FB}(s) \text{ for } \forall s > s_L.
\]

### 2.5 Extension with Resale

The basic model assumes that once a buyer buys the asset, he keeps it forever. If a buyer is financially constrained in the future, he has motives to sell his asset to exchange for cash and will then re-enter the market as sellers. Clearly, taking this into account, buyers’ expected profit will also depend on the resale value. To capture preference for asset ownership possibly switch overtime and the impact of liquidity shock on the equilibrium price and market liquidity, this section extends our model to allow for resale. To be precise, the flow value of owning the asset decreases, dropping from \( h(s) \) to \( u(s) \), when the owner hit by the liquidity shock which arrives at the Poisson arrival rate \( \delta \). In our basic model, this simply means that the owner now needs to pay the holding cost and hence he naturally becomes the seller in the market. Given the market is designed in an incentive-compatible way, the owner of the asset \( s \) then enters the market \( (p(s), \theta(s)) \) as a seller. The contingent value of the ownership can now be rewritten as:

\[
r J(s) = h(s) + \delta(V^*(s) - J(s))
\]

where \( V^*(s) \) is the expected value of a type-\( s \) seller at the equilibrium. All methods developed in our main model remain valid and the key difference is that the value of holding the asset, which is a function of the equilibrium resale value \( V^*(s) \), will then be determined in the equilibrium. Nevertheless, one can see that the monotonic condition
still holds given that \( h(s) + \delta V^*(s) \) strictly increases with \( s \)\(^{15} \). Hence, as shown in our previous discussion, a full-separated equilibrium is the unique outcome. Our previous approach can be applied directly with the modified free entry condition:

\[
p(s) = J(s) - \frac{k\theta}{m(\theta)} = \frac{h(s) + \delta V^*(s)}{r + \delta} - \frac{k\theta(s)}{m(\theta(s))} \tag{10}
\]

The only difference is that we now have a different differential equation and, of course, different first best solution, i.e., different initial condition. The differential equation can be derived by substituting the above price schedule into (??) and differentiate respect to \( s \) in both sides, which yields:

\[
[(h(s) - u(s) + \frac{k}{\rho}((\rho - 1)\theta - \frac{(r + \delta)\theta}{m(\theta)}))] \frac{d\theta}{ds} = -(\frac{r + \delta + m(\theta)}{r + \delta}) \cdot \frac{\theta}{\rho} (h_s(s) + \frac{\delta u_s(s)}{r + m(\theta)}) \tag{11}
\]

One can easily check that the basic version, i.e., equation (??), is simply the case when \( \delta = 0 \). Furthermore, (11) can be understood from (??) by setting the effective discount rate \( \tilde{r} = r + \delta \) and equilibrium buyers’ values \( \tilde{h}(s) = h(s) + \delta V^*(s) \), where \( \tilde{h}_s(s) \) then corresponds to \( h_s(s) + \frac{\delta u_s(s)}{r + m(\theta(s))} \) as shown in the RHS of (11). Same as before, the initial condition is then given by the first best solution in such an environment.

**Remark:** As before, the impact of adverse selection on the market tightness is downward distorted compared to the first best. However, the resale value adds an additional effect on the trading price. In particular, according to (10), price decreases as the equilibrium value of the asset decreases, \( V^{FB}(s) > V^*(s) \) for \( \forall s > s_L \). That is, taking into account the resale, adverse selection further leads to a decrease in the trading price.

### 2.6 Remarks on the Aggregate Dynamics

Although the key equilibrium object \( \{P^*, \theta(p), \mu(s|p)\} \) is stationary, the aggregate distribution does evolve over time and is very tractable in this dynamic framework. In particular, the transaction outflow for each type-\( s \) asset is determined by the matching rate \( \theta^*(s) : m(\theta(s))g^t(s) \), where \( g^t(s) \) is the size of sellers with asset-\( s \) at time \( t \). In order to maintain a positive market size, I now analyze the aggregate dynamics with resale, the law of motion for \( g^t(s) \) is then given by

\[
dg^t(s) = -m(\theta^*(s))g^t(s) + \delta q^t(s)
\]

\(^{15}\)This is true as \( V^*(s) \) is necessarily increasing in \( s \) according to Proposition 1.
where \( q^t(s) \) represent the measure of asset \( s \) owned by agent who do not need to pay the holding cost (holders). Those owners want to sell the asset only when receiving the liquidity shock. In that case, he becomes the seller. For any fixed supply of the asset \( F(s) \), the measure of holders and sellers must equal the total supply of asset-s: 
\[
q^t(s) + g^t(s) = F(s)
\]
Given the solution \( (p^*(s), \theta^*(s)) \) and the law of motion of \( g^t(s) \), it is straightforward to solve for the dynamic transition \( \{g^t(s), q^t(s)\} \) and the steady state as in such an environment. In particular, for any type of asset, the steady state ratio of the holders to the sellers is pinned down by \( \frac{m(\theta^*(s))}{\theta^*(s)} \). That is, due to the downward distortion of \( \theta^*(s) \), the better the asset, the larger portion of which stays in the bad hands. Given any
Hence, in this environment, the density of type-s sellers satisfies the following equation: 
\[
g^t(s) = g^0(s)e^{-m(\theta(s))t},
\]
where \( g^0(s) \) is the density of the initial distribution \( G_0(s) \). This then gives a non stationary aggregate-pricing dynamics, where the aggregate price is a weighted trading price,

\[
\bar{P}_t \equiv \int \frac{p(\tilde{s}) \cdot m(\tilde{\theta}(\tilde{s})) g^t(\tilde{s}) d\tilde{s}}{V_t}
\]

and \( V_t \) denotes the aggregate trading volume at time \( t \) :

\[
V_t = \int m(\tilde{\theta}(\tilde{s})) g^t(\tilde{s}) d\tilde{s}.
\]

For any distribution \( G^0(s) \), one can easily see that \( \bar{P}_t \) increases over time because a seller with a relatively low-quality asset exits faster. In order to maintain a positive market size, one can further extend the model with resale. In such an environment, one can analyze the market dynamics in response to a permanent information shock that increases the adverse selection problem. The model then predicts a drop in both the aggregate trading price and the trading volume during the transition dynamics as well as when the market reaches its new steady state.

3 Obscure Motives for Selling

In our baseline model, trading probability essentially acts as a screening mechanism. Given any holding cost, sellers’ willingness to wait is simply determined by the asset quality; therefore, an seller’s type is perfectly revealed by his choice of which market to trade in. The crucial assumption for this result is that sellers’ holding costs—which capture sellers’ distress level—are observed. However, it might be the case that investors cannot
perfectly verify whether the bank is solvent or the firm is financially distressed. One can interpret this environment, for example, when such distress is driven by a bank’s exposure to its counterparties and, due to the complexity of banks’ interconnection, investors cannot identify who are exposed to the counterparty risk.

I now analyze the environment in which sellers have heterogeneous distress level, and such information is unobservable to investors. In other words, there are two dimensions in sellers’ types: the asset quality (the common value component) and the sellers’ holding costs (the private value component). The goal of this section is to explain how trading probability and equilibrium prices are affected by the combination of these two components. I first show how this setup can be nested into our general model and then discuss how the equilibrium might behave differently because of the unobserved trading motives.

The setup is similar to our basic model but a seller’s type now has two components: \( z^i = (s^i, c^i) \in Z \equiv S \times C \). As before, the support of \( s^i \) is the real interval \( S \equiv [s_L, s_H] \subset \mathbb{R}_+ \), but the support of \( c^i \) is some arbitrary set \( C \) which can assume either discrete or continuous values. A seller’s payoff for holding an asset is then governed by both the cash flow \( s \) and his liquidity position \( c \). Define type \( x \) as \( x = s - c \in X \equiv \{ x \mid x = s - c, s \in S \text{ and } c \in C \} \), representing the seller’s value for holding an asset. Intuitively, the mechanism discriminates only on the basis of the sellers’ payoffs for owning an asset. Two agents with the same type \( x \) must obtain the same utility, irrespective of any other unobservable characteristics that might differentiate the two agents in terms of their attractiveness to buyers. Therefore, the general model in this paper can be applied to this setup as well, with the following two reinterpretations. First, \( x \) is now the effective seller’s type.\(^{16}\)

The utility of seller \( x \), when reporting his type to be \( \hat{x} \), and thus entering the market \((\theta(\hat{x}), p(\hat{x}))\), is then given by

\[
    rV(\hat{x}, x) = \frac{x + m(\theta(\hat{x})) \cdot p(\hat{x})}{r + m(\theta(\hat{x}))}.
\]

Second, since buyers care only about the asset quality (i.e., the common value component of the seller’s type), a buyer’s expected value for buying the asset from type \( x \) is given by \( h(x) = E[s|s - c = x] \), where \( h : X \to \mathbb{R}_+ \). With the above interpretation, one can now apply the analysis used previously. In particular, Lemma 1 and Lemma 2

\(^{16}\)The IC condition requires that \( V(x, s) = V(x, s') \) and imposes that the allocation \( \theta(x, s) \) varies with \( s \) only over a countable set of \( x \). In this particular setup, any mechanism in which the allocation is conditional on \( s \) in additional to \( x \) necessarily reduces traders’ utilities as it leads to more distortions on the sellers’ sides.
remain intact. Furthermore, as discussed in the general payoff section, as long as $h(\cdot)$ is monotonically increasing, there exists a unique fully separating equilibrium with respect to the effective type $x$. Hence, the equilibrium outcome crucially depends on the function $h(x)$, which is determined by the underlying joint distribution of $(s, c)$.

