Informational Lobbying and Competition for Access

Christopher Cotton
Cornell University

—Job Market Paper—

October 25, 2007

Abstract

There is substantial evidence that political contributions buy access to politicians. This paper incorporates access into a model of informational lobbying, then uses the access framework to analyze the impact of contribution limits on policy outcomes and representative citizen welfare. In the competition for access model, interest groups provide contributions to a politician and those that provide the highest contributions win access. A group with access can present verifiable evidence in favor of its preferred policy. Because equilibrium contributions are chosen endogenously, the politician learns about the evidence quality of all interest groups, even when he grants access to only some of the groups. A contribution limit reduces the amount of information available to the politician and tends to result in worse policy. Under a variety of assumptions, a limit has an unambiguously negative impact on representative citizen welfare. However, when the politician can choose whether to sell access or sell policy favors, a contribution limit can improve citizen welfare by making it more likely that the politician sells access.

\textit{(JEL D72, D44, D78)}

\textit{Keywords:} Lobbying, campaign contributions, contribution limits, bid caps, political access, all-pay auction, hard information

---

*Department of Economics, Cornell University, Ithaca, NY 14853; csc35@cornell.edu. I appreciate advice from Talia Bar, Kaushik Basu, Ted O’Donoghue, and especially Steve Coate, as well as comments from Dan Benjamin, Jayant Ganguli, Ben Ho, Justin Johnson, Joe Price, Josh Teitelbaum and seminar participants at Cornell, PET07, the 2006 Midwest Economic Theory Meeting, the 2006 North American Summer Meeting of the Econometric Society, and the 17th International Conference on Game Theory at Stony Brook. Mistakes are my own. An earlier version of this paper was titled “Informational Lobbying and Access When Talk Isn’t Cheap.” First version: February 2006.
Money doesn’t buy...a position. But it will definitely buy you some access so you can make your case.

- Thomas Downey (former US Congressman)

Doing what’s right isn’t the problem. It’s knowing what’s right.

- Lyndon B. Johnson

1 Introduction

In the United States, interest groups and lobbyists provide political contributions in an effort to gain access to politicians. Access allows a contributor to present evidence and arguments in favor of its preferred policy. Contributions typically are not provided in a quid pro quo exchange for policy favors. These statements not only summarize the claims of interest groups and policy makers (e.g., Herndon 1982, Schram 1995), they are also supported by empirical evidence (e.g., Langbein 1986, Milyo et al. 2000, Ansolabehere et al. 2002). Even campaign finance reform advocates argue that the current system is bad for society because it favors wealthy interests who can more easily buy access to politicians compared with less-wealthy interest groups and individuals (Makinson 2003).

This paper analyzes the impact of contribution limits under the assumption that political contributions buy access to politicians. I begin by incorporating access into a model of informational lobbying. The model relies on three fundamental assumptions that are consistent with the lobbying process as described by interest groups and politicians. First, interest groups have verifiable evidence that can influence a politician’s beliefs about the best course of action. If interest group evidence is completely unverifiable, or if the groups have no private evidence, then they do not have an incentive to gain access to the politician since access does not enable them to influence

---

1 For example, in Herndon (1982) an anonymous interest group representative stated: “About all you get [in exchange for a contribution] is a chance to talk to them... If you have a good case you can win them over. But you have to be able to talk to them.” Former US Senator Howard Metzenbaum said “Those who contribute may have more ready access and may at least be able to present their arguments with you whether you agree with them or not,” and former US Senator Dennis DeConcini said “What they got out of me for that contribution is access to come in... and to tell me why... it’s good ‘for America’” (Schram 1995).

2 See also Sabato (1984), Hall and Wayman (1990), Wright (1990), Clawson et al. (1992).

3 This view of political contributions and access is not limited to the United States. For example, Lee Rhiannon, an Australian politician, and Norman Thompson argue the following about their country: “Access is power, and money buys access to politicians in our country. This means large donors can influence governmental decisions, which benefit them and their companies” (Rhiannon and Thompson 2006).
the politician’s beliefs. Second, the politician controls who receives access. In order for an interest
group to present its information to the politician, the politician must grant the group access. Third,
the politician is constrained in his ability to provide access to all interest groups. Although the
analysis assumes that the politician simply cannot give access to all groups due to time constraints,
the results continue to hold if the politician can give access to everyone but faces a cost of doing
so. To my knowledge, no other paper makes all three assumptions.

Using the above assumptions as a foundation, Section 3 models competition for access in which
interest groups compete for access to a politician by providing political contributions. The groups
that provide the largest contributions win access, and can present evidence or arguments in sup-
port of their preferred policies. After observing the contributions from all groups and the evidence
presented by groups with access, the politician then implements the policies he believes are best.
The model shows how politicians may collect contributions from interest groups without sacrificing
voter welfare. Interest groups provide money to politicians even when the money buys access, not
explicit favors. Even more interesting is that in equilibrium interest groups with better evidence
provide larger contributions than those with worse evidence. The strict monotonicity of the equi-
librium contribution function means that the rational politician learns an interest group’s evidence
strength by observing its contribution, even if he does not grant the group access. By observing
contributions, the politician learns about the evidence of all interest groups, even when he only
provides access to some groups.\(^4\) When money buys access, allowing contributions moves policy
closer to the position that is best for the representative citizen.

A contribution limit distorts the signaling power of the contributions, and tends to result in
a less-informed politician and worse policy. This is the focus of Section 4, where, for the basic
competition for access model, I show that a contribution limit strictly reduces expected citizen
welfare. The main result is in contrast to much of the past lobbying literature in which money is
given to buy policy favors or to help preferred candidates win election (e.g., Grossman and Helpman
1994, Coate 2004a). In these previous models, a contribution limit tends to reduce the influence
interest groups have on policy, thereby improving expected citizen welfare.\(^5\)

\(^4\)However, it is the possibility of receiving access (in which case the politician learns one’s evidence quality for
sure) that drives the monotonicity of the contribution function. Therefore, it is essential that the politician gives
access to a positive number of groups.

\(^5\)See Section 2 for a review of the past literature.
The competition for access model analyzed in Sections 3 and 4 presents an optimistic view of money in politics, and stark results about the impact of contribution limits. However, the real world is far more complicated than the stylized competition for access model developed in Section 3. Sections 5 through 8 generalize the model in a variety of ways. Although some generalizations weaken the results, it remains clear that selling access tends to improve the politician’s ability to choose policies that benefit his constituents. In certain cases—when there is unobserved interest group heterogeneity or the politician can choose whether to sell access or sell policy favors—a carefully set contribution limit can improve expected citizen welfare.

One way to increase the realism of the model is to allow for interest group heterogeneity. For example, interest groups might differ in terms of wealth, opportunity costs of money, preference intensity, or distribution of evidence strength. By allowing for interest group heterogeneity, the model can directly address one of the most popular arguments in favor of contribution limits: that they level the playing field between wealthy interest groups and less-wealthy groups. This is the focus of Section 5.

Section 5.1 considers the case when interest group differences are known by the politician. This is the case for gun control policy, for example. Politicians recognize that the interest group against gun control, the National Rifle Association (NRA), is well financed and that interest groups in favor of gun control are relatively poor. Given that interest group asymmetries are observed by the politician, he can take these differences into account when updating his beliefs about an interest group’s evidence strength. The politician recognizes that a wealthy group chooses to contribute more than a less-wealthy group with the same quality evidence. Similarly, an interest group that cares intensely about an issue will contribute more than a group that cares less, all else equal. When there is interest group heterogeneity, each interest group may have a unique equilibrium contribution function that is determined by its own, individual characteristics. The rational politician, who is fully aware of these characteristics, can correctly determine the contribution functions for all agents (which continue to be strictly increasing in evidence quality), and he can thereby also correctly infer an interest group’s evidence strength given its contribution. In this case, contribution limits unambiguously result in a less informed politician who is less capable to identifying and implementing the socially optimal policy.

In Section 5.2, I allow for unobserved interest group heterogeneity. The analysis focuses on
wealth heterogeneity by assuming that some interest groups face a binding budget constraint while others do not. In this section, the politician is aware that interest groups may differ in terms of their ability to pay, but he is uncertain as to the characteristics of individual interest groups. The analysis shows that rich interest groups tend to realize higher equilibrium payoffs compared to poor groups, and that wealth differences may result in an equilibrium policy profile that is biased in favor of rich groups. Contribution limits can eliminate the rich-group advantage. However, I show that just because the contribution limit levels the playing field between rich and poor groups, this does not imply that a limit improves citizen welfare; although it can do so under certain conditions. Just as in the case without wealth differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about evidence quality. This can result in a less-informed politician and a policy profile that is worse for society. Although campaign finance reform advocates are correct in their claim that the exchange of contributions for access tends to favor wealthy interest groups, they are incorrect in concluding that contribution limits will therefore improve citizen welfare. Contribution limits do level the playing field between rich and poor interests, but they may also reduce the politician’s ability to identify and implement the welfare maximizing policy.

The initial model assumes a simple information structure in order to maximize intuition for the results. Section 6 discusses a variety of more general information structures that improve the realism of the model, including cases when an interest group’s evidence strength is not independent from other groups’ evidence, and when interest groups are uncertain as to how the politician will interpret their evidence. Section 7 endogenizes the politician’s choice of how many groups receive access.

Section 8 allows the politician to choose whether he sells access or sells policy favors. I show that selling policy maximizes contributions, but selling access results in higher policy utility for the politician (he is able to identify and implement the best policies). When the politician does not care enough about policy relative to contributions, he prefers to engage in the quid pro quo exchange of money for policy favors. When he puts enough weight on the policy outcome relative to contributions, the politician prefers to sell access.\(^6\) In this case, a carefully set contribution limit

\(^6\)The revenue results are consistent with Ansolabehere et al. (2003) who argue that total political contributions are significantly less than would be expected if money was given in the direct exchange for policy favors.
can improve expected citizen welfare by making it more likely that the politician sells access rather than policy. However, too strict of a limit is not optimal because it reduces the politician’s ability to identify and implement the socially optimal policy.

The paper concludes with Section 9 where I discuss the results, policy implications, and possible extensions and applications of the model.

2 Literature

This paper incorporates access into a model of informational lobbying. Although empirical and anecdotal evidence supports the idea that political contributions buy access, few theoretical papers incorporate access into their models. Past papers that do incorporate “access” into their models do so in a way that is largely inconsistent with the access story told by interest groups and politicians. Both Austen-Smith (1995) and Lohmann (1995) develop models in which interest groups receive private, unverifiable signals about the impact of a certain policy. In these models, an interest group who favors the policy will always claim it received a positive signal, independent of its true signal. However, groups that do receive positive signals have a higher expected benefit from the policy being implemented, and a higher expected benefit from the politician taking their signals into account when updating his beliefs. In equilibrium, the politician acts as if an interest group received a signal in support of its own position only if the group provides a large-enough contribution.

Although these models are considered “access” models, the possibility of face-time with politicians is completely irrelevant since interest groups will always claim that their signal was in favor of their own position. Equilibrium behavior in these models is clearly inconsistent with the story told

\footnote{For example, Langbein (1986) finds correlation between a politician’s contributions and the amount of time the politician spends meeting with constituents and interest groups. Ansolabehere et al. (2002) finds correlation between an organizations political contributions and their spending on lobbyists. This suggests that influencing policy requires both contributions to politicians (which secures access) and the careful presentation of information (which may require a lobbyist). Herndon (1982) surveys interest group representatives to determine their reason for contributing. All of the business interests in his survey emphasize access as a reason for contributing. Other interest groups, including labor unions, say that their main reason for contributing is to help their preferred candidates win election. Schram (1995) and Makinson (2003) interview retired politicians, as well as representatives from special interests and campaign finance reform advocacy groups. They offer substantial anecdotal support of the idea that money buys access to politicians.}

\footnote{In this way, these models are similar to the signaling models of Austen-Smith (1994) and Esteban and Ray (2006) in which interest groups provide contributions or engage in costly lobbying in order to provide a signal to a policy maker regarding their preferences.}

\footnote{In the basic model developed in Section 3 of this paper, interest groups do not care whether they win access in equilibrium since their evidence strength is fully revealed by their contributions. However, unlike in these other access models, it is essential that the politician grant access to a positive number of interest groups, otherwise, the}
interest groups and politicians, in which access itself plays an important role. Assuming that interest group information is verifiable results in a more realistic model of informational lobbying. To my knowledge, this paper presents the first access model in which access allows one to present verifiable information to a politician.

There is a significant theoretical literature that considers the role of money in politics, but ignores the issue of access. Most of the literature focuses on one of two motivations behind political contributions. First, contributions may help one’s favorite candidate compete for election (e.g., Coate 2004b). For example, the National Rifle Association may give money to candidates who hunt, and Planned Parenthood may give money to candidates who have made public statements against stricter abortion legislation. A politician (or political party) who requires money to fund his campaign may have an incentive to commit to policies that attract larger interest group contributions rather than the policies that are best for his constituents. Second, contributions may be intended to directly influence policy. In this case, politicians and interest groups engage in the quid pro quo exchange of contributions for favorable policy. In traditional rent-seeking models, interest groups compete (by providing contributions) for an explicit policy favor such as a government contract or favorable legislation (e.g., Tullock 1980, Baye et al. 1993, Che and Gale 1998). In the influential models of Bernheim and Whinston (1986) and Grossman and Helpman (1994, 1996), interest groups commit to a payment schedule that defines how much they contribute to a politician based on his policy choice. In Prat (2002a,b) and Coate (2004a), a politician who cares about winning election may provide policy favors to special interests in order to attract contributions to finance his campaign. In each of these models, the politician may choose policies that interest groups have no incentive to contribute in equilibrium. Furthermore, when there is a contribution limit or unobserved interest group wealth differences, certain interest groups do receive additional benefit from gaining access to the politician.

These models also make the unrealistic prediction that contributions are increasing in the distance between an interest group’s preferences and the preferences of the politician. However, data show that interest groups tend to give more money to politicians with similar preferences. Austen-Smith (1995) shows how this limitation of the model is eliminated when he assumes that politicians do not know the policy preferences of the interest groups. He argues that if one finds such an assumption unreasonable (which I do), then he may have to reject the belief that interest groups seek access in order to share evidence. I show that this conclusion is incorrect. In this paper’s competition for access model, access is present even though the politician knows the policy preferences of the interest groups.

For an excellent overview of the lobbying literature, see Grossman and Helpman (2002). See also other applications of an all-pay auction by Holt (1979), Holt and Sherman (1982), Baye et al. (1993, 1996), Anderson et al. (1998), Che and Gale (1998), and Moldovanu and Sela (2001). When applied to lobbying, these models require that a politician be willing to award policy favors to the highest contributors, even if the action hurts their constituents. In competition for access, interest groups bid for access rather than the prize itself, and the politician is free to choose his preferred policy at the end of the game. Interest groups are willing to pay for the opportunity to influence the politicians belief’s about the best policy.
favor wealthy special interests in order to attract contributions. In this way, contributions tend to distort policy away from that which is best for constituents. In the current paper, contributions have the opposite impact on policy, as they tend to move policy closer to the position that which is best for constituents.

There is also a significant literature on the disclosure of hard information; however, this literature ignores the issue of access all-together. Milgrom and Roberts (1986) consider conditions under which agents have an incentive to fully reveal their private information to a decision maker in a setting where all agents automatically receive access. Applied to the competition for access framework developed in this paper, their results establish that an interest group will always disclose its evidence once it receives access. Only an interest group with the worse possible evidence quality has the incentive not to reveal its evidence. Bennedsen and Feldmann (2002) consider the choice of interest groups to disclose information in a multiple-policy-maker setting. Bennedsen and Feldmann (2006) and Dahm and Porteiro (2006a,b) allow interest group to influence policy through both the disclosure of hard evidence and a quid pro quo exchange of contributions and policy favors. Papers such as Green and Laffont (1986), Lipman and Seppi (1995), and Bull and Watson (2004, 2007) formalize the concept of evidence. More-formal evidentiary structures may be incorporated into the competition for access model without changing the results.13 In these models, and throughout this literature, agents do not require access to disclose their information.

After showing that contributions enable to politician to choose policy that is better for his constituents, I formally consider the impact of a contribution limit on citizen welfare. This is not the first paper to analyze contribution limits, but it is the first paper to do so in a setting where contributions buy access to present hard evidence. Much of the previous literature on contribution limits considers the impact of limits in models in which politicians sell policy favors. In such a setting, limits decrease the monetary incentive to sell policy favors, and increase the likelihood that the politician chooses the policies preferred by his constituents (e.g., Prat 2002a,b, Coate 2004a). In this way, limits can have a welfare-improving affect on policy. However, not all papers suggest that contribution limits improve welfare. For example, in Drazen et al. (2007) limits can increase politically motivated government spending. In Wittman (2002) and Coate (2004b) limits decrease

13 The evidentiary structure must meet the evidentiary normality condition from Bull and Watson (2007), equivalently, the full reports condition from Lipman and Seppi (1995).
the amount of advertising, which results in a less-informed electorate. Coate (2004b) shows that a limit tends to redistribute welfare from ordinary citizens to interest groups. Where Wittman (2002) and Coate (2004b) show how a limit may result in less-informed voters, I show how a limit may also result in a less-informed politician. Additionally, a variety of other papers consider the impact that bid caps have on bidder behavior in auctions. For example, Che and Gale (1998) and Gavious et al. (2002) focus on the affect contribution limits have on total revenue.

3 Informational Lobbying and Access Game

In this section, I develop a simple model of competition between interest groups for access to a decision maker. By starting with a simple model (i.e., one with a simple information structure and without interest group heterogeneity), the analysis can maximize intuition for the results. As I show in later sections, one can improve the descriptive ability of the model without changing the primary results.

3.1 Model

There are $N$ independent policy issues. There is a risk-neutral politician who, for each issue, must choose a policy from a single-dimensional policy space defined by the interval $[-1, 1]$. There are a total of $2N$ interest groups, where for each of the $N$ issues one interest group prefers policy $-1$, and one group prefers policy 1. An interest group is denoted by the issue it is concerned with and its policy preference; therefore $(n, j)$ refers to the interest group concerned with issue $n \in \{1, \ldots, N\}$ and policy preference $j \in \{-1, 1\}$. Where it is clear which issue a group is concerned with, I refer to it as group $j$.

At the beginning of the game, each interest group draws private evidence in support of its preferred policy. The quality of $(n, j)$’s evidence is denoted by $e_{jn}$ and is the independent realization of a random variable distributed on the continuum $[0, 1]$. A higher $e_{jn}$ can be thought of as interest group $(n, j)$ having stronger evidence or a better argument in support of its preferred policy. The distribution of evidence quality is denoted by function $F$, with density function $f$, and is common knowledge.$^{14}$

$^{14}$The body of the paper assumes that the distribution of evidence quality is the same for all interest groups. This does not have to be the case. Alternatively, $F_{jn}$ could define the distribution of group $(n, j)$’s evidence. So long as
After the interest groups realize their evidence qualities, they independently provide contributions to the politician. Group \((n,j)\) provides contribution \(b^j_n \geq 0\). Interest groups receive access if they provide one of the \(K\) largest contributions, where \(K \in \{1, ..., 2N - 1\}\). If the \(K\)th and \((K + 1)\)th largest contributions are equal, then all of the groups that provide this same contribution receive access with equal probability.

Interest groups with access present their evidence to the politician. When a group presents its evidence, the politician becomes fully informed of the group’s evidence quality. Assuming that interest groups with access must present their evidence greatly simplifies the description of the game. However, the results do not change if groups are allowed to reject access. Let \(\Omega\) denote the vector of interest group evidence qualities revealed to the politician through access. After observing the contributions of all interest groups and the evidence quality of those with access, the politician chooses a policy for each of the \(N\) issues (by maximizing a payoff function defined later). Let \(p^*_n \in [-1, 1]\) denote the policy implemented by the politician for issue \(n\), and let \(p^* = \{p^*_1, ..., p^*_N\}\) denote the policy profile chosen for all issues.

The politician does not sell policy favors. Contributions determine whether an interest group receives access, but do not directly influence the policy choice of the politician. Contributions are non-refundable, and are not contingent on being granted access. Therefore, the exchange of access for political contributions is an all-pay auction: all bidders (interest groups) pay their bids (contributions) before the prizes (access) are allocated to the highest bidders.

The politician is fully informed about an issue if he is certain about the evidence quality of both the different distributions are common knowledge, the results of the analysis do not change.

Assuming that the evidence quality of the different interest groups is independently drawn simplifies the analysis. However, an alternative model can be developed in which the evidence quality of the two groups concerned with the same issue is correlated. For example, when one of the groups concerned with an issue has very strong evidence, it may be more likely that the other group concerned with the same issue has relatively low evidence. Under reasonable assumptions, such an alternative model would not change the results of the analysis.

\(^{15}\) If \(K = 0\) or \(K = N\), then interest groups provide no contributions in equilibrium. The basic model assumes that \(K\) is determined independently of the model. Section 6 discusses the case where \(K\) is endogenous. As Section 6 shows, the politician will commit to provide a positive amount of access \((K \geq 1)\) when \(K\) is endogenous.

\(^{16}\) It can be shown that interest groups with access will always accept. If a group rejects access, the politician believes that the group had lower evidence quality than he expected. This causes him to update his beliefs and lower his expectation. This results in an unravelling of beliefs until the politician believes any interest group that rejects an offer of access has the lowest possible evidence quality.

\(^{17}\) I use an all-pay auction to model competition for access because it seems the most realistic framework. The results continue to hold so long as the probability of winning access is non-decreasing in the size of a group’s contribution, which results in an equilibrium contribution function that is strictly increasing in an interest group’s evidence quality. The model could alternatively assume that the politician allocates access through another type of auction, a lottery in which one’s probability of gaining access is proportional to the relative size of a group’s contribution, or even if all groups receive access with equal probability independent of their contributions.
interest groups involved with the issue. When a politician is fully informed about an issue, he can determine the welfare-maximizing policy choice for that issue. Let \( p^o_n \) denote the socially optimal policy for issue \( n \), and the vector \( p^o = \{ p^o_1, ..., p^o_N \} \) define the vector of socially optimal policies across all issue. For simplicity, I assume \( p^o_n \) takes the form \( p^o_n = e^1_n - e^{-1}_n \). It is straightforward to incorporate a more complicated socially optimal policy function into the analysis, and reasonable changes to the function do not change the paper’s results.\(^{18}\)

**Payoffs**

Citizen welfare depends on the difference between the implemented policy and the socially optimal policy across all issues. The parameter \( \gamma_n > 0 \) represents the relative weight society places on issue \( n \). Citizen welfare is

\[
W(p^*, p^o) = - \sum_{n=1}^{N} \gamma_n \times |p^*_n - p^o_n|.
\]

Welfare is maximized when the politician implements the socially optimal policy profile \( p^* = p^o \). The politician can determine \( p^o \) with certainty only when he is fully informed about all issues.

The politician is concerned with citizen welfare and collecting political contributions. The parameter \( \rho \geq 0 \) represents how much the politician cares about revenue generation relative to citizen welfare, and \( b \) represents the profile of contributions from all interest groups. His payoff is

\[
U^P(p^*, p^o, b) = W(p^*, p^o) + \rho \sum_{n=1}^{N} (b^1_n + b^{-1}_n).
\]

An interest group’s payoff is decreasing in the size of its contribution and the distance between its preferred policy and the implemented policy for the issue it cares about. Group \((n,j)\)’s payoff is

\[
U^j_n (p^*_n, b^j_n) = V(|p^*_n - j|) - b^j_n.
\]

\(^{18}\)For any issue \( n \), the analysis requires that \( p^o_n (e^{-1}_n, e^1_n) \) be strictly decreasing in \( e^{-1}_n \), strictly increasing in \( e^1_n \), additively separable, and such that \(-1 \leq p^o_n (1,0) \leq p^o_n (0,1) \leq 1 \). The function \( p^o_n \) does not have to be linear in evidence quality. So long as the function is additively separable in terms of \( e^{-1} \) and \( e^1 \), the asymmetries between the impact of the two groups’ evidence on the optimal policy may be accounted for through a transformation of their evidence distribution functions \( F^j_n \). As stated previously, allowing for asymmetric distribution functions does not change the results of the analysis.

The model can be adapted to allow for a biased politician by changing the definition of the socially optimal policy. For each issue \( n \), let the fully informed politician implement policy \( p^o_n = \beta^*_n e^1_n - \beta^{-1}_n e^{-1}_n \), where each \( \beta^*_n \in (0,1] \) is common knowledge. When \( \beta^*_n > \beta^{-1}_m \), interest group \((n,j)\) will have a greater impact on the implemented policy than interest group \((m,i)\) by revealing the same quality evidence to the politician.
The function $V$ defines interest group policy utility where $V'(\cdot) < 0$, and $V(0) = 0$. Since interest group $(n, j)$ prefers policy $j$, $|p_n^* - j|$ denotes the distance between the implemented policy and the group’s preferred policy.\textsuperscript{19}

**States and Beliefs**

The realized state of the world is defined by the vector of realized evidence qualities $\{e_{jn}\}_{(n,j) \in \Omega}$. Let $S$ denote the set of all potential states of the world, and $s \in S$ denote an arbitrary state within the state space $S$. Each $s$ assigns a value $e \in [0, 1]$ to each interest group. Let $e_{jn}(s)$ denote the evidence quality of $(n, j)$ in state $s$. The function $\mu(\cdot | b, \Omega)$ defines the politician’s beliefs about the state of the world given the contribution vector $b$ and the vector of evidence revealed through access $\Omega$. These beliefs may be fully represented by the vector of all updated density functions $\{f_{jn}(\cdot | b, \Omega)\}_{(n,j) \in \Omega}$, where $\mu(s | b, \Omega) = \prod_{(n,j)} f_{jn}(e_{jn}(s) | b, \Omega)$. Also the operator $E$ represents the expectations given the ex ante distribution of states, and $E_\mu$ represents the politician’s expectations given his beliefs $\mu$.

**Solution Concept**

The analysis solves for the symmetric Perfect Bayesian Equilibrium of the game, which I label the contribution equilibrium. A complete description of the equilibrium must include the strategy profiles for the interest groups and the politician, as well as the politician’s beliefs about the state of the world at the time he chooses a policy profile. The politician’s beliefs must be consistent with using Bayes’ Rule on the ex ante distribution of evidence quality given the strategies of the interest groups. Each player’s strategy must be a best response to the strategies of the other players, given the player’s beliefs.

In the contribution equilibrium, all interest groups share the same contribution function $B : [0, 1] \rightarrow b$, where $B(e)$ defines the equilibrium contribution for an interest group with evidence quality $e$. The value $P_n^*(\mu)$ defines the politician’s equilibrium policy choice given his beliefs. A description of $P_n^*(\mu)$ for all possible $\mu$ and each $n$ fully describes the politician’s equilibrium strategy.\textsuperscript{20}

\textsuperscript{19} All interest groups have the same policy utility function. The results do not change if the utility functions differ, so long as they are common knowledge.

\textsuperscript{20} A formal definition of a contribution equilibrium requires some additional notation. Let $\Omega (b_n, s; B)$ define the vector of evidence qualities presented by interest groups with access in state $s$ when group $(n, j)$ contributes $b_n$, and all other groups contribute according to $B$. Let $\mu(s | e)$ denote the probability that an interest group puts on the world being in state $s \in S$ given that its own evidence quality is $e$. 