This section has two goals. The first goal is to provide the sufficient condition for the distribution such that the monotonicity condition holds. For those cases, the equilibrium outcome then follows immediately from the baseline model with general payoffs. The second goal is to provide a general manual for constructing equilibria for the non monotonicity cases. In particular, I focus on one type of equilibria which have an interesting interpretation of fire sales, and have a distinct prediction from the baseline model.

### 3.1 Monotonicity: A Fully Separating Equilibrium

According to Lemma 3, if the continuous function $h(x)$ is monotonically increasing in $x$, the equilibrium continues to exhibit full separation with respect to type $x$. Such an environment can be nested in the framework with a general payoff function: sellers with the same value $x$ will enter the market $\{\theta(x), p(x)\}$, while buyers in that market, taking into account that different sellers might have different motives for sales, pay the expected asset value given $x$: $h(x) = E[s|x = c = x]$. Given any monotonic increasing function $h(x)$, the equilibrium market tightness function $\theta(x)$ and the price $p(x)$ can be solved according to Proposition 2 (i.e., equation (8) and (9)). In this case, as before, it is hard for sellers to find a buyer due to the downward distorted market tightness.

To guarantee that the function $h(x) = E[s|x]$ is increasing in $x$, the following proposition provides the sufficient condition on the joint distributions. Let $f(s, c)$ denote the joint distribution density of the random variable $(s, c)$.

**Proposition 3** If the joint distribution $f$ satisfies the following condition:

$$f_1 \cdot f_2 + (f_2)^2 - (f_{12} + f_{22}) \cdot f \geq 0$$

where $f_i$ denotes the partial derivative respect to the variable, then $h(x)$ is increasing in $x$.

**Corollary 2** If $s$ and $c$ are independent and $f_c(c)$ is log-concave (where $f_c(c)$ denotes the marginal distributions for $c$), then $h(x)$ is increasing in $x$.

The condition on the distribution mainly borrows from the standard affiliation condition discussed in Milgrom and Weber (1982). Notice that when $(s, c)$ are independent,
a log-concave density function \( f_c(c) \) is sufficient to guarantee a fully separating equilibrium.\(^{17}\) For the purpose of illustration, consider the case in which asset quality \( s \) is uniformly distributed and \( s_H - c_H > s_L - c_L \). After some algebra, the function \( h(x) \) can then be simply expressed as:

\[
h(x) = x + E[c|x]^{18}
= \begin{cases} 
  s_L + \frac{Q_R(x)}{Q_R(x)} & \text{if } s_L - c_H \leq x < s_L - c_L \\
  x + \mu_c & \text{if } s_L - c_L \leq x \leq s_H - c_H \\
  s_H + \frac{Q_L(x)}{Q_L(x)} & \text{if } s_H - c_H < x \leq s_H - c_L
\end{cases}
\]

where \( Q_R(x) \equiv \int_{s_L-x}^{s_H-x} (1 - F(\tilde{c})) d\tilde{c} \) and \( Q_L(x) \equiv \int_{c_L}^{s_H-x} F(\tilde{c}) d\tilde{c} \) represent, respectively, the right-hand and the left-hand integrals of the cumulative distribution function \( F(\tilde{c}) \).

Given that \( s \) is uniform, when \( x \) is low (i.e., in the first case: \( s_L - c_H \leq x < s_L - c_L \)), the conditional probability that a seller type-\( x \) with holding cost \( c \) is given by: \( f_c(c|x) = \frac{f_c(\tilde{c})}{1 - F(s_L - x)} \) (the hazard function). That is, intuitively, buyers know that the holding cost of type-\( x \) seller must be drawn from the right-hand side of the distribution. Similarly, when \( x \) is high enough (i.e., the third case: \( s_H - c_H \leq x < s_H - c_L \)), the holding cost of type-\( x \) seller must be drawn from the left-hand side of the distribution. Given that the log-concavity is inherited by right-hand integrals as well as by left-hand integrals, a log-concave density function \( f_c(c) \) implies that \( \frac{Q_R(x)}{Q_R(x)} \) and \( \frac{Q_L(x)}{Q_L(x)} \) are increasing functions in \( x \), and therefore, that \( h(x) \) is increasing in \( x \).\(^{19}\)

### 3.2 Nonmonotonicity: Semi-pooling Equilibria

In this section, I analyze the case in which the monotonicity condition does not hold. For a simple illustration, consider the case in which there are two possible holding costs for sellers \( C \equiv \{c_H, c_L\} \), where \( c_H > c_L > 0 \), and let \( \lambda \) denote the probability that the seller who owns the asset \( s \) has liquidity position \( c_H \) (a higher holding cost). For simplicity, assume that \( c \) and \( s \) are independently distributed and \( s \) is uniformly distributed over the interval \([s_L, s_H]\). The value of \( h(\cdot) \) can then be understood as in figure 2. Observe that the

\(^{17}\)The commonly used distribution with log-concave density functions are, for example, uniform, normal, and exponential distribution. On the other hand, some distributions (such as the beta and gamma functions) have log-concave functions only if their parameters fall into certain ranges. (See more detailed discussions in Bergstrom and Bagnoli (2005)).

\(^{19}\)Note that the function \( h(\cdot) \) is continuous but might not be differentiable at \( x_1 = s_L - c_L \) and at \( x_2 = s_H - c_H \). However, the left and the right limit exist; hence, the differential equation (8) still applies. In other words, the previous method applies to any monotonically increasing continuous function \( h(\cdot) \).
buyers’ value function $h(\cdot)$ is not strictly increasing in $x$. In particular, the expected value of the asset drops in the overlapping region, since the asset could be either a low-quality one owned by a seller with a low holding cost or a high-quality one owned by a seller with a high holding cost. Observe that this drop in the expected value happens whenever $c$ is discrete, regardless of the underlying distribution of $s$.

This simple example shows that the type who is willing to wait longer (a higher $x$) does not necessarily have a better asset (which is a violation of assumption A1). I first prove that a fully separating equilibrium cannot exist and then characterize the semi-pooling equilibria. The intuition for why a fully separating equilibrium (respect to the effective type $x$) cannot exist is the following: a fully screening mechanism is a combination of a downward-distorted trading probability and an upward-rising price scheme. To generate such a price schedule, the market designer must make sure that buyers are willing to pay the price, given the expected value in each market. However, if the types who are willing to wait longer (a higher $x$) have assets that are worth less (a lower $h(x)$), buyers’ are no longer willing to pay a higher price for it, which undermines the full screening mechanism and leads to semi-pooling equilibria.

To facilitate the equilibrium construction for semi-pooling equilibria, define $\tilde{h}(x) \equiv H(p^*(x))$ and recall that $H(p)$ denotes the buyers’ expected value in the market with price $p : H(p) = E[h(x)|p^*(x) = p]$. That is, given any equilibrium $\{p^*(x), \theta^*(x)\}$, the function $\tilde{h}(\cdot)$ represents how buyers value type-$x$ in equilibrium. For example, if there exists a subset of sellers who are pooled in one submarket, the value of type-$x$ in the eye of

---

20 Notice that, in contrast to standard mechanism problems, the set of feasible mechanisms $A$ is solved subject to the free-entry condition. A fully separating allocation therefore has to solve (8). If the solution $\theta^*(s)$ to (8) does not satisfy (M), a fully separating scheme is no longer in the set of feasible mechanisms $A$. 


buyers is then the expected value in that submarket (instead of the real value \( h(x) \)). Note that \( \tilde{h}(\cdot) \) coincides with the underlying value \( h(\cdot) \) if and only if there is full separation with respect to type-\( x \).

**Proposition 4** For any equilibrium outcome \( \{ p^*(x), \theta^*(x) \} \), the corresponding buyers function \( \tilde{h}(x) \) cannot have a strict local maximum.

**Proposition 5** Given a non monotonous function \( h(\cdot) \), if \( h(\cdot) \) has a strict local maximum, then a fully separating equilibrium cannot exist.

**Proof.** A full separating equilibrium implies that \( \tilde{h}(x) = h(x) \). Hence, it follows immediately from Proposition 4.

### 3.2.1 Equilibria with upward-distorted market tightness: the Fire Sale

I now characterize equilibria for the nonmonotonicity case and demonstrate how different liquidity distortions will arise because of the multidimensional information problem. In this section, I consider a general nonmonotonic function \( h(\cdot) \) (as demonstrated in figure 3); I characterize the equilibrium in which upward-distorted market tightness occurs, and provide the sufficient condition for its existence. Such an equilibrium exhibits the following distinct feature: certain types of sellers sell their assets *quickly* at a price below the fundamental value. I refer to this situation as a *fire sale* of the assets. In particular, an upward-distorted market tightness (that is, where some sellers sell even faster than the first-best outcome) makes a different prediction for the aggregate trading volume and the trading price, compared to the monotonicity case.