12
3.2 Equilibrium

I first determine the politician’s policy choice at the conclusion of the game, then analyze the all-pay auction in which interest groups choose the size of their contributions, and the highest contributors receive access.

Policy Choice

At the time the politician chooses policy, the interest groups have already given their contributions. This means the policy choice can only impact citizen welfare. The politician chooses the policy profile that maximizes expected citizen welfare given his beliefs. For each issue $n$, his policy choice $p_n^*$ is defined by the function

$$P_n^*(\mu) = E_\mu p_n^0 = E_\mu e_n^1 - E_\mu e_n^{-1},$$

where $E_\mu$ denotes the politician’s expectations given beliefs $\mu$. When the politician is fully informed,

$$E_\mu e_j^n = e_j^n, \quad \text{and} \quad E_\mu p_n^0 = p_n^0.$$ A politician who is fully informed about all issues implements the socially optimal policy profile and maximizes citizen welfare.

Interest Group Contributions

In equilibrium, all interest groups contribute according to the contribution function $B$. I start with the assumption that the contribution function is strictly increasing in a group’s evidence quality. After solving for $B$, I show that this assumption holds. Since $B$ is strictly increasing, it is invertible, where $e(b) = B^{-1}(e)$, and there is a one-to-one mapping between the group’s contribution and its evidence quality. It immediately follows that a rational agent can determine an interest group’s evidence quality if he observes its contribution.

To solve for the contribution function, the analysis considers the contribution decision of interest group $(n,j)$ assuming that all other groups contribute according to the equilibrium function. Because the other groups contribute according to $B$, the politician is certain regarding all other groups.

Definition 1 The interest group contribution function $B$, politician strategy $\{P_n^*\}_{n=1}^N$, and politician beliefs $\mu$ constitute a contribution equilibrium if

1. For all $e_n^1 \in [0,1],$
   $$B(e_n^1) \in \arg \max_{b_n} \int_{s \in S} \hat{\mu}(s | e_n^1) U_n^1 (P_n^* (\mu), b_n) \, ds$$
2. For any possible $b$ and $\Omega,$
   $$\{P_n^* (\mu)\}_{n=1}^N \in \arg \max_{p^*} \int_{s \in S} \mu(s | b, \Omega) U^{p^*} \left( p^*, \{p_n^0 (e_n^1(s), e_n^{-1}(s))\}, \{B(e_n^1(s))\}_{(n,j)} \right) \, ds$$
3. Beliefs $\mu$ meet the requirements of Perfect Bayesian Equilibrium, given the equilibrium strategy profile.

For a detailed description of Perfect Bayesian Equilibrium belief requirements, see Fudenberg and Tirole (1991, pp. 324-326).

$^{21}$ Also, $E_\mu e_n^1 = \int_{s \in S} \mu(s | b, \Omega) e_n^1(s) \, ds.$
groups’ evidence qualities. Let $\Theta \left( e; e_n^{-j} \right)$ be the probability that fewer than $K$ other interest groups have evidence quality greater than $e$, given that group $(n, -j)$ has $e_n^{-j}$. Therefore, $\Theta \left( e(b); e_n^{-j} \right)$ denotes the probability that group $(n, j)$ receives access given contribution $b$. Interest group $(n, j)$ chooses its contribution $b$ to maximize the expression:

$$
\int_0^1 f(e_n^{-j}) \left[ \left( 1 - \Theta \left( e(b); e_n^{-j} \right) \right) V \left( 1 - e(b) + e_n^{-j} \right) 
+ \Theta \left( e(b); e_n^{-j} \right) V \left( 1 - e_n^{-j} + e_n^{-j} \right) \right] de_n^{-j} - b.
$$

(1)

With probability $\Theta \left( e(b); e_n^{-j} \right)$ group $(n, j)$ receives access and presents its evidence to the politician who then chooses policy $e_n^{j} - e_n^{-j}$. This results in policy utility $V \left( 1 - e_n^{j} + e_n^{-j} \right)$ for the interest group. Alternatively, with probability $\left( 1 - \Theta \left( e(b); e_n^{-j} \right) \right)$ the group does not receive access and the politician believes the interest group has evidence quality $e(b)$ rather than its true evidence quality $e_n$. This results in interest group policy utility $V \left( 1 - e(b) + e_n^{-j} \right)$.

First order conditions for the interest groups’ problem are given by

$$
\int_0^1 f(e_n^{-j}) \left[ \left( 1 - \Theta \left( e(b); e_n^{-j} \right) \right) V' \left( 1 - e(b) + e_n^{-j} \right) \frac{\partial e}{\partial b} (1) 
+ \Theta \left( e(b); e_n^{-j} \right) V \left( 1 - e_n^{-j} + e_n^{-j} \right) \right] de_n^{-j} - 1 = 0.
$$

The first row of notation represents the marginal impact of a change in an interest group’s contribution on the politician’s beliefs about the group’s evidence quality provided it does not win access, and the corresponding change in the group’s policy utility. The second row represent the marginal impact of a change in a group’s contribution on the probability the group wins access.

In equilibrium, all interest groups contribute according to the function $B$, which implies $e(b) = e_n^{j}$ for all $(n, j)$. Strict monotonicity of the function means $(\frac{\partial e}{\partial b})^{-1} = \frac{\partial B}{\partial e}$. The first order conditions simplify to

$$
\frac{\partial B(e)}{\partial e} = -\int_0^1 f(e_n^{-j}) \left( 1 - \Theta \left( e_n^{j}; e_n^{-j} \right) \right) V' \left( 1 - e_n^{j} + e_n^{-j} \right) de_n^{-j}.
$$

It is straightforward to show that $\frac{\partial B}{\partial e}$ is positive.\(^{22}\) Therefore, the contribution function $B$ is strictly increasing in a group’s evidence quality. This also means that a group’s evidence quality is increasing in the size of its equilibrium contribution, and that the politician can correctly infer an

\(^{22}\)This follows because $f(e) > 0$, $(1 - \Theta(e)) \geq 0$ (with strict inequality for some $e$), and $V'(\cdot) < 0.$
interest group’s evidence quality by observing its contribution.

The closed-form solution for the contribution function is

\[ B(e) = -\int_0^e \int_0^1 f(e^{-j}) \left( 1 - \Theta(y, e^{-j}) \right) V' \left( 1 - y + e^{-j} \right) dy. \]  

In equilibrium, for any evidence quality \( e \), the benefit an interest group receives from bidding more than \( B(e) \) in an attempt to convey higher-quality evidence is completely offset by the cost of doing so.

**Game Equilibrium**

The above analysis derives the unique contribution equilibrium of the game. The first lemma summarizes the results.

**Lemma 1** In the contribution equilibrium,

1. \( b_n^j = B(e_n^j) \) for all \((n, j)\)

2. \( P^*_n(\mu) = E_\mu p_n^1 - E_\mu e_n^{-1} \) for all \( n \in \{1, \ldots, N\} \), and

3. the politician’s beliefs \( \mu \) are such that for any \((n, j)\), \( f_n^1(e_n^j | b, \Omega) = 1 \) if group \((n, j)\) has access, and \( f_n^1(e(b_n^j) | b, \Omega) = 1 \) if group \((n, j)\) does not have access.

In the contribution equilibrium, all interest groups contribute according to the same function \( B \), and the politician chooses the policy profile that he believes maximizes citizen welfare. Because the monotonicity of the contribution function allows the politician to learn the evidence quality of all interest groups, the politician knows the socially optimal policy profile at the time he chooses policies. It immediately follows that the politician implements the socially optimal policy profile. This result is stated by the first proposition.

**Proposition 1** In the contribution equilibrium, \( p^*_n = p_n^0 \) for all \( n \in \{1, \ldots, N\} \).

The first lemma and proposition follow directly from the above analysis.

Previous models of political contributions imply that contributions result in policies that benefit special interests, but decrease overall welfare (e.g., Grossman and Helpman 1994). This paper suggests that contributions can have the opposite impact on citizen welfare. In competition for
access, political contributions can have a positive impact on overall welfare because they enable the politician to recognize and implement better policies.

**Importance of Access**

In equilibrium, the politician becomes fully informed about the evidence quality of all groups by observing their contributions alone. This does not imply that the politician can provide no access. If the politician does not provide access to any group, then the contributions become uninformative. Without access, all interest groups face the same incentives when choosing their contributions; groups with high qualifications are no longer willing to provide larger contributions than groups with low qualifications. The politician recognizes this and does not take the size of the contributions into account when updating his beliefs. This means that \( E \mu e^j_n = E^j_n \) for all \((n,j)\), and the politician chooses \( p^*_n = 0 \) for all \( n \). Because contributions have no impact on the politician’s beliefs, the interest groups are unwilling to contribute anything, and the politician receives nothing.

If the politician provides no access, the interest groups provide no contributions and the politician does not learn anything about the groups’ evidence. However, providing access to at least one group allows the politician to become fully informed about the evidence quality of all groups. Section 5 endogenizes the amount of access.

4 **Contribution Limit**

The previous section assumes that there are no limits to the maximum size of interest group contributions. This section considers how the analysis changes if contributions are constrained.\(^{23}\)

The parameter \( \bar{b} \) denotes the maximum allowed size of a contribution, where \( b^j_n \in [0, \bar{b}] \) for all \((n,j)\). Assume \( 0 < \bar{b} < B (1) \), which implies that the contribution limit is lower than the maximum possible contribution in the game without contribution limits, and high enough such that contributions exist.\(^{24}\) If \( \bar{b} \geq B(1) \), the limit has no affect on interest group contributions when the politician sells access.

\(^{23}\)Che and Gale (1998) consider the impact of a contribution limit in a game where bidders in an all-pay auction compete for a policy favor (e.g., a government contract) rather than for access (as is the case in this paper). They show that even in the competition for policy favors, contribution limits can have a negative impact on social surplus.

\(^{24}\)When more interest groups provide the maximum contribution than the politician provides access to, I assume that the politician allocates access between each of the groups with equal probability. Alternatively, he could provide access to the interest groups involved with the issues that he cares the most about (those with the largest \( \gamma_n \)’s). This alternative assumption does not change the results, although it complicates the analysis.
When there is a contribution limit, the politician continues to implement the policy profile he believes is best for society, or \( P_n^* (\mu) = E_\mu e_n^1 - E_\mu e_{n-1}^1 \) for each \( n \). However, because contribution limits change the politician’s beliefs \( \mu \), they change the resulting policy profile.

**Impact of Contribution Limits**

If there did not exist a contribution limit, then groups with high-enough evidence quality would contribute more than \( \bar{b} \) in equilibrium. With a limit in place, groups with high \( e \) prefer to provide the maximum contribution \( \bar{b} \) compared to a lower amount; although some groups may prefer to contribute more than \( \bar{b} \) if it was allowed. Groups with relatively low \( e \) prefer to contribute less than \( \bar{b} \).

The appendix formally derives the equilibrium of the game with a contribution limit. Here, I describe the results. The equilibrium contribution function \( B_{CL} : [0, 1] \rightarrow b \) is a discontinuous function comprised of two parts: a continuous function \( \tilde{B} \) for low enough \( e \), and the constant \( \bar{b} \) for higher \( e \). Let \( \bar{e} \) denote the evidence quality of the interest group that is indifferent between contributing according to \( \tilde{B} \) and contributing amount \( \bar{b} \). Therefore,

\[
B_{CL} (e) = \begin{cases} 
\tilde{B} (e) & \text{for } e \in [0, \bar{e}) \\
\bar{b} & \text{for } e \in [\bar{e}, 1].
\end{cases}
\]

The function \( \tilde{B} \) is derived in the same way that \( B \) was derived in the game without contribution limits (I formally derive \( \tilde{B} \) in the appendix). \( \tilde{B} \) is strictly increasing in a group’s evidence quality. Therefore \( \tilde{B} \) is invertible and the politician becomes fully informed of the evidence quality of any group that provides a contribution according to this function. In contrast, when the politician observes contribution \( \bar{b} \), he cannot infer which value of \( e \in [\bar{e}, 1] \) resulted in such a contribution. The politician only learns with certainty the evidence quality of an interest group that provided the maximum contribution if he grants that group access. It is possible that more interest groups provide the maximum contribution than the politician grants access to. When this happens, the politician remains less than fully informed about the evidence quality of some groups, and cannot determine the policy profile that maximizes citizen welfare.\(^{25}\)

\(^{25}\)In the equilibrium of the no-limit game, all interest groups are indifferent between gaining access and not gaining access to the politician after they submitted their contribution. This is because their contributions communicate their evidence quality to the politician, and gaining access does not allow them to further impact the politician’s beliefs. This is not the case in the game with a contribution limit. The politician acts as if all interest groups that
An example of a contribution function $B_{CL}$ is provided by Figure 1.

The politician may no longer know each group’s evidence quality with certainty. The following proposition interprets this difference in terms of citizen welfare. Expected citizen welfare is strictly lower when there is a contribution limit than when there is no limit.

**Proposition 2** In the informational lobbying game, $EW(p^*, p^o)$ is strictly higher when there is no contribution limit than if there exists a contribution limit $\bar{b} \in [0, B(1))$.

Since the politician provides access to fewer than the total number of interest groups ($K < 2N$), there is a positive probability that the number of interest groups with $e \geq \bar{e}$ (the number that contribute $\bar{b}$) is larger than the number of groups that receive access. Since the politician only learns the evidence quality of a group that provides $\bar{b}$ if the group receives access, there is a positive probability that the politician is less than fully informed when he chooses a policy profile. A less than fully informed politician almost certainly chooses a policy profile that is different from the social optimal. Therefore, contribution limits strictly reduce expected welfare. This does not mean that the realized welfare is necessarily lower. Rather, contribution limits never improve realized citizen welfare, and they reduce realized welfare with positive probability.

provide the maximum contribution (and do not gain access) have the same expected evidence quality. The groups that have evidence quality above this expected level are made better off if they gain access, since access results in the politician learning that their evidence quality is higher than his expectations.
5 Interest Group Asymmetries

The previous sections assume that interest groups all have the same wealth, preference intensity, and distribution of evidence quality. In reality interest groups are not homogeneous. Section 5.1 allows for interest group asymmetries when these differences are observed by the politician. In many situations, assuming that the politician is aware of interest group asymmetries seems the most reasonable approach. However, it may not always be the case, and in Section 5.2 I consider the case in which differences in interest group budget constraints are unobserved by the politician.

Throughout Sections 5.1 and 5.2, I focus on wealth differences instead of preference intensity, evidence distribution, or other possible heterogeneities because it allows me to address issues central to the policy debate. Campaign finance reform advocates often argue that the contributions-for-access system favors wealthy interest groups relative to less-wealthy interest groups and individuals. They claim that limiting or banning contributions reduces the rich-group advantage and therefore increases citizen welfare. The analysis illustrates the flaw in this logic. In Section 5.1, limits never improve citizen welfare. In Section 5.2, a contribution limit can level the playing field between rich and poor interest groups; however, I show that this does not imply that the limit also improves citizen welfare.

5.1 Known differences

If the politician observes asymmetries between the interest groups, then the main results of the model remain unchanged. Contributions continue to provide the politician with information about interest group evidence strength, and contribution limits tend to reduce the accuracy of the politician’s beliefs.

There are two simple ways to incorporate wealth inequality into the model. First, interest group utility functions may put different weights on contributions. In this case, rich group utility

---

26So long as the politician observes the differences between the interest groups, it is not required that interest groups themselves are aware of the characteristics of other groups.

27An implicit assumption here is that there are multiple rich interest groups. When this is the case, the politician may continue to grant access to the highest bidders. Alternatively, the politician may grant access to the interest groups who’s contributions signal the highest evidence strength, controlling for the groups’ wealth. Under this alternative mechanism, a rich group that gave the highest contribution may not receive access if a poor group’s contribution represents higher information quality. In the formulation of wealth in this section, either mechanism is sufficient to generate the results. However, if wealth differences are drawn from a continuum, the alternative mechanism will continue to generate the results, while the original mechanism may not continue to do so.
functions put a smaller weight on money compared with poor groups.\textsuperscript{28} Second, some interest groups may face binding budget constraints.

Consider the first way of incorporating wealth differences. Let $\beta^j_n$ denote the weight that interest group $(n,j)$’s utility function puts on its contribution, where $\beta^j_n \in \{\beta_R, \beta_P\}$ and $\beta_R > \beta_P$. Therefore, the interest group’s payoff is given by

$$U^j_n(p^*_n, b^j_n) = V(|p^*_n - j|) - \beta^j_n b^j_n.$$ 

The value $\beta^j_n$ is observed by group $(n,j)$ and the politician.\textsuperscript{29} One can show that, in equilibrium, a group’s contribution is decreasing in $b^j_n$, all else equal. The more costly an interest group finds providing a contribution, the less the group will contribute given the same quality evidence. Because the politician knows how much interest groups care about money (he knows each group’s utility function), he can account for these differences when updating his beliefs. When two groups give the same contribution in equilibrium, and the politician knows that one of the groups cares more about money than the other group, then the politician will correctly infer that the interest group that cares more about money must have higher-quality evidence compared to the group that cares less about money. The rational politician can correctly derive each group’s individual contribution function, and he can therefore also infer each group’s evidence quality. Without a contribution limit, the politician remains fully informed about interest group evidence quality, and can choose the socially optimal policy in equilibrium. A contribution limit has a similar impact on each individual contribution function as it has on the common contribution function in Section 4.

The above explanation applies to cases when group asymmetries do not limit the ability of some groups to provide a contribution. Conceivably, interest groups might differ in terms of their budget constraints. In this alternative case, the politician may be less than fully informed about interest group evidence quality even when there is no limit. However, one can show that even though the politician may not be able to identify and implement the socially optimal policy, imposing a contribution limit tends to result in an even less informed politician and worse policy choice than when there is no limit.

\textsuperscript{28}Incorporating wealth differences in such a way has similar implications as allowing groups to have different policy utility functions or ex ante distributions of evidence quality.

\textsuperscript{29}The analysis does not require that an interest group’s characteristics are observed (or unobserved) by the other groups, so long as the distribution of the characteristic is common knowledge.
In both of the cases considered here, a contribution limit reduces the ability of groups with relatively high-quality evidence to communicate their evidence strength to the politician. This affect is greatest for rich groups. Since poor groups contribute less than similar rich groups, the contribution limit is less likely to prevent a poor group from contributing its preferred amount than it is to prevent a rich group from doing so. Although a contribution limit can improve the expected payoff of poor groups relative to rich groups, a limit reduces expected citizen welfare. As in Section 4, a contribution limit make the politician less informed about evidence quality.

5.2 Unknown wealth differences

In this section, some interest groups (the poor) face a binding budget constraint, while other groups (the rich) do not. The politician does not know which groups are rich and which are poor, although he knows the distribution of types. Unlike in the case when asymmetries are known to the politician, here it is possible for a contribution limit to improve citizen welfare. However, this is not generally true, and a contribution limit often reduces citizen welfare even when it eliminates biases in favor of rich groups.30

The model with unobserved wealth differences differs from the game presented in Section 3 as follows. Each interest group is rich with probability $\alpha$, and poor with probability $(1 - \alpha)$, where the parameter $\alpha \in (0, 1)$ is common knowledge. A poor group faces a binding budget constraint such that its contribution must be less than $\omega$.31 Each interest group knows its own wealth, but does not know the wealth of other interest groups. The rest of the model is unchanged.

The first subsection describes the equilibrium of the game; first for when there is no contribution limit, then for when there is a limit. The second subsection shows how rich groups tend to receive higher payoffs compared to similar poor groups, and how a contribution limit can eliminate this payoff inequality. In the third subsection, I show that contribution limits often reduce citizen welfare even when they eliminate the rich group advantage. The details regarding the analysis is provided in the appendix, Section 10.2.

---

30To keep things simple, I focus on the case when wealth differences are represented by differences in budget constraints. To consider the case with differences in utility function weight on contributions ($\beta$ from Section 5.1), one needs to formulate and solve a complex multi-dimensional mechanism design problem.

31For the budget constraint to be binding, $\omega$ must be less than the amount an interest group with the highest quality evidence ($e_j = 1$) would want contribute in equilibrium if there was no budget constraint. Formally, $\omega < B(1)$ where $B$ is the contribution function derived in Section 2.
5.2.1 Equilibrium Contribution Functions

No Contribution Limit

I consider the symmetric Perfect Bayesian Equilibrium of the game, which I label the contribution equilibrium with wealth differences. A description of the new equilibrium requires an explanation of two different contribution functions: one for rich groups, and one for poor groups. The following lemma describes the contribution functions of the game with wealth differences. The functions $B_P$ and $B_R$ respectively denote the poor and rich group contribution functions.

**Lemma 2** In the contribution equilibrium with wealth differences, there exists cut off values $\bar{e}_a \geq 0$ and $\bar{e}_b \in (\bar{e}_a, 1)$, and functions $B_a$ and $B_b$, where $B_a (0; \omega) = 0$, $B_b (\bar{e}_b; \omega) = 0$, $\frac{\partial B_a}{\partial e} > 0$, and $\frac{\partial B_b}{\partial e} > 0$ such that

\[
B_P (e; \omega) = \begin{cases} 
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a) \\
\omega & \text{for } e \in [\bar{e}_a, 1], \text{ and} 
\end{cases}
\]

\[
B_R (e; \omega) = \begin{cases} 
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a) \\
\omega & \text{for } e \in [\bar{e}_a, \bar{e}_b) \\
\omega + B_b (e; \omega) & \text{for } e \in [\bar{e}_b, 1]. 
\end{cases}
\]

Figure 2 illustrates example rich and poor group contribution functions.

The wealth constraint has a similar impact on the poor group contribution function $B_P$ as a contribution limit had on the contributions of all interest groups in Section 4. For low enough $e$, a wealth constrained interest group’s contribution is strictly increasing in its evidence quality, and for higher $e$ the group contributes $\omega$.\(^{32}\) This means that when the politician observes a contribution not equal to $\omega$, he can accurately infer the interest group’s $e$. However, when he observes a contribution equal to $\omega$ and does not grant the interest group access, he remains uncertain about the true evidence quality of the group and acts as if the group has $e = E_{\mu} (e \mid \omega)$.\(^{33}\)

\(^{32}\)The cutoff value $\bar{e}_a \geq 0$ represents the evidence quality at which an interest group is indifferent between providing a contribution according to function $B_a$, and providing contribution $\omega$. If the group provides contribution $B_a (\bar{e}_a)$, then the politician can correctly infer the group’s evidence quality. If instead the group provides contribution $\omega > B_a (\bar{e}_a)$, then the politician learns the group’s true evidence quality only if the group receives access. If the group does not receive access, which happens with positive probability, the politician overestimates the interest group’s evidence quality. For an interest group with evidence quality $\bar{e}_a$, the benefit from contributing $\omega$ is completely offset by the cost of doing so.

\(^{33}\)The paper generally assumes that the highest $K$ contributors receive access. This assumption makes the analysis more straightforward. Similar results will follow from an analysis that allows interest groups who contribute $\omega$ to receive access before groups who contribute more than $\omega$. 

22
For $e \leq E_\mu (e \mid \omega)$ rich groups provide the same contributions as poor groups with similar evidence in the symmetric equilibrium. This is because rich and poor groups only differ in terms of the budget constraint. Therefore, a group’s preferred contribution is independent of its wealth, and when a poor group prefers its chosen contribution to any other $b \geq 0$, a rich group with the same $e$ prefers to contribute the same amount. Alternatively, for $e > E_\mu (e \mid \omega)$ both rich and poor groups prefer to provide more than $\omega$, but only the rich group can afford to do so. Therefore, rich interest groups with high enough $e$ contribute more than similar poor groups. Poor interest groups cannot afford a higher contribution, therefore poor groups with $e > E_\mu (e \mid \omega)$ provide $\omega$ in equilibrium instead. It follows that $\bar{e}_b = E_\mu (e \mid \omega)$.

**Contribution Limit**

In considering relative payoffs between the different wealth types, I focus on the case where contribution limit $\bar{b}$ is no greater than $\omega$. This ensures that all interest groups can provide the maximum contribution if they choose to do so. Focusing on this range of limits is most consistent with the policy debate in which limits are intended to eliminate the ability of wealthy interest groups to outspend less wealthy groups. The appendix describes the impact that a higher limit has
on the contribution functions.

Since $\bar{b} \leq \omega$, both rich and poor interest groups have the same equilibrium contribution constraint. This means that in the symmetric equilibrium, the contribution function is independent of wealth, and is the same as $B_{CL}$ in Section 3. See Figure 1 for an illustration. There exists a cut off value $\bar{e} \in [0, 1)$ such that an interest group with $e < \bar{e}$ contributes $\bar{B}(e)$, and an interest group with $e \geq \bar{e}$ contributes $\bar{b}$.

## 5.2.2 Rich Group Advantage

When there is no contribution limit, in equilibrium, poor interest groups with evidence quality greater than $\bar{e}$ prefer to contribute more than $\omega$ (just as the rich groups do), but they are prevented from doing so by their budget constraint. Therefore, rich groups with high-quality evidence have a higher probability of communicating their evidence to the politician compared with poor interest groups with similar quality evidence. This can result in policies that are on average biased in favor of the rich groups, and it does result in rich groups having higher expected payoffs than similar poor groups.

If a rich interest group and a poor interest group have opposite policy preferences involving the same issue, the politician will tend to choose a policy that favors the rich group, all else equal. This is stated by the following lemma.

**Lemma 3** If $(n, j)$ is rich, $(n, -j)$ is poor, and $e_n^j = e_n^{-j}$, then $E\pi_n^* > 0$ when $j = 1$ and $E\pi_n^* < 0$ when $j = -1$.

Furthermore, a group’s expected payoffs are higher when it is rich than when it is poor. This is because rich interest groups are always able to provide the contribution that maximizes their expected payoffs, but poor interest groups face a budget constraint that prevents them from doing so for some values of $e$. This result is stated by the following proposition.

**Proposition 3** $EU_n^j$ is at least as great, and is strictly greater for some $e_n^j \in [0, 1]$, when $(n, j)$ is rich than when $(n, j)$ is poor.

A contribution limit can eliminate the bias in favor of rich groups. When there is a contribution limit, an interest group’s contribution and the probability that the group receives access are
independent of its wealth. This means that policies do not tend to favor rich groups, and that an interest group’s expected payoff is independent of its wealth. This is stated by Proposition 4.

**Proposition 4** If $\bar{b} \leq \omega$, then (1) $E_{p_n^*} = 0$ for all $n$ and (2) $EU_{n,j}$ is independent of wealth for all $(n,j)$.

A contribution limit can completely eliminate the rich group advantage. This is consistent with the argument made by many campaign finance reform advocates. However, it is important to recognize that these results says nothing about citizen welfare. I discuss the welfare implications in the following section.

### 5.2.3 Impact on Welfare

This section considers the impact that a contribution limit has on expected citizen welfare when interest groups have different levels of wealth. I show that under certain parameter values, contribution limits can improve expected welfare. However, this is not generally the case. Just because a contribution limit eliminates the impact of wealth on interest group payoffs, this does not imply that the same limits improve citizen welfare.