Consider a nonmonotonic \( h(\cdot) : [x_L, x_H] \rightarrow [h_L, h_H] \), which is strictly increasing in \( x \) after some point \( \hat{x} \), \( h(x) \) is therefore invertible if we restrict to the domain \( x \geq \hat{x} \). Let \( \phi(h) \) denote its inverse function as it maps \( h \) to \( x \geq \hat{x}_1 \). For any \( \hat{h} \in [h(\hat{x}), h_H] \), define \( q(\hat{h}) \equiv \int_{x_L}^{\phi(\hat{h})} h(x) \frac{dG(x)}{G(\phi_1(\hat{h}))} \), which represents the expected asset quality for all sellers’ types below the cutoff type \( x = \phi(\hat{h}) \). Given that \( h(x) \) is strictly increasing after \( \hat{x} \), then if \( q(h) = h \) admits a solution \( h^* \), it follows that for any \( \hat{h} \in [h(\hat{x}), h^*] \), it must be true that \( q(\hat{h}) > h^* \). That is, the expected asset quality among those who want to sell relatively faster (i.e., all sellers’ types below the cutoff type \( x = \phi_1(\hat{h}) \)) is higher than the asset quality of the cutoff type-\( x \) (the value of which is \( \hat{h} \) by construction). This is an important condition for which that a semi-pooling equilibrium with upward distortion exits, which I will discuss in detail later.
Now I show that a semi-pooling equilibrium can be constructed for each \(h \in [h(\hat{x}), h^*]\), according to the following three steps:

1. Given \(\hat{h}\), the marginal type \(x^*\) is given by \(x^* = \phi(\hat{h})\). Pool all types below the marginal type into one submarket.

2. Set the marginal type \(x^*\) to be indifferent between trading in the pooling market and trading in his own market, with the first-best outcome \((p^{FB}(x^*), \theta^{FB}(x^*))\).

3. Separate markets for each type above the marginal type by using downward-distorted market tightness, which solves (8) as before.

In other words, this semi-pooling equilibrium has two parts: one is a pooling submarket for all type below the marginal type and, a full separating market for all type above the marginal type. Given that we have learned how to construct full separating equilibrium from the baseline model, the new task here is to determine the price and trading probability in the pooling market so that the marginal type is indeed indifferent between trading in the pooling market and trading in his own market (Step 2). To this end, define the pair of functions \((p_q(x, \hat{h}), \theta_q(x, \hat{h}))\) which solves the following equations for any \(q(\hat{h}) > h(x)\):

\[
\begin{align*}
\theta_q(x, \hat{h}) &= \max_\theta \left\{ \theta | V(x, p, \theta) = V^{FB}(x) \text{ and } p = \frac{q(\hat{h})}{r} - \frac{k\theta}{m(\theta)} \right\} \\
p_q(x, \hat{h}) &= \frac{q(\hat{h})}{r} - \frac{k \cdot \theta_q(x, \hat{h})}{m(\theta_q(x, \hat{h}))}.
\end{align*}
\]
That is, the pair \((p_q(x, \hat{h}), \theta_q(x, \hat{h}))\) is solved subject to two constraints. The first constraint requires that a type-x seller obtains his first-best utility when going to the market \((p_q(x, \hat{h}), \theta_q(x, \hat{h}))\). The second constraint is the free-entry condition for buyers, when buyers’ value in this pooling is given by \(q(\hat{h})\). The solution pair \((p_q(x, \hat{h}), \theta_q(x, \hat{h}))\) to (12) is then the intersection point of \(V^{FB}(x)\) and the free-entry condition which gives an upward-distorted market tightness \(\theta_q(x, \hat{h}) > \theta^{FB}(x)\). A construction for equilibrium with upward-distorted market tightness is formally described in the following proposition:

**Proposition 6** If there exists an \(\hat{h}\) satisfying both of the following inequalities:

\[
q(\hat{h}) > \hat{h} \quad \text{(H1)}
\]
\[
V^*(x_L, \theta_q(x^*, \hat{h}), p_q(x^*, \hat{h})) \geq V^{FB}(x_L) \quad \text{(H2)}
\]

where the marginal type is given by \(x^* = \phi_1(\hat{h})\), then a semi-pooling equilibrium \(\hat{h}\) with upward-distorted market tightness exists. Such an equilibrium is characterized by the following equilibrium market tightness function and price function, respectively:

\[
\theta^*(x) = \begin{cases} 
\theta_q(x^*, \hat{h}) & \forall x \in [x_L, \phi_1(\hat{h})] \\
\theta(x; \theta^{FB}(x^*)) & \forall x \geq \phi_1(\hat{h}) = x^*
\end{cases}
\]
\[
p^*(x) = \begin{cases} 
p_q(x^*, \hat{h}) & \forall x \in [x_L, \phi_1(\hat{h})] \\
\frac{\hat{h}(x)}{\tau} - \frac{k\theta(x)}{m(\theta(x))} & \forall x \geq \phi_1(\hat{h}) = x^*
\end{cases}
\]

where \(\theta(x; \theta^{FB}(x^*))\) denotes the solution of (8) with the initial condition \(\theta^{FB}(x^*)\).

**Proof.** I now apply the method developed in the previous section to verify the above proposition. Recall that an equilibrium needs to satisfy sellers’ optimality condition \((E1)\), the free-entry condition \(E2(a)\), and buyers’ optimal price-posting \(E2(b)\). First of all, one can easily see that this construction satisfies Lemma 1, which guarantees sellers’ optimality condition \((E1)\). In particular, the market tightness in the pooling market is upward-distorted (i.e., \(\theta_q(x^*, \hat{h}) > \theta^{FB}(x^*)\)) so that monotonic condition (M) is satisfied. Since the marginal type is indifferent between the pooling market and his first-best market, all sellers’ types below the marginal type then strictly prefer the pooling market. Given that buyers’ valuation is strictly increasing after the marginal type \((h'(x) > 0 \text{ for } \forall x > x^*)\), a separating market for each \(x > x^*\) can then be solved as before, which guarantees that both \((E1)\) and the free-entry condition \(E2(a)\) are satisfied for each \(x > x^*\). Furthermore, as discussed earlier, the free-entry condition \(E2(a)\) holds for the pooling market by construction. Hence, the above construction satisfy both \(E1\), the free-entry condition \(E2(a)\).
What is left to show is that $E2(b)$ is satisfied: buyers do not find it profitable to post any price $p' \notin P^*$. In particular, notice that there is an upward jump in the equilibrium price at $x^*$ from $p_q(x^*, \hat{h})$ to $p^{FB}(x^*)$.\(^{21}\) According to Lemma 2, a buyer will only attract the marginal type $x^*$ if he posts a price $p' \in (p^{FB}(x^*), p_q(x^*, \hat{h}))$. Since the marginal type has already obtained his first-best utility, any price $p' > p^{FB}(x^*)$ and its corresponding $\theta(p')$ necessarily generates distortion. This can be seen clearly from fig X. Given that $\theta(p')$ is set to be the market tightness which guarantees the marginal seller the same level of utility, $V^{FB}(x^*)$, a pair $(p', \theta(p'))$ for $p' \in (p^{FB}(x^*), p_q(x^*, \hat{h}))$ will be a point on the marginal types’ first best utility curve, and above the buyers’ free entry condition: $p' > \frac{\hat{h}}{r} + \frac{kq(p')}{m(\theta(p'))}$. That is, a buyer suffers this additional distortion. Furthermore, a buyer will not benefit from lowering the price $(p' = p_q - \varepsilon)$ to attract $x_L$, since he cannot do better given condition $(H2): V(x_L, \theta_q, p_q) \geq V^{FB}(x_L)$.\(^{22}\) Evidently, for the same reason as in the baseline model, any price $p' = p(x_H) + \varepsilon$ is not profitable since it attracts $x_H$ while resulting in more distortion. As a result, the scheme in the above proposition also satisfies $E2(b)$; therefore, it can be decentralized as a competitive equilibrium outcome.

One special property of this type of semi-pooling equilibria is that there exists a large trading market where the market tightness is upward distorted (i.e., $q(x^*, \hat{h}) = p^{FB}(x^*)$). Any fire-sale equilibrium $(x_L, \theta_q, p_q)$ leads to an upward jump in the price.\(^{21}\) In the pooling market, the lowest type gets subsidies from the high type and therefore can achieve higher utility than his first best.

These phenomenon are consistent with the canonical model of fire sales (see Shleifer and Vishny (1992) and the recent survey by Shleifer and Vishny (2010)), where the undervalued price arises because high valuation buyers are assumed to be sidelined and distressed sellers are forced to sell at such a dislocated price. Nevertheless, the model generates such patterns without relying on an exogenous assumption about market pa-

\(^{21}\) Only when $\hat{h} = h^*$, there is no jump in the price $p_q(x^*, \hat{h}) = p^{FB}(x^*)$. Any fire-sale equilibrium $\hat{h} \in [h(\hat{x}), h^*)$ leads to an upward jump in the price.

\(^{22}\) In the pooling market, the lowest type gets subsidies from the high type and therefore can achieve higher utility than his first best.
anticipation, and moreover, it is indeed for a distressed seller to unwind their asset quickly at a undervalued price.

### 3.2.2 Equilibria with Ironing

Besides the equilibria discussed above, there exists another type of equilibrium which involves pooling but for which the market tightness is downward-distorted. The basic idea of such a construction is straightforward: given a non-monotonic function $h(x)$, bunch certain types of sellers and reconstruct a new function $\tilde{h}(x)$ so that the buyers’ valuation function $\tilde{h}(x)$ is weakly increasing. As this method is straightforward, the detail of such construction is left to the appendix. Nevertheless, one can understand the basic idea from the example shown below: there is a pooling region $x \in [x_1, x_2]$, and buyers’ expected value in this pool is exactly the marginal type: $h(x_1) = h(x_2) = E[h(\bar{x})|x_1 \leq \bar{x} \leq x_2]$. Since buyers’ value is strictly increasing in the first $(x \in [x_L, x_1])$, the market tightness function can be obtained as before. At the pooling interval, given $\theta^*(x_1)$ obtained from the first region, set $\theta^*(x) = \theta^*(x_1)$ for $\forall x \in [x_1, x_2]$. This means that the allocation is the same among this set of sellers, and buyers pay the expected asset quality in this pool. Hence, the free entry condition is satisfied automatically. In the last region, where $\tilde{h}(x)$ is strictly increasing again, so one can let $\theta^*(x)$ solve (8) with the initial condition $\theta^*(x_2) = \theta^*(x_1)$.\(^{23}\)

\(^{23}\)There is an additional condition for which $\theta^*(x)$ to be non-increasing in the last region. See appendix for details.
4 Will distress lead to a Fire Sale?

The empirical evidence on fire sales is mixed. Contrary to the canonical model of a fire sale, where, by assumption, distressed sellers are forced to sell at a price discount, the fire sale equilibrium in my framework is endogenous. Hence, the model is able to show when fire sales would actually take place under different information structures and different distress shocks.

I now use two simple examples of distress shocks to demonstrate how the market responds differently to these shocks. Both shocks are unique singularities, permanent, and unanticipated changes in the distribution of sellers’ holding costs. Before receiving the distress shocks, it should be assumed that all sellers have a low holding cost $c_L$. One can interpret that, during normal times, all banks are relatively solvent. After the shock, sellers’ distress positions are represented by $\tilde{c}$ and there are two types of shock: (1) Local distress shock: $\tilde{c} = c_H > c_L$ with probability $\lambda$ and $\tilde{c} = c_L$ with probability $(1 - \lambda)$; (2) Contagious distress shock: $\tilde{c} = c_L \cdot \varepsilon$, where $\varepsilon$ is uniformly distributed with with support $[1, \frac{c_H}{c_L}]$.

First of all, if there is no adverse selection (as in benchmark) and if such distress shocks can be perfectly observed (as in Sec. 3), trading price should reflect the fundamental asset. In particular, when there is adverse selection but such shock is public information, one can solve the equilibrium for each holding cost separately: there will be a fully separate equilibrium respect to asset quality, given $c$. Because of the nature of separation, there is no individual fire sale in the sense that all trading price reflects the asset fundamental. Yet, the model predicts a low aggregate trading price and volume compared with the case without adverse selection.

Now, consider the situation when the market participation cannot perfectly verify sellers’ holding costs. One can interpret this environment, for example, when such distress is driven by a bank’s exposure to its counterparties and, due to the complexity of banks’ interconnection, investors cannot identify who are exposed to the counterparty risk. A seller who wants to sell faster is driven by two forces: (1) he or she has a low quality asset, or (2) he or she is financially distressed.

Now, first consider the second type of distress shock, which affects everyone in a rather uniform way. As a result, the first force dominates: investors know that the ones who want to sell faster still have a lower asset quality on average. Hence, the market exhibits a full separation respect to sellers’ trading motives. In such an equilibrium, a seller may also sell at a price below what he or she would obtain without adverse selection, since the
price for each submarket is valued at the average asset quality given x. Nevertheless, due to the downward market tightness, sellers still sell their assets at a slower rate.

On the other hand, the first type of distress generates non-monotonicity, as illustrated in figure 2. Intuitively, investors know that sellers may want to sell fast because they are highly distressed and, on average, their asset quality is actually worth more than some patient sellers. The second force is therefore stronger than the first one, leading to the non-monotonicity case. I now apply the developed result to characterize the equilibrium outcome and prove the existence of the fire sale equilibrium.

4.1 An Example of Fire-Sales

In this simple example, the buyers’ value over x is then \( h(x) = E[s|s - c = x] \), where \( h: [s_L - c_H, s_H - c_L] \to [s_L, s_H] \). Since \( h(\cdot) \) is strictly increasing in x after \( x_1 \equiv s_H - c_H \), then, in light of Proposition 6, the semi-pooling equilibrium \( \hat{h} \) can be found when there exists an \( h^* \) which solves \( q(h) = h \). I now provide the restrictions on the parameters for the existence of such an equilibrium and then illustrate the equilibrium outcome when such parameter restrictions are satisfied. Notice that the set \( X \) is the domain of both functions \( h(\cdot) \) and \( G(\cdot) \). I focus on the case in which \( s_H - s_L > c_H - c_L \) so that both \( h(\cdot) \) and \( G(\cdot) \) have full support.

**Proposition 7** Given \( \{G(s), \lambda\} \) and \( \pi = \frac{c_L}{k} \), there exists a corresponding \( \bar{\Delta} \in (0, s_H - s_L) \) and \( \bar{k} > 0 \) such that a fire sale equilibrium exists when \( c_H - c_L > \bar{\Delta} \) and \( k > \bar{k} \).

**Proof.** See details in the appendix. According to Proposition 6, the conditions \((H1)\) and \((H2)\) are sufficient to guarantee the existence of a fire-sale equilibrium. Note that \( \bar{\Delta} \) and \( \bar{k} \) are all a function of the parameters (such as \( \{G(s), \lambda\} \) and \( \pi \)).
Since the existence of a fire-sale equilibrium is guaranteed by the above parameter restrictions, I can now construct a fire-sale equilibrium $\hat{h} \in (h(x_1), h^*)$ according to Proposition 6. That is, one can construct a semi-pooling equilibrium by pooling all sellers below the marginal type $x^* = \phi(\hat{h})$. An example of such a pooling scheme is shown in figure 5. Figure 6 shows the corresponding price function and market tightness ($p : X \to \mathbb{R}_+$ and $\theta : X \to \mathbb{R}_+$, respectively), with respect to the relevant type $x$. The flat schedule then represents the pooling submarket. As stated in proposition 6, the price $p_q$ and the market tightness $\theta_q$ have to solve simultaneously (1) the free-entry condition and (2) $V(x^*, \theta_q, p_q) = V^{FB}(x^*)$, subject to $\theta_q \geq \theta^{FB}(x^*)$. In this equilibrium, sellers entering the pooling market unload their assets quickly at the pooling price $p_q = \frac{q(h)}{r} - \frac{kgp_m}{m(\theta_q)}$, which is the buyers’ expected value from purchasing an asset in this pooling market minus the buyers’ expected searching cost.

5 Empirical Implications

5.1 On Adverse Selection

I now use the theoretical result to provide a testable implication on the effect of transparency of an asset on the trading volume and price in an environment with private information on the asset quality only (i.e., the holding cost is observable). Consider that different types of asset are subject to different levels of the adverse selection problem,
denoted by the parameter $\sigma$. In particular, the cash flow of assets is given by $d = y + \sigma s$, where $(y, \sigma)$ are the common observable components of one type of asset and $s$ is the sellers’ private information within this type of asset. Let $s$ follows a symmetric distribution $G(s)$ with mean zero, with support $s \in [-1, 1]$. For example, one can think of a Treasury bond as an asset without adverse selection by setting $\sigma = 0$, while there is some dispersion for the AAA mortgage-backed security $\sigma > 0$.

From equation (8), one can see that an asset with a lower trenchancy (a higher $\sigma$) leads to a more distorted market tightness. As a result, the model predicts that both the aggregate volume and the aggregate price will be lower for the asset with a higher degree of adverse selection. The comparator’s statics on transparency are summarized in the following proposition:

**Proposition 8** The asset with a higher trenchancy has a higher buyer-seller ratio, and therefore a higher aggregate trading volume. Formally, let $\theta^*(s; \sigma)$ denote the solution of (6) for asset $\sigma$, the following inequality holds when $\sigma' > \sigma > 0$:

$$\theta^*(s; \sigma') < \theta^*(s, \sigma) \text{ for } \forall s.$$ 

This prediction therefore provides an interpretation of the drop in the volume of AAA-MBS. Before 2007, the appreciation of the housing price and the senior tranches structure made AAA-MBS effectively a safe asset before the crisis. However, the drop in housing price has made some AAA-MBS, which were previously treated as safe assets, no longer risk free: certain assets, depending on the underlying composition, no longer deliver a high cash flow as promised.\(^\text{24}\)

The fact that the originators have more information of the credit quality and the underlying composition has been documented in the literature. In particular, Agarwal et al. (2012) examined the quality of mortgage loans that originated between 2004 and 2007. They showed that, in the subprime market, the only year where the evidence of adverse selection is found is 2007, when the real estate bubble started to burst. Such evidence therefore supports the view that the drop in the housing price generates a higher dispersion $\sigma$ within the MBS. The model then predicts that one will observe a large drop in the trading volume after 2007.