In the game with wealth differences, as in the game without differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about their $e$. Without a limit, the politician learns with certainty the $e$ of any rich group with high enough evidence, or any group (rich or poor) with $e < \bar{e}_a$ even if these groups do not receive access. A contribution limit can reduce the range of $e$ for which an interest group contributes according to a strictly increasing function, and for which the politician learns a group’s evidence quality regardless of access. This means that a limit can result in the politician being less informed when he chooses a policy profile, which tends to reduce citizen welfare.

A limit does not necessarily have this affect, and can improve welfare under certain parameter values. For example, a contribution limit at $\bar{b} = \omega$ can increase the probability that a group who gave $\omega$ in the no limit game receives access. Depending on the model parameters, the benefit of increasing the probability that the groups who had given $\omega$ receive access may be greater than the expected costs of not always learning the evidence quality of the rich groups that gave more than $\omega$ before the limit. This is because when there is no contribution limit, the politician may tend to
have significantly inaccurate beliefs about the e of those that give ω and do not receive access, and when there is a contribution limit, the politician may have relatively accurate beliefs about the e of groups for whom he does not learn the e with certainty but for whom he would have learned e if contributions were not limited (the rich interest groups that give more than ω when there is no limit). When this is true, the contribution limit can result in the politician having more accurate beliefs and choosing better policies, on average. Although this is possible, the conditions that must be met for a limit to improve citizen welfare may be very specific, and one should not conclude that contribution limits generally have a positive impact on welfare.

To illustrate the potential welfare impact of a contribution limit, I consider a very simple competition for access game that allows for the explicit solution of different equilibrium variables and payoffs. Suppose that evidence quality is uniformly distributed on [0, 1], and that interest group policy utility is linear, or \( V(1 - e^j + e^{-j}) = -(1 - e^j + e^{-j}) \). There is a single issue with two interest groups, and one group receives access, or \( K = 1 \). The appendix provides details regarding the solution of this game. Here, I briefly describe the results.

In this simplified game, there exists cut off values \( \alpha' \), \( \omega'_L(\alpha) \), and \( \omega'_H(\alpha) \) such that contribution limit \( \bar{b} \leq \omega \) improves expected citizen welfare if and only if \( \alpha \in (\alpha', 1) \), \( \omega \in [\omega'_L(\alpha), \omega'_H(\alpha)] \), and \( \bar{b} \in \left[\frac{1}{4}, \omega\right] \). The values \( \omega'_L(\alpha) \) and \( \omega'_H(\alpha) \) depend on the value of parameter \( \alpha \), and determining the values \( \alpha' \) and \( \omega'_H(\alpha) \) require calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. \( \omega'_H(\alpha) \) is greater than \( \omega'_L(\alpha) \) only when \( \alpha \) is high enough, implying that \( \alpha' \) is approximately 0.750427. This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any \( \alpha \), the range of \( \omega \) for which a limit can have a positive impact is even more restrictive. For any \( \alpha > \alpha' \), \( \omega'_H(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260199, and \( \omega'_L(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260751. At its maximum, the difference between \( \omega'_H(\alpha) \) and \( \omega'_L(\alpha) \) is approximately 0.00215, meaning that for any value \( \alpha \), only a very narrow range of \( \omega \) results in the contribution limit being beneficial. Furthermore, because \( \omega'_H(\alpha) \) is close to \( \frac{1}{4} \), even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected citizen welfare.

The results have two important implications. First, unlike in the game without wealth differ-
ences, a contribution limit can improve citizen welfare when there is wealth inequality. For the limit to actually do so may require that a large enough portion of interest groups are poor, and that the contribution limit is high enough (among other possible restrictions). Imposing a low limit, or banning all contributions, has a strictly negative impact on expected citizen welfare in the simple example above. Second, the result that a contribution limit eliminates the advantage that rich groups tend to have over poor groups does not imply that the limit improves citizen welfare. This means that the logic behind the popular argument in support of contribution limits is flawed. As the above example illustrates, the contribution limit often reduces welfare even when it eliminates the bias in favor of wealthy groups.

6 Alternative Information Structures

Up to this point, the analysis assumes a simple information structure. Interest groups know their own evidence, and their evidence quality is the independent realization of a random variable. Although the information structure presented in Section 3 is simple, the model and analysis are robust to a variety of generalizations. Here, I discuss some of these refinements, which help improve the real-world representation of the model.

6.1 Comments on the Evidence Structure

An interest group’s evidence quality represents the impact that presenting its evidence has on the policy the politician believes is best. I have not modeled evidence itself, only the impact that evidence has on policy. However, one could incorporate a more formal model of evidence into the paper without changing the results of the analysis. For example, one could assume that an interest group’s evidence is made up of a collection of verifiable documents or facts, and when groups receive access they choose which pieces of their evidence to reveal to the politician. Such a definition of hard evidence has been formalized in a variety of papers (e.g., Green and Laffont 1986, Lipman and Seppi 1995, Bull and Watson 2004, 2007), although it has not been applied to games in which revealing evidence requires access. Under general assumptions about the evidentiary structure, incorporating such a formal model of evidence into this paper will not change the results.\footnote{For the results of the analysis to hold, the evidentiary structure must meet Bull and Watson (2007)’s normal condition, or, equivalently, Lipman and Seppi (1995)’s full reports condition. When interest group \((n, j)\)’s type is}
Additionally, it is important that each interest group knows all of the evidence in favor of its own preferred policy. Otherwise, the equilibrium outcome is not first-best in terms of citizen welfare. In equilibrium, an interest group with access has an incentive to reveal all of the evidence in favor of its preferred policy position, and none of the evidence against its preferred position. No interest group besides \((n, j)\) has an incentive to present evidence in favor of policy \(j\) for issue \(n\). If \((n, j)\) does not have all of the evidence in its favor to present, even if that evidence is known to some other group, then the politician will not learn about the evidence in equilibrium. This also implies, however, that the politician becomes fully informed about the socially optimal policy whenever each group has all of the evidence in favor of its own position, even if groups also have information against their own positions. Evidence against \((n, j)\)’s preferred policy is evidence in favor of \((n, -j)\)’s preferred policy. Therefore, although \((n, j)\) will not reveal evidence against its position, this evidence will be revealed by group \((n, -j)\) and the politician remains fully informed.

6.2 Correlated Evidence Quality

The main analysis assumes that an interest group’s evidence quality is independent of other group evidence quality. It is reasonable, however, that if one group has a strong case in favor of its preferred policy, then the opposite group concerned with the same issue may tend to have a weaker case. Incorporating correlation (or negative correlation, as the case may be) of evidence quality into the model does not change the results.

To see this, let the function \(F(\cdot | e^j)\) denote the expected distribution of group \(-j\)’s evidence quality from the perspective of group \(j\), given that \(j\) realized its own evidence quality \(e^j\). Denote the density of this distribution by function \(f(\cdot | e^j)\). For any \(e^j\) and \(e \in [0, 1]\), \(f(e | e^j) > 0\). The density function is determined by the ex ante distribution of evidence quality, and the correlation between group \(j\) and \(-j\)’s evidence quality.

This alteration of the model has minimal impact on the interest groups’ optimization problem. In the game without evidence correlation, interest group \((n, j)\) chooses a contribution to solve Eq. 1. When evidence is correlated, \((n, j)\) chooses a contribution to maximize an equation that is identical to Eq. 1, except that the ex ante density function \(f\) is replaced by the revised density given by the variable \(e^j_n\) (for any \(e^j_n \in (0, 1]\)). These conditions ensure that \((n, j)\)’s evidence is sufficient to distinguish it from groups with lower types (i.e., \(e < e^j_n\)).
function $f(\cdot | e^j)$. Solving the problem in the same way that Section 3 solved Eq. 1, one can show that with correlated evidence, an interest group with information $e$ contributes according to the function

$$B(e) = -\int_0^e \int_0^1 f(e_n^j | y) (1 - \Theta(y, e_n^j)) V'(1 - y + e_n^j) \, de_n^j \, dy.$$ 

Just as in Section 3, the contribution function is strictly increasing in an interest group’s evidence strength. Therefore, the results from the earlier analyses continue to hold. The politician can correctly infer an interest group’s evidence strength from its contribution, and imposing contribution limits tends to reduce the information available to the politician and result in worse policy decisions.

### 6.3 Uncertain Evidence Quality

Up to this point, I assume that interest groups know exactly how the politician will react to their evidence. Alternatively, one may assume that interest groups observe a signal that is correlated with their evidence quality, but they remain uncertain as to its true value until they reveal their evidence to the politician. One can show that, in this case, an interest group’s contribution is strictly increasing in its signal; just as it was increasing in its true evidence quality in the previous sections. In equilibrium, the politician correctly learns the interest groups’ signals by observing their contributions, but he remains uncertain regarding a group’s actual evidence quality unless he provides the group access. When interest groups are uncertain regarding their true evidence quality, contributions lead to the politician having more accurate expectations about the socially optimal policy than he does without contributions or if contributions are limited.\footnote{Unlike in Section 3, citizen welfare is not independent of the level of access. In this case, citizen welfare is increasing in the number of groups that receive access. This is because access allows the politician to learn a group’s evidence with certainty, rather than only learn the signals received by the groups.} Similar to the case when groups know their $e$ values with certainty, a contribution limit decreases the expected accuracy of the politician’s beliefs, and results in lower expected citizen welfare than when contributions are unconstrained.
7 Endogenous Access Choice

The previous sections assume that the number of interest groups that receive access $K$ is determined exogenously. In this section, I relax this assumption, and allow the politician to choose the amount of access, where $K \in \{0, 1, ..., K_{\text{max}}\}$.\textsuperscript{36} The politician commits to $K$ at the beginning of the game, before the interest groups provide contributions.\textsuperscript{37}

First, I show that the politician always provides access to some groups ($K \geq 1$). If the politician provides no access ($K = 0$), there is no possibility an interest group gets caught if it signals higher quality evidence than it actually has. When there is no possibility of receiving access, interest groups with high quality evidence no longer have an incentive to provide larger contributions than groups with low quality evidence. Instead, contributions are independent of evidence quality and the politician can no longer infer anything about a group’s evidence quality by observing its contribution. This eliminates any incentive for the interest groups to provide contributions. Therefore, if the politician chooses no access, he learns nothing about the interest groups’ evidence, and receives no contributions. By choosing a positive $K$ he receives contributions and learns about group evidence quality (as I show in the previous sections). Thus, the politician will always commit to providing a positive amount of access.

When there are no contribution limits and interest groups are certain about their evidence quality, the politician becomes fully informed about the evidence quality of all interest groups independent of the amount of access (see Section 3). In this case, the amount of access he chooses to provide only impacts total contributions since he becomes fully informed so long as $K \geq 1$. One can show that expected total contributions are strictly decreasing in the number of interest groups that receive access.\textsuperscript{38} Therefore the politician chooses the minimum, positive amount of access.

\textsuperscript{36}Moldovanu and Sela (2001) consider the optimal choice of $K$ in a traditional all-pay auction in which bidders benefit from receiving a prize (not from the act of bidding, which is the case here).

\textsuperscript{37}If the politician chooses $K$ following the contribution decisions of the interest groups, there exists $K_{\text{max}} + 1$ contribution equilibria. In each of these equilibria, the politician becomes fully informed of all interest groups’ evidence quality, and implements the policy profile that maximizes citizen welfare. When there are contribution limits, the politician is less than fully informed with positive probability. This means that contribution limits strictly reduce expected citizen welfare, independent of which contribution equilibrium is achieved.

\textsuperscript{38}Access can be interpreted as the politician’s monitoring of interest group contributions in order to limit the groups’ ability to overrepresent their true evidence quality. When an interest group increases its contribution, it benefits from the increase in the evidence quality the politician believes it has only when it does not win access. When the politician reduces the number of groups that receive access, the action increases the probability an interest group does not win access given any contribution, thereby increasing the potential benefit to an interest group from increasing its contribution. This is true for all groups independent of their evidence quality. Therefore, when the politician reduces the number of interest groups that receive access, all groups increase their contributions, except
\(K = 1\), which results in the politician maximizing contributions while becoming fully informed about interest group evidence quality.

This result is inconsistent with what one observes in reality: politicians provide access to more than one interest group. The result is driven by the simplifying assumptions of the basic model, and is eliminated when there are contribution limits (Section 4), interest groups have unobserved wealth differences (Section 5.2), or interest groups are uncertain about their evidence quality (Section 6.3). When the model makes any of these more realistic assumptions, the politician does not necessarily choose the lowest amount of access. When the politician lowers \(K\), he receives higher contributions, but also learns with certainty the evidence quality of fewer interest groups. The politician’s access decision trades off contributions for evidence, and it is not possible to determine the equilibrium choice of \(K\) without making further assumptions regarding the weight the politician places on contributions relative to citizen welfare, or the ex ante distribution of evidence quality. Although one does not know which \(K\) the politician will choose in the game with contribution limits, it is likely more than one. Just as in the previous sections, it is straightforward to show that a contribution limit strictly reduces expected citizen welfare.

Furthermore, when politicians spend time meeting with a greater number of interest groups, they may do so at the cost of spending less time in their home districts meeting with constituents. When the politician can choose the amount of access to provide, he may choose to meet with a greater number of interest groups when there are contribution limits compared with when there are no limits. To the extent that this choice reduces the amount of time the politician spends meeting with constituents, contribution limits may further reduce citizen welfare. However, I do not explicitly model this trade off here.

### 8 Selling Access Versus Selling Policy

This section considers the conditions under which a politician prefers to sell access rather than sell policy favors. The results establish that selling access maximizes the politician’s policy utility but does not maximize expected revenue. Selling policy favors, on the other hand, maximizes revenue but not policy utility. Therefore, a politician who cares enough about the policy outcome relative to for those that already provide the maximum contribution amount (in the case of contribution limits). This is also the reason why revenue equivalency (of auctions) does not apply to in a competition for access.
revenue prefers to sell access, otherwise he prefers to sell policy. This suggests that politicians are more likely to sell access to interest groups concerned with issues which constituents are passionate about (those with a high $\gamma$ in the citizen welfare function), and are more likely to explicitly sell policy favors regarding issues about which constituents care less. When the politician is free to choose whether to sell access or policy favors, a contribution limit can make selling access relatively more favorable.