Furthermore, there is price dispersion within an asset class $\sigma$, as there is a different transaction price associated with each asset $s$. Hence, the model gives the following

\(^{24}\)See Gorton (2009) for a detailed description about how the chain of interlinked securities was sensitive to house prices and how asymmetric information was created via complexity.
testable prediction conditional on observable characteristics: (1) Across different asset class, an asset class with a higher price dispersion will have a lower aggregate trading volume; (2) Within an asset class, a higher individual transaction price is associated with a lower trading frequency. Both effects exist only in an environment with adverse selection. With complete information (i.e., $s$ is public information), the equilibrium buyer-seller ratio is only the function of the gain from trade (which is simply the holding cost). Hence, all assets should trade at the same rate. As a result, an increase in $\sigma$, by construction, will neither change the aggregate volume nor the aggregate trading price. Along with this line, Jankowitsch et al. (2011) shows that there is indeed a correlation between the price dispersion in OTC market and the trading volume (as well as common liquidity measures). Similarly, the second prediction will not hold without adverse selection.

5.2 On Fire Sales

The distinguishing feature of a fire sale equilibrium is that there exists a large market in which sellers trade fast (the market tightness can be upward distorted) at a common but depressed price. Such an outcome will not exist in a fully separating equilibrium or in an equilibrium with ironing, where market tightness is always downward-distorted: most sellers will hold on to their assets and trade more slowly. As a result, the equilibrium with fire sales predicts a larger amount of volume at a distressed price at the aggregate level.

Furthermore, in the non-monotonicity example constructed above, all sellers with a high holding cost sell their asset at a higher rate and at a common price. That is, the model predicts a strong relationship between sellers’ distress position and the propensity to sell. In this sense, distress does result in a fire sale. On the other hand, in the monotonicity case, the correlation between sellers’ distress positions and the trading probability is lower.

6 Discussion

6.1 On Efficiency

From the solution method, the set of feasible mechanisms $\alpha \in A$ is defined by Lemma 1, $E1$, and the buyers’ free entry condition, $E2(a)$. Within this set, the decentralized outcome is the one satisfying the additional condition $E2(b)$ on buyers’ optimal price-posting. Hence, to study efficiency, we must ask whether there exists an allocation $\alpha \in A$ that gives all traders weakly higher utilities than the decentralized outcome? that is,
whether the decentralized outcome is necessarily Pareto efficient.

The results show that as long as the monotonicity condition is satisfied (Assumption 1), the decentralized outcome has to be the least-costly fully separating equilibrium. A fully separating equilibrium, which implies distortions on market tightness, depends only on the range of the distribution. On the other hand, any kind of pooling equilibrium belonging to the set $A$, which involves distortions on the price, would obviously depend on the shape of the distribution. Whether a high-type seller is better off in a pooling equilibrium or a separating equilibrium then depends on which type of distortion is more costly. If a distribution has a wide range (implying a larger distortion on the market tightness) but a high expected mean (implying a higher expected price), a high-type seller is then better off in a pooling market. Hence, depending on the distribution, it is possible to construct a partial pooling equilibrium $\alpha \in A$ such that everyone is strictly better off.\footnote{Note that buyers always get zero and a low-type seller is obviously better off in the pooling market since he get subsidies from a high-type seller.}

This point then explains why the competitive search equilibrium is Pareto-inefficient for some parameter values in Guerrieri et al. (2010). The reason is that, depending on the distribution, the distortion on market tightness can be more costly. However, (partial) pooling cannot be sustained even when it is desirable.

For the same reason, when the monotonicity condition does not hold, the decentralized outcome is not necessarily Pareto-efficient. That would depend on which type of distortion is more costly and therefore would also depend on the underlying distribution.

### 6.2 Policy Implication

#### 6.2.1 Asset Purchase Program

This section focuses on the buyback policy, as we know that cleaning up a toxic asset will show a significant improvement in overall market liquidity. The idea of cleaning up a toxic asset in the market is not new. In particular, Tirole (2011) shows that intervention needs to take into account traders’ participation constraints and the government always strictly overpays for the worst assets. In our framework, since traders can choose either to trade their assets in the over-the-counter market specified earlier or to join the government’s scheme, traders join the scheme if and only if they are leaving the market. Hence, though we do know that cleaning up toxic assets would improve the market liquidity, what is important is to understand what price has to be paid in order to clean the market. Suppose
now the government offers the price $p_g$ to whomever shows up in the discount windows. Anticipating that traders’ utilities would increase in the future after government intervention, the original price and market tightness is then no longer incentive-compatible. Intuitively, a seller can now choose to hold on to the asset and claim a higher type in the future. Therefore, to solve for the equilibrium, a mechanism designer needs to take into account that sellers participate in the scheme only if they get at least as much as what they would have obtained in the decentralized market.

In the equilibrium, the sets of types who join in government intervention, $\Omega_g$, and of those who stay in the decentralized market, $\Omega_d$, are disjointed and satisfy $\Omega_g \cup \Omega_d \equiv S$. Agents’ utilities can be expressed as:

$$V(s) = \max\{ \frac{s-c}{r}, \max_{p'} U(p', \theta(p'), s), p_g \}$$

One can therefore pin down the condition for the marginal participant type $s^*$:

$$V(s^*) = p_g$$

That is, the marginal type has to be indifferent between trading in the OTC market and obtaining the transfer from the government right away. Let $(p^*, \theta^*)$ denote the price and the market tightness that the marginal participant type will be facing if he goes to the market. Obviously,

$$U_g(s) = p_g > \max_{p'} U(p', \theta(p'); \Omega_d) \text{ for } \forall s < s^*$$

$$U_g(s) = p_g < \max_{p'} U(p', \theta(p'); \Omega_d) \text{ for } \forall s > s^*$$

Also, from the previous discussion, we know how to solve the equilibrium outcome $p(\cdot), \theta(\cdot)$, given $\Omega_d$. The key task is to pin down the marginal type, which is characterized as follows:

**Proposition 9** A competitive search equilibrium with government buyback price $p_g$, is a marginal participant type $s^*$ which solves

$$V(s^*) = p_g = V^{FB}(s^*)$$

and a pair of $(p(s; \Omega_d), \theta(s; \Omega_d))$ that satisfies (5), (6),(7), as a solution to the market maker’s constrained incentive-efficient problem.
The figure below shows traders’ utilities with the buyback policy $p_g$, represented by the red line. Given any price offered by the government, the marginal types $s$ has to solve $V^{FB}(s^*) = p_g$, represented by the intersection of $p_g$ and the green line, which in turn is represented by $V^{FB}(s)$.

![Equilibrium with Buyback Policy $P_g$](image)

6.2.2 Disclosure of Sellers’ Trading Motives?

Is it necessarily better to have more information on sellers’ trading motives? From the previous analysis, we see that if sellers’ trading motives are observed, a high-type seller then suffers a trading delay; however, when the trading motives are unobserved, a high-type seller may suffer a different type of distortion. In particular, as shown in Section 4, a (semi-) pooling equilibrium can be sustained when the monotonicity condition does not hold. Hence, in an environment in which pooling is more desirable, traders can be better off when the private value of holding an asset is unobserved.

For illustration, consider the case in which there are two types of holding cost. When the holding cost is unobserved, as shown in the fire sale example, only the types with a low holding cost and relatively good asset qualities are separated while the rest are pooled. Sellers’ utilities in such an equilibrium are shown in figure 7, corresponding to the red line. Now compare this to the environment where the sellers’ holding costs are observable. The outcome can then be solved as in the baseline model but with a different holding cost. The utility of a type-$s$ seller with holding cost $c$ is then denoted by $V^*(s; c)$, which is represented by the black line in figure 7.

First of all, all sellers with a low holding cost ($c_L$) are better off when the holding
cost is unobserved: a seller with relatively high-quality asset \((s - c_L > x^*)\) is better off as the underlying range effectively decreases and therefore implies a smaller distortion; and a seller with relatively low-quality asset \((s - c_L < x^*)\) is also better off since each effectively receives subsidy from sellers with better assets in the pooling market. For a similar reason, a seller with a relatively low-quality asset but a high holding cost \((c_H)\) is also better off. Hence, the only type that might suffer when the holding cost is unobserved is the type with good assets but a high holding cost.

By the same logic discussed earlier, whether this high-type seller is better off or not depends on which distortion is more costly: a price discount or a distortion in market tightness. If the distribution is such that pooling is more desirable, the high-type seller is then better off in the pooling equilibrium. If that is the case, then, counterintuitively, all types of sellers achieve higher equilibrium utilities when the holding cost is unobserved (that is, when the red line is strictly higher than the black line for all types as shown in 7).