Throughout this section, I make a couple of assumptions that simplify the analysis. Relaxing these assumptions will complicate the analysis, but will not change the results. First, interest group policy utility is linear, where $V(x) = -v x$. Therefore, $U_j^n(p^*_n, b^i_j) = -v |p^*_n - j| - b^i_j$. Additionally, there is only one issue or $N = 1$.

In this section, the politician chooses whether to sell access, or to explicitly sell a policy position. He commits to the policy mechanism at the beginning of the game before interest groups submit bids or contributions. If he sells access, he does so using the competition for access mechanism developed earlier in the paper. Since $N = 1$, he awards access to a single interest group. Alternatively, if he sells policy favors, I assume that he does so using a winner-pay auction in which the high bidder gets to select the policy. Obviously, the high bidder chooses its own (extreme) preferred policy. One could incorporate a more complex money-for-prize mechanism such as a menu auction (e.g., Bernheim and Whinston 1986, Grossman and Helpman 1994), lottery (e.g., Tullock 1980), or all-pay auction (e.g., Che and Gale 1998) into the game in place of the winner-pay auction. Under these alternative mechanisms, the politician still prefers to sell access when he cares enough about the policy outcome relative to revenue. However, changing the way in which the politician sells policy will alter the parameter values for which the politician is indifferent between selling access and selling policy favors.

### 8.1 Selling Access

When the politician chooses to sell access, the game precedes as it did in the earlier sections of the paper. In equilibrium, both interest groups contribute according to the function

$$B(e) = v \int_0^e (1 - F(y)) dy$$  \hspace{1cm} (3)
which is a restatement of Eq. 2 given that there are only two interest groups and policy utility is linear. Because the contribution function is strictly increasing in $e$, there exists a one-to-one mapping between each group’s contribution and its evidence quality. In equilibrium, the politician correctly infers the evidence quality of both interest groups, even though he only gives access to one of them. Therefore, the politician identifies and implements the socially optimal policy, where $p^* = p^o$. No other mechanism can result in higher policy utility for the politician. However, this does not imply that selling access necessarily maximizes the politician’s utility since he also cares about collecting contributions.

The expected revenue from selling access is

$$2 \int_0^1 f(e)B(e)de.$$  

Given that the politician’s policy utility is maximized at 0 when $p^* = p^o$, the politician’s expected utility when he sells access is equal to his expected revenue, or

$$EU^P = 2vp \int_0^1 f(e) \int_0^e (1 - F(y)) dyde.$$  

(4)

### 8.2 Selling Policy

Now, consider the case when the politician sells a policy position rather than access. In the symmetric equilibrium of the winner-pay auction, each of the interest groups bid $2v$, which equals the benefit that an interest group receives when the politician implements its preferred policy compared to the preferred policy of the other group. The bids are independent of whether the auction is a first-price or second-price auction. In this case, the politician earns $2v$ in equilibrium.\(^{39}\)

When the politician sells policy, his expected revenue is higher than if he sells access. However, achieving this higher amount of revenue comes at the cost of no longer being able to identify or implement the socially optimal policy. Instead, the politician implements an extreme policy. Given

\(^{39}\)Remember that in the setting analyzed here, interest groups differ in terms of their qualifications rather than their valuations. Because interest groups share the same benefit from winning the auction, and this benefit is known to both interest groups, the bids and expected payments are the same for both a first-price and second-price winner-pay auction.

\(^{40}\)In fact, choosing a policy by selling the policy choice to the highest bidder is a expected-revenue maximizing mechanism. For this framework, there does not exist a mechanism for choosing policy that results in higher expected revenue for the politician than $2v$. I leave the proof of this to the interested reader.
that both interest groups share the same distribution of evidence quality and that \( p^o = e^1 - e^{-1} \), the policy implemented when the politician sells a policy favor (either \(-1\) or \(1\)) averages a distance of \(1\) from the socially optimal policy. This results in expected citizen welfare and politician policy payoff of \(-\gamma\).\(^{41}\) In equilibrium, the politician’s expected utility when he sells a policy favor is \(2v\rho - \gamma\).

### 8.3 Selling Access v. Selling Policy

Given the politician’s payoffs when he sells access and when he sells policy, it is straightforward to determine when he prefers each course of action.\(^{42}\) The politician prefers to sell access when

\[
2v\rho \int_0^1 f(e) \int_0^e (1 - F(y)) dy de \geq 2v\rho - \gamma.
\]

This simplifies to

\[
\frac{\gamma}{\rho} \geq 2v \int_0^1 f(e) \left[ 1 - e + \int_0^e F(y) dy \right] de.
\]

This result is restated in the following proposition, where \( \bar{z} \) is the right hand side of Eq. 5.

**Lemma 4** Consider the game in which the politician chooses whether to sell access or policy favors. There exists a cut-off value \( \bar{z} > 0 \) such that

1. if \( \frac{\gamma}{\rho} \geq \bar{z} \), then the politician sell access, and

2. if \( \frac{\gamma}{\rho} < \bar{z} \), then the politician explicitly sells policy.

The fraction \( \frac{\gamma}{\rho} \) represents how much the politician cares about citizen welfare (\( \gamma \)) relative to how much he cares about political contributions (\( \rho \)). This means that the politician prefers to sell access rather than policy favors when he cares enough about citizen welfare (or the policy outcome) relative to contributions. If he does not care enough about policy, then he will choose to sell policy favors which results in higher revenue (and lower policy utility).

\(^{41}\)If the politician uses a different type of auction (such as a menu auction) to sell policy, then the expected distance between the socially optimal and implemented policies may be less than 1. However, the expected distance will be greater than 0, which means that selling policy will continue to result in lower policy utility for the politician.

\(^{42}\)I assume that the indifferent politician sells access.
8.4 Contribution Limit

In Section 4, a contribution limit could never improve citizen welfare. When the politician can choose whether to sell access or sell policy, however, a carefully set contribution limit may be advantageous because it makes explicitly selling policy relatively less attractive to the politician. This analysis focuses on the contribution limit that maximizes citizen welfare.

If Eq. 5 is not met, then the politician will sell policy favors rather than access. When the politician gives the highest bidder the right to choose policy, the policy ends up being extreme. On average the distance between the implemented policy and the socially optimal policy is 1, and the expected citizen welfare from selling policy is $EW = -\gamma$. In contrast, if the politician sells access, he becomes fully informed and implements the socially optimal policy, and $W = 0$.

Given that expected citizen welfare is strictly lower when the politician sells policy rather than access, the policy question then becomes: does there exist a limit that entices the politician to sell access and improves citizen welfare? In short, the answer is yes. To see this, consider a complete contribution ban (i.e., $\bar{b} = 0$). In this case, expected citizen welfare is $EW = -\gamma \int_0^1 f(e) | e - \int_0^1 f(e) ede | de$ which is strictly higher than $-\gamma$. Because the politician collects zero contributions regardless of whether he sells policy or access, he will choose to sell access which results in strictly higher expected citizen welfare.\(^{43}\)

This does not, however, imply that banning contributions is ideal. When there are no contributions, the politician is unable to infer anything about the evidence quality of the interest group that does not receive access. If one can impose a strictly positive contribution limit such that the politician continues to sell access rather than policy, then the politician can infer something about interest group evidence quality from the contributions, even if the limit prevents contributions from being fully revealing. This results in the politician being better informed than when contributions are completely banned. The socially optimal limit for any single policy choice is the maximum limit under which the politician chooses to sell access.

When $\frac{\gamma}{\rho}$ is high enough (when Eq. 5 holds), no limit is needed to entice the politician to sell access, in which case any $\bar{b} \geq B(1)$ is optimal including no limit ($\bar{b} = \infty$), where $B(1)$ is the highest possible contribution when interest groups compete over access. Any $\bar{b} \geq B(1)$ has no impact on

\(^{43}\)Although I use the term "sell", when contributions are banned the politician chooses between randomly assigning access to an interest group, or randomly choosing an interest group to win the policy favor.
the contributions in the competition for access, and will not limit the politician’s ability to infer evidence quality from contributions.

When the value $\frac{\gamma}{\rho}$ is known at the time one chooses a contribution limit $\bar{b}$, then the limit $\bar{b}$ can be set just low enough to entice the politician to sell access instead of policy. The assumption that $\frac{\gamma}{\rho}$ is known, however, is largely unjustified in a world in which a contribution limit applies to all interest groups, not only those concerned with a single issue. To better represent a situation in which the politician must choose many policies, while maintaining the simplifying assumption that $N = 1$, I assume that $\frac{\gamma}{\rho}$ (or at least $\gamma$) is observed after $\bar{b}$ is set, and before the politician chooses whether to sell access or policy. In this case imposing (or decreasing) a contribution limit makes it more likely that the politician sells access, but also decreases the accuracy of the politician’s beliefs about the socially optimal policy when he does sell access. The socially optimal contribution limit is such that the expected welfare costs of increasing or decreasing the limit outweighs the expected benefits of doing so.

One cannot determine the value of the socially optimal contribution limit without additional assumptions regarding the distribution of $e$ and $\frac{\gamma}{\rho}$. Even without further assumptions, however, one can conclude that a carefully set contribution limit strictly improves expected citizen welfare in the case when $\frac{\gamma}{\rho}$ is unknown ex ante. When $\frac{\gamma}{\rho}$ is known, a carefully set limit can improve (and never reduces) expected citizen welfare.

For a number of different evidence quality distributions, including the Uniform$[0, 1]$ distribution, setting $\bar{b} = B(1)$ guarantees that a politician will sell access. This is because, given the parameters, selling access results in higher expected revenue, as well as higher policy utility, compared with selling policy. When this is the case, $\bar{b} = B(1)$ results in the first-best citizen welfare outcome. For other distributions of evidence quality, whether $\bar{b} = B(1)$ is optimal depends on the distribution of $\frac{\gamma}{\rho}$. When a low $\frac{\gamma}{\rho}$ is sufficiently unlikely, $\bar{b} = B(1)$ is optimal. On the other hand, when the politician tends to care relatively little about citizen welfare (i.e., a low $\frac{\gamma}{\rho}$ is likely), setting the limit below $B(1)$ is optimal.

**Proposition 5** Consider the game in which the politician chooses whether to sell access or policy

---

44Technically, the optimal limit makes the politician indifferent between selling policy and selling access, and in equilibrium the politician chooses to sell access.

45Such as the Beta$[x, y]$ distribution where $x = 1/2$ and $y > 1$.  

36
favors. There exists a $\bar{b}^* \in [0, B(1)]$ such that expected citizen welfare is maximized when $\bar{b} = \bar{b}^*$.

In contrast to the results from Section 4, this proposition formally establishes that, when the politician is free to choose whether he sells access or sells policy, contribution limits tend to improve citizen welfare. This is because a limit reduces the expected revenue from selling policy by more than it reduces the expected revenue from selling access, thereby making selling policy relatively less attractive to the politician. However, although a contribution limit tends to improve citizen welfare by making it more likely the politician sells access, a limit can also reduce the information value of contributions which results in an access-selling politician making less-informed policy decisions. Therefore, one must be cautious about setting an overly-strict limit.

9 Conclusion

This paper incorporates access into a model of informational lobbying. When the politician sells access to the highest bidders, contributions help him learn about the impact of different policies. In contrast to much of the previous literature, I show how political contributions may move policy closer to the platform that is best for constituents. The model illustrates how a contribution limit may reduce the amount of information available to the politician, and results in worse policy decisions.

In no way are these results irrefutable evidence in favor of allowing unlimited payments from special interests to politicians. Instead, the results should be viewed as evidence that political contributions may not be as bad as generally portrayed by the media and campaign finance reform groups. When the politician can sell access, he may choose better policies than if he could not sell access.

Certainly, the real world is much more complicated than the competition for access model developed in this paper. When I generalize the model to allow for unobserved interest group wealth differences, or allow the politician to choose whether to sell access or sell explicit policy favors, contribution limits can improve welfare. However, even in these cases, setting too low of a limit has averse effects on welfare.

Also, the competition for access framework likely applies to some issues better than others. Certain interest groups likely give to candidates because they want to help the candidate win
election, not because they want to secure access. In this way, competition for access probably does not apply to an issue likely abortion, for which politicians are already well informed or likely to publicly commit to a position. Instead, an access model is likely a better fit for an issue such as steel tariffs in which the domestic auto producers have arguments against a tariff, the domestic steel industry has arguments in favor of a tariff, and most politicians are not well informed as to the optimal level of tariff for their constituents.

This paper may serve as a starting point for future research on hard information and access. Future work may incorporate the competition for access model into more complete models of the political process. For example, interactions between the politician and interest groups are often repeated over time, policy is often chosen by a group of legislators rather than a single decision maker, and politicians use contributions to complete against other politicians for election. Additional work may also consider alternative mechanisms through which the politician may award access, or apply the competition for access model to non-political settings.46

10 Appendix

10.1 Contribution Equilibrium with Limits

Here, I derive the details regarding the contribution equilibrium with limit $\bar{b} < B(1)$, where $B(1)$ is the contribution of an interest group with the highest-possible quality evidence in the equilibrium without a limit. Let $B_{CL}(e;\bar{b})$ describe the equilibrium contribution of an interest group with evidence quality $e$, when there is a contribution limit $\bar{b}$. Since $\bar{b} < B(1)$, a group with high-enough $e$ prefers to contribute more than $\bar{b}$ but cannot do so. Instead, it provides the maximum contribution $\bar{b}$. There exists a cut off value $\bar{e}(\bar{b}) \in [0,1)$ such that for any $e \in [\bar{e}(\bar{b}),1]$ an interest group provides contribution $\bar{b}$ in equilibrium. For any $e \in [0,\bar{e}(\bar{b}) )$, an interest group's contribution is strictly increasing in $e$. There exists a function $\bar{B}$, where $\bar{B}(0;\bar{b}) = 0$ and $\frac{\partial \bar{B}}{\partial e} > 0$, such that in the contribution equilibrium with limit $\bar{b}$

$$B_{CL}(e;\bar{b}) = \begin{cases} \bar{B}(e;\bar{b}) & \text{for } e \in [0,\bar{e}(\bar{b}) ) \\ \bar{b} & \text{for } e \in [\bar{e}(\bar{b}),1] \end{cases}$$

46For example, instead of allocating access through an auction, the politician may set individual prices for access. If an interest group pays the price assigned to them, they receive access, otherwise they do not receive access. In related work, Cotton (2007) develops such a model.
For the rest of the analysis, I assume $\tilde{b}$ is fixed, and ignore its value when writing the functions. So, $B_{\text{CL}}(e; \tilde{b}) = B_{\text{CL}}(e)$ and $\tilde{B}(e; \tilde{b}) = \tilde{B}(e)$. Let $\tilde{e}(b) = \tilde{B}^{-1}(e)$.

In equilibrium, for every issue the politician chooses the policy he expects will maximize citizen welfare, or $p_n^* = E_{\mu}e_{n1}^1 - E_{\mu}e_{n1}^{-1}$ for every $n$. A complete description of the equilibrium must also describe the politician’s beliefs $\mu$. In equilibrium, if interest group $(n, j)$ receives access, then $E_{\mu}e_{n}^1 = e_{n}^1$. If $(n, j)$ does not receive access and provides contribution $\tilde{b}$, then $E_{\mu}e_{n}^1 = \frac{1}{1 - F(e(\tilde{b}))} \int_{\tilde{e}(\tilde{b})}^{1} f(y) y dy$. If $(n, j)$ does not receive access and provides contribution $b_{n}^j < \tilde{b}$, then $E_{\mu}e_{n}^1 = \tilde{e}(b_{n}^j)$.

The function $\tilde{B}$ is derived just as $B$ was derived in the no-limit game. Interest group $(n, j)$ chooses a contribution $b$ to maximize the expression

$$f_{\text{CL}}(e_{n}^{-j}) \left[ (1 - \Theta_{\text{CL}}(\tilde{e}(b) ; e_{n}^{-j})) V(1 - \tilde{e}(b) + e_{n}^{-j}) + \Theta_{\text{CL}}(\tilde{e}(b) ; e_{n}^{-j}) V(1 - e_{n}^j + e_{n}^{-j}) \right] de_{n}^{-j} - b.$$

This differs from a group’s no-limit maximization problem in that $b$ must now be on the interval $[0, \tilde{b}]$, the functions $\Theta_{\text{CL}}$ and $f_{\text{CL}}$ take into account the fact that all other interest groups with $e \geq \tilde{e}(\tilde{b})$ provide the same contribution $\tilde{b}$. The function $\Theta_{\text{CL}}(e(b) ; e_{n}^{-j})$ defines the probability that $(n, j)$ is granted access in equilibrium given that it contributes $b$. I leave the formal derivation of $\Theta_{\text{CL}}$ to the reader, as it is not required for the analysis. Note that $\Theta_{\text{CL}}(e; e_{-j})$ is increasing in $e$ for all $e \leq \tilde{e}$. In equilibrium, all groups with $e \in [\tilde{e}, 1]$ have the same ex ante probability of being offered access. I denote this probability by $\tilde{\Theta}_{\text{CL}}$ when nothing is assumed about the evidence quality of another interest group, and by $\Theta_{\text{CL}}(e_{-j})$ when one other group has evidence quality $e_{-j}$. The value $f_{\text{CL}}(e)$ denotes the ex ante probability that $E_{\mu}e_{n}^{-j} = e$.

Therefore,

$$f_{\text{CL}}(e) = \begin{cases} f(e) & \text{for } e \in [0, \tilde{e}) \\ \tilde{\Theta}_{\text{CL}} f(e) & \text{for } e \in [\tilde{e}, 1] \cap e \neq \int_{\tilde{e}}^{1} f(y) y dy \\ (1 - \tilde{\Theta}_{\text{CL}}) \int_{\tilde{e}}^{1} f(y) y dy & \text{for } e = \int_{\tilde{e}}^{1} f(y) y dy. \end{cases}$$

Using the interest group’s above maximization problem, one can solve for $\tilde{B}$ using the technique from Section 3. It follows that

$$\tilde{B}(e) = - \int_{0}^{e} \int_{0}^{1} f_{\text{CL}}(e^{-j}_{n}) (1 - \Theta_{\text{CL}}(y ; e_{n}^{-j})) V'(1 - y + e_{n}^{-j}) de_{n}^{-j} dy.$$

\footnote{Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of $b$ that are on the path of play for any $e \in [0, 1]$ (Fudenberg and Tirole,1991, pp. 324-326). The concept of Perfect Bayesian Equilibrium does not restrict beliefs off the equilibrium path of play. For those $b$ that are never played in equilibrium, such assumptions regarding the politician beliefs guarantees that a pure strategy equilibrium exists; other less restrictive beliefs produce the same result.}
The cut-off evidence quality \( \bar{\epsilon} (\bar{b}) \) is the evidence quality at which an interest group is indifferent between contributing according to \( \tilde{B} \) and contributing \( \bar{b} \), and solves the following equation for \( \bar{\epsilon} \)

\[
\int_0^1 f_{CL} (\epsilon_n^{-j}) \left[ (1 - \tilde{\Theta}_{CL} (\epsilon_n^{-j})) V \left( 1 - \frac{1}{1 - F(\epsilon)} \int_\epsilon^1 f(y) \, dy + \epsilon_n^{-j} \right) + \tilde{\Theta}_{CL} (\epsilon_n^{-j}) V (1 - \epsilon + \epsilon_n^{-j}) \right] \, d\epsilon_n^{-j} - \bar{b} = 0
\]

For an interest group with evidence quality \( \bar{\epsilon} \), the left hand side of the equality denotes the group’s expected utility from providing a contribution according to the increasing equilibrium function \( \tilde{B} \), and the right hand side of the equality denotes the expected utility from providing the maximum contribution. It follows that \( \tilde{B} (\bar{\epsilon} (\bar{b})) < \bar{b} \). In equilibrium, the group with the cut-off evidence quality is indifferent between the two actions. If the solution to this equality is negative, then \( \bar{\epsilon} (\bar{b}) = 0 \) and all interest groups contribute \( \bar{b} \) independent of their evidence qualities.

Next, I show that interest groups that do not have an incentive to deviate from the contribution function \( B_{CL} \). For an interest group with \( e < \bar{\epsilon} (\bar{b}) \), providing \( \bar{b} \) rather than \( b < \bar{b} \) gives an expected benefit of

\[
\int_0^1 f_{CL} (\epsilon_n^{-j}) \left[ (1 - \tilde{\Theta}_{CL} (\epsilon_n^{-j})) V \left( 1 - \frac{1}{1 - F(\epsilon)} \int_\epsilon^1 f(y) \, dy + \epsilon_n^{-j} \right) + \tilde{\Theta}_{CL} (\epsilon_n^{-j}) V (1 - \epsilon + \epsilon_n^{-j}) \right] \, d\epsilon_n^{-j} - \bar{b}
\]

which is strictly increasing in an interest groups evidence quality. To see this, take the derivative of the benefit with respect to \( e \).

\[
\int_0^1 f_{CL} (\epsilon_n^{-j}) (1 - \tilde{\Theta}_{CL} (\epsilon_n^{-j})) V' (1 - \epsilon + \epsilon_n^{-j}) \, d\epsilon_n^{-j} + \frac{\partial \tilde{B} (e)}{\partial e}
\]

Since \( \tilde{\Theta}_{CL} (\epsilon_n^{-j}) - \Theta_{CL} (e; \epsilon_n^{-j}) \) for any \( e < \bar{\epsilon} (\bar{b}) \), simplifying gives

\[
- \int_0^1 f_{CL} (\epsilon_n^{-j}) \left[ \tilde{\Theta}_{CL} (\epsilon_n^{-j}) - \Theta_{CL} (e; \epsilon_n^{-j}) \right] V' (1 - \epsilon + \epsilon_n^{-j}) \, d\epsilon_n^{-j} > 0.
\]

From the derivation of \( \tilde{B} \), one already knows that an interest group with \( e < \bar{\epsilon} (\bar{b}) \) prefers to provide \( \tilde{B} (e) \) rather than any other \( b < \bar{b} \).

From the derivation of \( \tilde{B} \), it also follows that any group with \( e \geq \bar{\epsilon} (\bar{b}) \) prefers to provide \( b \to \bar{b} \) instead
of any other $b < \bar{b}$. The expected benefit to an interest group with $e > \bar{e}(\bar{b})$ from giving $\bar{b}$ instead of $b < \bar{b}$ is

\[
\lim_{b \to \bar{b}} \int_{0}^{1} f_{CL}(e_{n}^{-j}) \left[ (1 - \bar{\Theta}_{CL} (e_{n}^{-j})) V \left( 1 - \frac{1}{1 - F(e_{n}^{j})} \int_{e_{n}^{j}}^{1} f(y) y dy + e_{n}^{-j} \right) \right.
\]

\[
+ \bar{\Theta}_{CL} (e_{n}^{-j}) V (1 - e + e_{n}^{-j}) \bigg] \left. de_{n}^{-j} - \bar{b} \right]
\]

\[
- \int_{0}^{1} f_{CL}(e_{n}^{-j}) \left[ (1 - \Theta_{CL} (\bar{e}(b); e_{n}^{-j})) V (1 - V (1 - \bar{e}(b) + e_{n}^{-j} + e_{n}^{-j})) \right.
\]

\[
+ \Theta_{CL} (\bar{e}(b); e_{n}^{-j}) V (1 - e + e_{n}^{-j}) \bigg] \left. de_{n}^{-j} + b \right]
\]

which is strictly increasing in the group’s $e$. To see this, take the derivative with respect to $e$:

\[
\lim_{b \to \bar{b}} - \int_{0}^{1} f_{CL}(e_{n}^{-j}) \left[ \bar{\Theta}_{CL} (e_{n}^{-j}) - \Theta_{CL} (\bar{e}(b); e_{n}^{-j}) \right] V' (1 - e + e_{n}^{-j}) \bigg] de_{n}^{-j} > 0.
\]

Since the benefit of providing $\bar{b}$ rather than $b < \bar{b}$ is strictly increasing in a group’s evidence quality, and $\bar{e}(\bar{b})$ is the evidence quality at which a group is indifferent between the two actions, it follows that interest groups with $e < \bar{e}(\bar{b})$ strictly prefer to contribute according to $\bar{B}$, and groups with $e > \bar{e}(\bar{b})$ strictly prefer to contribute $\bar{b}$.

### 10.2 Contribution Equilibrium with Wealth Differences

In this section, I derive the contribution equilibrium of the model presented in Section 5.2. Poor interest groups have wealth constraint such that their contributions must be no greater than $\omega$, and rich group contributions are unconstrained. Assume $\omega < B(1)$. A poor group with high-enough $e$ prefers to contribute more than $\omega$ but cannot do so. Instead, it provides $\omega$.

The functions $B_P$ and $B_R$ respectively denote the equilibrium poor and rich group contribution functions. There exists cut off values $\bar{e}_a \geq 0$ and $\bar{e}_b \in (\bar{e}_a, 1)$, and functions $B_a$ and $B_b$, where $B_a(0; \omega) = 0, B_b(\bar{e}_b; \omega) = 0, \frac{\partial B_a}{\partial e} > 0, \text{ and } \frac{\partial B_b}{\partial e} > 0$ such that

\[
B_{P} (e; \omega) = \begin{cases}
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_b) \\
\omega & \text{for } e \in [\bar{e}_a, 1], \text{ and } \frac{\partial B_a}{\partial e} > 0
\end{cases}
\]

\[
B_{R} (e; \omega) = \begin{cases}
B_a (e; \omega) & \text{for } e \in [0, \bar{e}_a)
\omega & \text{for } e \in [\bar{e}_a, \bar{e}_b)
\omega + B_b (e; \omega) & \text{for } e \in [\bar{e}_b, 1].
\end{cases}
\]
The cut off values are dependent on poor group wealth $\omega$, and the probability that a group is poor, $\alpha$. Also, through the rest of the analysis, I take $\omega$ as fixed and ignore it in writing the contribution functions $B_P(e)$, $B_R(e)$, $B_a(e)$, and $B_b(e)$. Let $e_a(b) = B_a^{-1}(e)$ and $e_b(b) = B_b^{-1}(e)$.