6.3 On the Searching Cost and the Elasticity of Matching Function

In this section, I investigate how the degree of the distortion on market liquidity corresponds to the change in the searching cost \(k\) and the parameter \(\rho\), which controls the concavity of the matching function \(m(\theta) = \theta^\rho\) and \(0 < \rho < 1\). For simplicity, the following comparative statics is done in the baseline model.
Note that from the sellers’ viewpoint, what matters is the arrival rate of a buyer, given by \( m(\cdot) \equiv m(\theta(s)) \); neither \( \kappa \) nor \( \rho \) play direct roles in sellers’ IC constraints, as shown in (??). However, these two parameters control the buyers’ marginal rate of substitution of \( m \) and \( p \): \( \frac{dp(s)}{dm}|_{\theta(s)=0} = -\frac{k(1-\rho)}{m}. \) Due to the nature of the matching, the equilibrium functions \( p(\cdot) \) and \( m(\cdot) \) are solved subject to the buyers’ free-entry condition; therefore, through this channel, \( \kappa \) and \( \rho \) have impacts on the resulting distortion of \( m \).

The level of the first-best liquidity is a function of \( k \) and \( \rho \), according to equation (1). However, the central question is how the degree of distortion relative to the first-best changes. To this end, I define \( d(s, \rho, k) \equiv 1 - \frac{m^*(s, \rho, k)}{m^{FB}(\rho, k)} \), which represents the percentage decrease of market tightness from its first-best level. Note that, in our basic model, \( m^{FB}(\rho, k) \) is independent of type.

As shown in the proposition below, the degree of the liquidity distortion decreases with \( k \) and increases with \( \rho \). The formal proof is in the appendix. The intuition is the following: A higher search cost \( k \) (or a lower parameter \( \rho \)) means a buyer is more willing to pay a higher price in exchange for a lower \( m \). Hence, when facing the schedule \( \{p(\cdot), m(\cdot)\} \), if buyers with a low \( k \) are indifferent among all submarkets, buyers with a higher \( k \) will be strictly better off in the market with lower \( m \). Hence, the decrease in the distortion has to be lower in the economy with a higher \( k \).

**Proposition 10** The degree of the liquidity distortion \( d(s, \rho, k) \equiv 1 - \frac{m^*(s, \rho, k)}{m^{FB}(\rho, k)} \) decreases with searching cost \( k \) and increases with \( \rho \).

The same intuition holds for the parameter \( \rho \). Notice that in the model of rationing, as in Guerrieri and Shimer (2011), buyers match with probability one when they are on the short side of the market. That is, buyers do not care about the market tightness since there is no congestion effect on the buyers’ side (the short side of the market). One can therefore understand the result in Guerrieri and Shimer (2011) as the limit economy in our model when \( \rho \to 1 \). As implied by the above proposition, the degree of the resulting distortion is expected to be larger in Guerrieri and Shimer (2011).

7 Conclusion

Using a competitive-search framework, this paper analyzes how asymmetric information on the common value, as well as the private value, leads to limited market participation, a trading delay, and an undervalued price. It provides an explicit and precise definition of
liquidity that integrates these two dimensions: trading price and trading rate. Contrary to the standard lemon model, in which trade is usually assumed to take place at one price, in my framework trade is allowed to take place at different prices, and the possible set of prices offered and their corresponding trading speed (which is controlled by the equilibrium market tightness) are jointly determined. In equilibrium, market thickness works as a screening device since owning assets of different quality generates different waiting preferences. Furthermore, how much the market can screen the agents depends on the degree to which sellers’ waiting preferences match up with buyers’ willingness to pay. If the types willing to wait longer are necessarily the ones with better assets (i.e., only the information on the common value is asymmetric), then a unique fully separating equilibrium is obtained. On the other hand, if a seller’s private value of the asset is also unobserved by the market and the types who are willing to wait longer are not necessarily the ones with better assets (which will depend on the underlying distribution), then the full screening cannot be sustained. Instead, a set of semi-pooling equilibria arise. In particular, we are interested in the equilibrium which corresponds the phenomenon of a fire sale. Compared to the previous literature, the main contribution of this work is to show how different market distortions on price and market thickness arise in different informational settings. It therefore separately identifies the effects of adverse selection on trading price, trading volume, and market segmentation, and further sheds light on the limited market participation of buyers and the infrequent trading observed in the recent asset markets.

A Appendix

A.1 Proof of Lemma 1:

Proof. Let \( V(\theta, s) \) denote \( V(p(\theta), \theta, s) \). Observe that \( V(\theta, s) \) satisfies the following properties: (1) \( V_2(\theta, s) \) exists and has an integrable bound: \( \sup_{s \in S} |V_2(\theta, s)| \leq \frac{M}{r} \) for all \( s \), where \( M = u'(s_L) \); (2) \( V(\theta, \cdot) \) is absolutely continuous (as a function of \( s \)) for all \( \theta \); (3) \( \theta^*(s) \) is nonempty. Following the mechanism literature, (see Milgrom and Segal (2002)), let

\[
V^*(s) = \max_{\tilde{s}} V(\theta(\tilde{s}), s) = \max_{\tilde{s}} \frac{u(s) + p(\theta(\tilde{s}))m(\theta(\tilde{s}))}{r + m(\theta(\tilde{s}))}.
\]

For any selection \( \theta(s) \) from \( \theta^*(s) \in \arg \max_{\theta'} V(\theta', s) \), it follows that:

\[
V^*(s) = V^*(s_l) + \int_{s_l}^{s} V_\theta(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s}.
\]
This means that (23) is the necessary condition for any IC contract. To prove the sufficiency, define the function: 

\[ x = q(\theta) = \frac{1}{r + m(\theta)}, \]

and \( q^{-1}(x) = \theta. \) Also, since \( \theta \geq 0, \) it follows that \( 0 < x \leq \frac{1}{r}. \) One can then easily see that \( V(x, s) \) satisfies the strict single crossing difference property (SSCD) under the assumption \( u'(s) > 0. \) For any \( x' > x \) and \( s' > s: \)

\[ V(x', s') - V(x', s) + V(x, s) - V(x, s') = x'(u(s') - u(s)) - x(u(s') - u(s)) > 0. \]

Therefore, \( V(x', s') - V(x, s') > V(x', s) - V(x, s). \) Given that \( V(x, s) \) satisfies the SSCD condition, then any non decreasing \( x(s) \) combined with (23) provides a sufficient conditions for the achievable outcome. Hence, \( x(s) = \frac{1}{r + m(\theta(s))} \) has to be solved subject to the non decreasing constraint. That is, the market tightness function \( \theta^*(\cdot) \) has to be non-increasing.

It is useful to characterize the set of allocations \( \{p(\cdot), \theta(\cdot)\} \) which satisfy the free entry condition. To facilitate the rest of the proofs, I now introduce the following notations: since the price must follow the free-entry condition \( p = \frac{H(p)}{r} - \frac{k\theta}{m(\theta)}, \) where \( H(p) \equiv E[h(s)|p(s) = p], \) sellers’ utilities can be rewritten as a function of market tightness and buyers’ expected value \( \tilde{h} \) in that market, which is denoted by \( \tilde{V}(s, \theta, \tilde{h}): \)

\[
\tilde{V}(s, \theta, \tilde{h}) \equiv \frac{u(s) + m(\theta)(\tilde{h} - \frac{k\theta}{m(\theta)})}{r + m(\theta)}. \tag{13}
\]

That is, \( \tilde{V}(s, \theta, \tilde{h}) \) denotes sellers’ seller’s utility of type-\( s \) when he enters the market with market tightness \( \theta \) and the expected value to buyers is \( \tilde{h}. \) Given a concave matching function, one can show that \( \tilde{V}(s, \theta, \tilde{h}) \) is a concave function in \( \theta. \) For given any \( \tilde{h} > u(s), \) let \( \theta^{FB}(h, s) \) denote the global maximizer of \( \tilde{V}(s, \theta, \tilde{h}) : \theta^{FB}(h, s) \equiv \arg\max_{\theta} \tilde{V}(\theta; s, \tilde{h}) \) and \( V^{FB}(h, s) \equiv \max_{\theta} \tilde{V}(\theta; s, \tilde{h}). \) Since \( \tilde{V}(s, \theta, \tilde{h}) \) is strictly increasing for \( \forall \theta \in [0, \theta^{FB}(s, \tilde{h})], \) let \( \theta^{D}(V; s, \tilde{h}) \) be the market tightness such that the type-\( s \) seller’s enjoys the utility level \( V \) with buyers’ value \( \tilde{h}, \) where the function \( \theta^{D}(\cdot; s, \tilde{h}) \) maps sellers’ utility level \( V \) to market tightness \( \theta : [u(s), V^{FB}(h, s)] \rightarrow [0, \theta^{FB}(s, \tilde{h})]. \) Similarly, since \( \tilde{V}(s, \theta, \tilde{h}) \) is strictly decreasing for \( \forall \theta > \theta^{FB}(s, \tilde{h}), \) define \( \theta^{U}(\cdot; s, \tilde{h}) : [u(s), V^{FB}(h, s)] \rightarrow (\theta^{FB}(s, \tilde{h}), \theta(s, \tilde{h})), \) where \( \theta(s, \tilde{h}) \) solves: \( \frac{\tilde{h}}{r} = \frac{k\theta}{m(\theta)} = \frac{u(s)}{r}. \)

To check buyers’ optimal price-posting condition \( E2(b), \) I first establish the following Lemma:

Lemma 6 Given a seller’s utility \( V < V^{FB}(s, \tilde{h}), \) any market tightness \( \theta' \in (\theta^{D}(s, \tilde{h}, V), \theta^{FB}(\tilde{h}, s)] \) and \( \theta' \in [\theta^{FB}(\tilde{h}, s), \theta^{U}(s, \tilde{h}(s), V)] \) gives a buyer a strictly positive profit.
Proof. Observe that $\theta^D(s, \tilde{h}, V)$ is decreasing in $\tilde{h}$ and $s$. Such a function can be interpreted as follows: By providing type-$s$ seller with utilities $V$, a buyer gets exactly zero payoff when he gets the asset with expected quality $\tilde{h}$ and faces the market tightness $\theta^D(s, \tilde{h}, V)$. For any $\varepsilon > 0$, take $\theta' = \theta^D(s, \tilde{h} - \varepsilon, V)$. The inequality $\theta^D(s, \tilde{h} - \varepsilon, V) > \theta^D(s, \tilde{h}, V)$ then implies $p(\theta') = \frac{\tilde{h} - \varepsilon}{r} - \frac{k\theta'}{m(\theta') < \frac{\tilde{h}}{r} - \frac{k\theta'}{m(\theta')}}$. That is, facing a higher market tightness $\theta'$, buyers can get a positive profit while providing a type-$s$ seller with utilities $V$, given a fixed asset quality $\tilde{h}$. Intuitively, due to the violation of the tangency condition (a downward-distorted market tightness $\theta^D(s, \tilde{h}, V) < \theta^{FB}(s, h)$), an increase in the market tightness $\theta' \in (\theta^D(s, \tilde{h}, V), \theta^{FB}(s, h))$ is Pareto-improvement. On the other hand, further downward-distorted market tightness leads to a negative payoff of buyers: $\theta' = \theta^D(s, \tilde{h} + \varepsilon, V) < \theta^D(s, \tilde{h}, V) \Rightarrow p(\theta') = \frac{\tilde{h} + \varepsilon}{r} - \frac{k\theta'}{m(\theta')} > \frac{\tilde{h}}{r} - \frac{k\theta'}{m(\theta')}$. In contrast, for any upward-distorted market tightness $\theta^{D'}(s, \tilde{h}, V)$, the function $\theta^D(s, \tilde{h}, V)$ is increasing in $\tilde{h}$. That is, a buyer will obtain positive/negative payoff with a decrease/increase in the market tightness. This proves the Lemma. Note that, this Lemma is only the first step to check buyer’s optimal price-posting condition, where $(s, \tilde{h})$ is now fixed. To complete the analysis, one needs to pin down the type who is most likely to come, which then gives prediction on $s$ as well as the asset quality $\tilde{h}$. ■

A.2 Proof of Lemma 2

Notice that $p^\alpha(\cdot)$ is non decreasing for all $\alpha \in A$, given the monotonic condition $(M)$ and the inequality $\theta^\alpha(s) > 0 \forall s$. Therefore, $T(p')$ is a singleton.\(^{26}\) That is, the type who is most likely to come is unique. For any $p'$ outside of the range of $p^\alpha$, $\theta(p', s) \equiv \inf\{\tilde{\theta} > 0 : U^s(p, \tilde{\theta}, s) \geq V^*(s; \alpha)\}$. Therefore, for any $p' > V^*(s; \alpha)$, which is the relevant case\(^{27}\) the

\(^{26}\)The only exception is when some types of sellers are out of the market. In this case, there then exists a marginal type $s^*$ such that $\theta(s) = 0$ for $\forall s > s^*$. For any $s > s^*$ and $p' > \frac{\theta(s)}{r}$, type-$s$ will come to the market even when $\theta(p', s) \rightarrow 0$. Hence, $T(p')$ is then a set of these types of sellers. Nevertheless, such an exception is not relevant for the equilibrium result since a buyer will deviate even when he expects the worst type within this set, as will become clear later.

\(^{27}\)In the case where $p' < V^*(s; \alpha)$, the function $\theta(p', s) = \infty$ because $U^s(p', \theta, s) \geq V(s; \alpha)$ has no solution. In words, if the deviating price is lower than a type’s equilibrium utility, this type will not come to this market.
market tightness value \( \theta(p', s) \) solves: \( G(p', \tilde{\theta}, s) \equiv U^*(p', \tilde{\theta}, s) - V^*(s; \alpha) = 0 \),

\[
\frac{d\theta(p', s)}{ds} = -\left( \frac{dG/ds}{dG/d\theta} \right) \propto \frac{1}{r + m(\theta^\alpha(s))} - \frac{1}{r + m(\theta(p', s))} = \begin{cases} < 0 \text{ if } p' > \theta^\alpha(s), \left( \therefore \theta(p', s) < \theta^\alpha(s) \right) \\ > 0 \text{ if } p' < \theta^\alpha(s), \left( \therefore \theta(p', s) > \theta^\alpha(s) \right) \end{cases},
\]

Recall that, when posting a new price \( p' \), a buyer should expect the lowest market tightness, \( \theta(p') = \inf_s \{ \theta(p', s) \} \), and the type most likely to come, \( T(p') = \arg \inf \{ \theta(p', s) \} \). The above result then implies that \( T(p') = s^+ \cup s^- \).

### A.3 Proof of Lemma 3: No Pooling

**Proof.** Proof by contradiction. Take a mechanism \( \{ p^\alpha(\cdot), \theta^\alpha(\cdot) \} \) which satisfies Lemma 1 and free entry condition, while \( p^\alpha(s) = p_p \) for \( s \in S' = [s_1, s_2) \subset S \). That is, there exists a subset of sellers \( s \in S' \) are in the same market \( (p_p, \theta_p) \). First of all, denote \( \tilde{h}(s) \equiv H(p(s)) \) and recall that \( H \) denotes the buyers’ expected value in the market with price \( p : H(p) \equiv E[h(s)|p(s) = p] \). Hence, given that \( h(s) \) is strictly increasing (Assumption 1), a pooling equilibrium implies that the function \( \tilde{h}(s) \) must exhibit an upward jump at \( s_2 \). Given that \( s_2 \) must be locally indifferent and \( \theta^\alpha(s_2, \tilde{h}, V^\alpha(s_2)) \) is continuous in \( \tilde{h} \), it follows that \( \theta^\alpha(\cdot) \) must jump downward and \( p^\alpha(\cdot) \) must jump upward at \( s_2 \).

Given that there is a jump at the price, I now show that buyers can deviate by posting a new price \( p' = p_p + \varepsilon \). Define the market tightness \( \hat{\theta}(p, V, s) \) which ensures that the seller of type \( s \) stays on his indifference curve \( U^*(s, p, \theta) = V \). That is, \( \hat{\theta}(p, V^\alpha(s), s) \) solves: \( m(\theta) = \frac{rV^\alpha(s) - (s - \varepsilon)}{p - V^\alpha(s)} \). Observe that \( \hat{\theta}(p, V, s) \) is unique and continuous in \( p \). Now, let \( p' = p_p + \varepsilon \). According to Lemma 2, the function \( T(p') = s_2 \). From the free entry condition and \( \frac{H(p_p)}{r} < \frac{s_2}{r} \):

\[
\frac{H(p_p)}{r} - \left( \frac{k\hat{\theta}(p_p, V^\alpha(s_2), s_2)}{m(\hat{\theta}(p_p, V^\alpha(s_2), s_2))} \right) - p_p = 0 \Rightarrow \frac{s_2}{r} - \left( \frac{k\hat{\theta}(p_p, V^\alpha(s_2), s_2)}{m(\hat{\theta}(p_p, V^\alpha(s_2), s_2))} \right) - p_p > 0.
\]

By continuity, there exists \( p' \in B_\varepsilon(p_p) \) such that

\[
\frac{s_2}{r} - \left( \frac{k\hat{\theta}(p', V^\alpha(s_2), s_2)}{m(\hat{\theta}(p', V^\alpha(s_2), s_2))} \right) - p' > 0.
\]

In other words, by positing the price \( p' \), buyers will attract \( s_2 \) and expect the market tightness \( \hat{\theta}(p', V^\alpha(s_2), s_2) \). The above inequality shows that such a deviation is profitable.
Finally, note that this result also holds even when there are no other markets open for all \( s > s_2 \). In that case, although \( T(p') \) is no longer uniquely defined, the possible types who will come to this market are all weakly better than \( s_2 \) (i.e., \( T(p') = [s_2, s'] \), where \( s' \) solves \( p' = \frac{u(s')}{r} \)). Hence, posting \( p' = p_{p} + \varepsilon \) is also profitable. ■

### A.4 Proof of Lemma 4

**Proof.** First, in a fully separating equilibrium, I claim that \( \theta^*(s) \) must be downward-distorted (that is, \( \theta^*(s) \leq \theta^{FB}(s) \)), and therefore \( \theta^*(s) = \theta^D(s, \tilde{h}(s), V^*(s)) \). Given that a separating equilibrium implies that \( \tilde{h}(s) = s \) and, furthermore, that \( V^*(s) \) must be Lipschitz continuous in \( s \), it follows immediately that \( \theta^*(s) \) is continuous in \( s \) and so is the price function \( p^*(s) = \frac{\tilde{h}(s)}{r} - \frac{k \theta^*(s)}{m \theta^*(s)} \). To understand why \( \theta^*(s) \) cannot be upward-distorted, observe that \( \theta^U(s, \tilde{h}(s), V(s)) \) is increasing in \( s \) and therefore it violates the monotonicity condition \((M)\) if \( \theta^*(s) = \theta^U(s, \tilde{h}(s), V(s)) \) for some interval \([s', s'']\) \( \subset S \). Furthermore, by definition, \( \theta^U(s, \tilde{h}(s), V(s)) > \theta^{FB}(s) > \theta^D(s, \tilde{h}(s), V(s)) \). Therefore, the only possibility for having the equality \( \theta^*(s) = \theta^U(s, h(s), V(s)) \) for some \( s \) is to set the value \( \theta^*(s_L) \) equal to \( \theta^U(s_L, h(s_L), V^*(s_L)) \), and after which the function exhibits a downward jump \( \theta^*(s_L^+) = \theta^D(s_L, h(s_L), V^*(s_L)) \). In words, the lowest type is indifferent between \( \theta^U(s_L, h(s_L), V^*(s_L)) \) and \( \theta^D(s_L, h(s_L), V^*(s_L)) \). However, this allocation necessarily imposes a distortion on the lowest type and therefore \( V^*(s_L) < V^{FB}(s_L) \).

The corresponding price schedule then also jumps upward at the point of \( s_L: p^*(s_L) = \frac{\tilde{h}(s_L)}{r} - \frac{k \theta^U(s_L, h(s_L), V^*(s_L))}{m \theta^U(s_L, h(s_L), V^*(s_L))} < p^*(s_L^+) = \frac{\tilde{h}(s_L)}{r} - \frac{k \theta^D(s_L, h(s_L), V^*(s_L))}{m \theta^D(s_L, h(s_L), V^*(s_L))} \). Now, consider a deviation of opening a new market \( p' = p(s_L^+) - \varepsilon \). According to Lemma 1, \( T(p') = s_L \), that is, a buyer can open a new market with a lower price and expect the lowest type to come; therefore, \( \theta' = \hat{\theta}(p', V^*(s_L), s_L) > \theta^D(s_L, h(s_L), V^*(s_L)) \). Hence, by Lemma 6, such a deviation is profitable. Clearly, the above argument holds whenever \( \theta^*(s_L) < \theta^{FB}(s_L) \). Hence, it follows immediately that \( \theta^*(s_L) = \theta^{FB}(s_L) \) ■

### A.5 Proof of Lemma 5

**Proof.** From the FOC of the first-best solution, one can solve \( \frac{d \theta^{FB}(s)}{ds} = \frac{h'(s) - u'(s)}{k(\frac{r}{J})^{1+\theta'}} \equiv f_2(\theta, s) \). Observe from the differential equation, that \( \frac{d \theta^*(s)}{ds} = f(\theta, s) \rightarrow -\infty \) at \( f(\theta^{FB}(s), s) \) given \( h_s > 0 \). Hence, we know that \( \theta^*(s) \leq \theta^{FB}(s) \) for some \( s_1 > s_L \). Suppose now that \( \theta^*(s) > \theta^{FB}(s) \) for some \( s \); this implies that these two functions must cross at some point \((\hat{s}, \theta^{FB}(\hat{s}))\) and that the slope of the crossing point must satisfy the following inequality:
\[ f_2(\theta^{FB}(\hat{s}), \hat{s}) < f(\theta^{FB}(\hat{s}), \hat{s}) = -\infty. \quad \text{Contradiction.} \]

### A.6 Proof of Lemma 3

**Proof.** Set \( z = s \) and \( x = s - c \). By the Jacobian of the transformation, the joint density of \( (z, x) \) yields:

\[
g(z, x) = f(s, c)\left| \frac{\partial(s, c)}{\partial(z, x)} \right| = f(z, z - x)
\]

\[
\frac{\partial}{\partial z \partial x} \ln g(z, x) = \frac{1}{f_2} \cdot \{f_2(f_1 + f_2) - f \cdot (f_1 + f_2)\}.
\]

It is well known that \( E[s|x] \) is increasing in \( x \) if \( \frac{\partial}{\partial z \partial x} \ln g(z, x) \geq 0 \), that is, if the variables \( s \) and \( x \) are affiliated (see Milgrom and Weber (1982)). When \( (s, c) \) are independent, \( \frac{\partial}{\partial z \partial x} \ln g(z, x) \propto [f'(c)]^2 - f''(c)f(c) \geq 0 \iff \ln f(c) \) is concave. ■

### A.7 Proof of Proposition 10:

**Proof.** Let \( \Delta(s; \rho, k) = \frac{m^*(s, \rho, k)}{m^{FB}(s, \rho, k)} \) and rewrite (6) with respect to \( m \), as follows:

\[
\frac{dm(s)}{ds} = \frac{-h_s(s) \cdot m(r + m)/r}{[g(s) - \frac{\xi}{\rho}(m^{1/\rho}(1 - \rho) + rm^{(1-\rho)/\rho})]} = F(m; \rho, k).
\]

Given (a) \( \frac{dF(\Delta, m^{FB}(\rho, k), \rho, k)}{dk} = \frac{\partial F}{\partial k} + \frac{\partial F}{\partial m^{FB}} \frac{dm^{FB}}{dk} > 0 \), and (b) the initial condition \( \Delta(s_L; \rho, k') = \Delta(s_L; \rho, k) = 1 \), then, by the comparison theorem, \( \Delta(s; \rho, k') > \Delta(s; \rho, k) \) for all \( s > s_L \). Hence, the distortion \( 1 - \frac{m^*(s, \rho, k)}{m^{FB}(s, \rho, k)} \) decreases with \( k \). Similarly, \( \frac{\partial F(\Delta, m^{FB}(\rho, k), \rho, k)}{d\rho} = \frac{\partial F}{\partial \rho} + \frac{\partial F}{\partial m^{FB}} \frac{dm^{FB}}{d\rho} < 0 \), and therefore \( 1 - \frac{m^*(s, \rho, k)}{m^{FB}(s, \rho, k)} \) increases with parameter \( \rho \). ■

### A.8 Proof of Proposition 4

**Proof.** Let \( \hat{x} \) be the strict local maximum: there exists some \( \varepsilon > 0 \) such that \( \hat{h}(\hat{x}) > \tilde{h}(x) \) when \( |x - \hat{x}| < \varepsilon \). Consider \( x_0 = \hat{x} - \frac{\varepsilon}{2} \), (given that \( \hat{h}(x_0) < \tilde{h}(\hat{x}) \) from the previous discussion), the market tightness for type-\( \hat{x} \) must therefore be downward distorted: \( \hat{\theta} \equiv \theta(\hat{x}) < \theta^{FB}(\hat{x}, h(\hat{x})) \). Given that \( \theta^{FB}(\hat{x}, h(\hat{x})) \) is continuous, exist \( x_1 > \hat{x} \) such that \( \theta(\hat{x}) < \theta^{FB}(x_1, h(\hat{x})) \) and \( x_1 \in B_\varepsilon(\hat{x}) \). The utility of seller-\( x_1 \) entering the market for seller-\( x_1 \) is as
follows:

\[
V(x_1, h(\hat{x}), \hat{\theta}) = \frac{u(x_1) + m(\hat{\theta})(\frac{\hat{h}(\hat{x})}{r} - \frac{k\hat{\theta}}{m(\hat{\theta})})}{r + m(\hat{\theta})} \\
\geq \frac{u(x_1) + m(\theta(x_1))(\frac{\hat{h}(\hat{x})}{r} - \frac{k\hat{\theta}}{m(\hat{\theta})})}{r + m(\theta(x_1))} \\
> \frac{u(x_1) + m(\theta(x_1))(\frac{\hat{h}(x_1)}{r} - \frac{k\hat{\theta}}{m(\hat{\theta})})}{r + m(\theta(x_1))} = V^*(x_1)
\]

The first inequality uses these facts: (1) \( \hat{\theta} < \theta^{FB}(x_1, h(\hat{x})) \) and (2) the monotonicity conditions ( \( \theta(x_1) \leq \theta(\hat{x}) \)). (Recall that \( V(x_1, h(\hat{x}), \theta) \) is decreasing in \( \theta \) when \( \theta < \theta^{FB}(x_1, h(\hat{x})) \)). The second inequality is given by \( \hat{h}(\hat{x}) > \hat{h}(x_1) \). This shows a contradiction to the IC condition: \( V^*(x_1) \geq V(x_1, h(\hat{x}), \hat{\theta}) \). Note that this proof relies on the fact that \( \hat{x} \) is a strict local maximum. Otherwise, if \( \tilde{h}(x_0) = \hat{h}(\hat{x}) \), then \( \hat{\theta} \) doesn’t need to be downward-distorted.

### A.9 Proof of Proposition 8

Let \( \theta^*(s_L; s'_L) \) be the equilibrium liquidity of market \( s_L \) when the worst asset quality is \( s'_L \). Given that \( \theta^*(s_L; s'_L) < \theta^{FB}(s_L) = \theta^*(s_L; s_L) \), the result follows from the Comparison Theorem.

### References