In equilibrium, the politician chooses the policy profile he expects will maximize citizen welfare, or $p_n^* = E\mu^j_n - E\mu_e^{-1}_n$ for every $n$. A complete description of the equilibrium must also describe the politician’s beliefs $\mu$. In equilibrium, if interest group $(n,j)$ receives access, then $E\mu^j_n = e^j_n$. If $(n,j)$ does not receive access and provides contribution $\omega$, then

$$E\mu^j_n = E(e | \omega) = \frac{(1 - \alpha) \int_{\bar{e}_a}^1 f(y) y dy + \alpha \int_{\bar{e}_b}^1 f(y) y dy}{(1 - \alpha) (1 - F(\bar{e}_a)) + \alpha (F(\bar{e}_b) - F(\bar{e}_a))}.$$ 

If $(n,j)$ does not receive access and provides contribution then $E\mu^j_n = e_a(b^j_n)$ if $b^j_n < \omega$, and $E\mu^j_n = e_b(b^j_n)$ if $b^j_n > \omega$.\(^{48}\)

The function $B_a$ is derived just as $B$ was derived in the no-limit game. Interest group $(n,j)$ chooses a contribution $b$ to maximize

$$\int_0^{1} f_{WL} (e_n^{-j}) \left[ (1 - \Theta_{WL}(e_a(b);e_n^{-j})) V (1 - e_a(b) + e_n^{-j}) + \Theta_{WL}(e_a(b);e_n^{-j}) V (1 - e_n^j + e_n^{-j}) \right] de_n^{-j} - b.$$ 

This differs from the maximization problem in Section 2 since $\Theta_{WL}$ and $f_{WL}$ take into account the fact that all other interest groups contribute according the the contribution function described above. The value $\Theta_{WL} (e(b);e_n^{-j})$ defines the probability that $(n,j)$ is granted access in equilibrium given that it contributes $b$. In equilibrium, all groups that provide contribution $\omega$ have the same ex ante probability of being offered access. I denote this probability by $\bar{\Theta}_{WL}$ when nothing is assumed about the evidence quality of another interest group, and by $\bar{\Theta}_{WL} (e^{-j})$ when one other group has evidence quality $e^{-j}$. The value $f_{WL} (e)$ denotes the ex ante probability that $E\mu e_n^{-j} = e$.

Using the interest group’s above maximization problem, one can solve for $\bar{B}$ using the technique from Section 2. It follows that

$$B_a(e) = - \int_0^e \int_0^1 f_{WL} (e_n^{-j}) (1 - \Theta_{WL}(y;e_n^{-j})) V' (1 - y + e_n^{-j}) de_n^{-j} dy.$$ 

\(^{48}\)Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of $b$ that are on the path of play for any $e \in [0,1]$. For those $b$ that are never played in equilibrium, such assumptions regarding the politician beliefs guarantees that a pure strategy equilibrium exists; other less restrictive beliefs produce the same results.
Similarly, \( B_b \) is derived from the maximization problem of an interest group with \( e > \tilde{e}_b \).

\[
B_b (e) = - \int_{\tilde{e}_a}^{e} \int_{0}^{1} f_{WL} (e_n^{-j}) (1 - \Theta_{WL} (y; e_n^{-j})) V' (1 - y + e_n^{-j}) de_n^{-j} dy.
\]

The cut-off evidence quality \( \tilde{e}_a \) is the evidence quality at which an interest group is indifferent between contributing according to \( B_a \) and contributing \( \omega \). The equilibrium value of \( \tilde{e}_a \) solves the following equation for \( \tilde{e} \)

\[
\int_{0}^{1} f_{WL} (e_n^{-j}) V (1 - \tilde{e} + e_n^{-j}) de_n^{-j} - B_a (\tilde{e}) = \int_{0}^{1} f_{WL} (e_n^{-j}) \left[ (1 - \Theta_{WL} (e_n^{-j})) V (1 - e_{WL} (\omega) + e_n^{-j}) + \Theta_{WL} (e_n^{-j}) V (1 - \tilde{e} + e_n^{-j}) \right] de_n^{-j} - \omega
\]

where \( e_{WL} (\omega) \) denotes the politician’s equilibrium expectations about the evidence quality of an interest group that provides \( \omega \) and does not receive access. If the solution to the equality is negative, then \( \tilde{e}_a = 0 \) and all poor interest groups contribute \( \omega \) independent of their evidence qualities.

The cut-off evidence quality \( \tilde{e}_b \) is the evidence quality at which an interest group is indifferent between contributing according to \( B_b \) and contributing \( \omega \). In equilibrium,

\[
\tilde{e}_b = E (e | \omega) = \frac{(1 - \alpha) \int_{\tilde{e}_a}^{\tilde{e}_b} f (y) y dy + \alpha \int_{\tilde{e}_b}^{\tilde{e}} f (y) y dy}{(1 - \alpha) (1 - F (\tilde{e}_a)) + \alpha (F (\tilde{e}_b) - F (\tilde{e}_a))}.
\]

Just as I do in the previous section of the appendix where I derive the equilibrium for the case with contribution limits, it is straightforward to show that none of the interest groups have an incentive to deviate in the equilibrium. Groups with evidence quality \( e > \tilde{e}_b \) strictly prefer to contribute \( \omega + B_b (e) \) compared to \( \omega \), and \( \omega \) compared to any amount less than \( \omega \). Groups with \( e > \tilde{e}_a \) strictly prefer to provide \( \omega \) than any amount less than \( \omega \), and those with \( e \in (\tilde{e}_a, \tilde{e}_b) \) strictly prefer to provide \( \omega \) compared with any amount greater than or less than \( \omega \). Similarly, groups with \( e < \tilde{e}_a \) strictly prefer to provide \( B_a (e) \) to any other contribution.

**With Contribution Limits**

Using the techniques developed earlier in this paper, one can incorporate contribution limits into this game. I describe the equilibrium contribution function here, but leave the details of the derivation to the reader.

The paper focuses on the case where \( \bar{b} \leq \omega \), which is consistent with the contribution limits proposed in the policy debate. In this case, the contribution limit has the same impact on the equilibrium contribution functions as the limit would have in the game without wealth differences. See Sections 3 and the first section.
of the appendix for a derivation and illustration. Although the body of the paper does not discuss the other cases, I describe them here. When $\bar{b} > \omega$, there are two cases. First, if the limit is high-enough, then the contribution limit impacts function $B_b$ in the same manner that the limit impacted $B$ in the case without wealth differences. In this case, rich interest groups with $e > E(e | \omega) = \bar{e}_b$ contribute $\omega + B_b(e)$ for relatively low $e$ (as $e \to \bar{e}_b$) and contribute $\bar{b}$ for higher $e$ (as $e \to 1$). The contributions of the poor interest groups are not impacted. This case is illustrated in Figure 3.1. Second, if $\bar{b} > \omega$ and the limit is relatively close to $\omega$, then there exists a function $\tilde{B}_a$ that is strictly increasing in $e$ and where $\tilde{B}(0) = 0$, and values $\bar{e}, \bar{e}'_b \in (0,1)$ such that when $(n,j)$ is rich, $b'_n = \tilde{B}(e_n')$ for $e'_n \in [0,\bar{e})$, $b'_n = \omega$ for $e'_n \in [\bar{e}, \bar{e}'_b)$, and $b'_n = \bar{b}$ for $e'_n \in [\bar{e}'_b, 1]$, and when $(n,j)$ is poor, $b'_n = \tilde{B}(e_n')$ for $e'_n \in [0,\bar{e}_a)$, $b'_n = \omega$ for $e'_n \in [\bar{e}_a, 1]$. It can be shown that $\bar{e} < \bar{e}'_b < \bar{e}_b < E(e | \omega)$, where $\bar{e}_a$ and $\bar{e}_b$ are the cutoff values when there is no contribution limit, and $E(e | \omega)$ is the politician’s expectations about an interest group’s evidence quality when that group contributes $\omega$ but does not receive access. This case is illustrated by Figure 3.2.

**Impact of limits on welfare in a simple game**

Here, I solve the simple example from Section 5.2. Evidence quality is uniformly distributed on $[0,1]$, interest group policy utility is linear, or $V(1 - e^j + e^{-j}) = -(1 - e^j + e^{-j})$, and both $N = 1$ and $K = 1$. Using the methods described previously, it is straightforward to solve for the game’s equilibrium.

For the case when there is no contribution limit. First, one can find that $B_a(e) = e - \frac{1}{2} e^2$, and calculate the expected payoff to an interest group from contributing less than $\omega$, which equals $\frac{1}{2} e^2 - \frac{3}{2}$. Note that $\omega$ must be less than $\frac{1}{2}$ to have an impact on contributions. If the same group contributed $\omega$ instead, its
expected payoff equals

\[- \int_{\tilde{e}_a}^{1} (1 - \alpha) (1 - \bar{e}_b + e') + \alpha \left( \frac{1}{2} (1 - e + \bar{e}_b) + \frac{1}{2} (1 - \bar{e}_b + e') \right) de' - \int_{\tilde{e}_a}^{\tilde{e}_b} \left( \frac{1}{2} (1 - e + \bar{e}_b) + \frac{1}{2} (1 - \bar{e}_b + e') \right) de' - \int_{0}^{\tilde{e}_a} (1 - e + e') de' - \omega.\]

This simplifies to

\[- \frac{3}{2} - \frac{1}{4} \tilde{e}_a^2 + (1 - \alpha) \left( \tilde{e}_b - \frac{3}{4} \tilde{e}_a^2 \right) + \frac{1}{4} \alpha + \frac{1}{2} e (\tilde{e}_a + \alpha + \bar{e}_b (1 - \alpha)) - \omega.\]

Because \(\tilde{e}_b = E_\mu (e \mid \omega)\) it follows that \(\tilde{e}_b = \frac{1 + \tilde{e}_a}{1 + \alpha}\). When an interest group has \(e = \tilde{e}_a\), these expected payoffs from contributing \(\omega\) equals the expected payoff from giving \(B_a(e)\). Therefore, one can solve for \(\tilde{e}_a\), where

\[\tilde{e}_a = 1 - \frac{1 + \alpha}{\sqrt{2 (1 - 2 \omega)}} \frac{\sqrt{2 - \alpha + 3 \alpha^2}}{\sqrt{2} \sqrt{1 - \alpha}}\]

when \(1 - (1 + \alpha) \sqrt{2 (1 - 2 \omega)} / \sqrt{2 - \alpha + 3 \alpha^2} \geq 0\). If the expression is negative, \(\tilde{e}_a = 0\). Solving for \(B_b(e)\) is not required for the results.

It holds that \(\tilde{B} (e) = B_a (e)\) when there is no contribution limit.\(^4\) Since rich and poor groups give \(b\) for the same range of \(e\), it follows that \(\tilde{e}_b' = \frac{1 + \tilde{e}_a}{2}\), where \(\tilde{e}'_b = E (e \mid \tilde{b})\). Solving for \(\bar{e}\) in the same way that \(\tilde{e}_a\) was found above gives

\[\bar{e} = 1 - \frac{2 \sqrt{1 - \bar{b}}}{\sqrt{3}}\]

when \(1 - \left(2 \sqrt{1 - \tilde{b}} / \sqrt{3}\right) \geq 0\), and \(\bar{e} = 0\) otherwise.

When there is no contribution limit, expected citizen welfare is

\[-\alpha^2 f_{\tilde{e}_a}^{1} f_{e_b}^{1} |e_1 - \tilde{e}_b| d_e_1 d_e_2 - 2 \alpha (1 - \alpha) f_{\tilde{e}_a}^{\tilde{e}_b} f_{e_b}^{1} \left( \frac{1}{2} |e_1 - \tilde{e}_b| + \frac{1}{2} (|\tilde{e}_b - e_2|) \right) d_e_1 d_e_2 - 2 \alpha (1 - \alpha) f_{\tilde{e}_a}^{\tilde{e}_b} f_{e_b}^{1} \left( \int_{\tilde{e}_a}^{\tilde{e}_b} (\tilde{e}_b - e_1) d_e_1 + f_{e_b}^{\tilde{e}_b} (\tilde{e}_b - e_2) d_e_2 \right) - (1 - \alpha)^2 f_{\tilde{e}_a}^{\tilde{e}_b} f_{e_b}^{1} (\tilde{e}_b - e_1) d_e_1 d_e_2.\]

Substituting in for \(\tilde{e}_b\) and \(\tilde{e}_a\), when \(1 - (1 + \alpha) \sqrt{2 (1 - 2 \omega)} / \sqrt{2 - \alpha + 3 \alpha^2} > 0\) expected citizen welfare simplifies to

\[EW_{\text{no limit}} = - \frac{2 \alpha (1 - 2 \omega) (1 + \alpha) \sqrt{2 (2 - \alpha + 3 \alpha^2) (1 - 2 \omega)}}{(2 - \alpha + 3 \alpha^2)^2}. \quad (7)\]

If \(1 - (1 + \alpha) \sqrt{2 (1 - 2 \omega)} / \sqrt{2 - \alpha + 3 \alpha^2} \leq 0\), then \(EW_{\text{no limit}} = - \frac{\alpha}{(1 + \alpha)^2}\).

\(^4\)This is true because \(V\) is linear.
When there is a contribution limit, expected citizen welfare is \( \int_\tilde{e}^1 \int_\bar{e}^1 |e_1 - \tilde{e}| de_1 de_2 \). When \( 1 - \left( \frac{2\sqrt{1 - \bar{b}}}{\sqrt{3}} \right) > 0 \),
\[
EW_{\text{limit}} = -\frac{2(1 - \omega)^{2/3}}{3\sqrt{3}}.
\]
If \( 1 - \left( \frac{2\sqrt{1 - \bar{b}}}{\sqrt{3}} \right) \leq 0 \), then \( EW_{\text{limit}} = -\frac{1}{4} \).

When both \( \tilde{e}_a \) and \( \bar{e} \) are 0, the contribution limit never improves expected citizen welfare since \( EW_{\text{no limit}} = -\frac{\alpha}{(1+\alpha)^2} > EW_{\text{limit}} = -\frac{1}{4} \) for \( \alpha \in (0, 1) \). Similarly, one can show that when \( \tilde{e}_a > 0 \) and \( \bar{e} = 0 \), the contribution limit cannot improve welfare since \( -\frac{2\alpha(1 - 2\omega)(1 + \alpha)\sqrt{2(2 - \alpha + 3\alpha^2)(1 - 2\omega)}}{(2 - \alpha + 3\alpha^2)^2} > -\frac{1}{4} \). There are no parameter values for which \( \tilde{e}_a = 0 \) and \( \bar{e} > 0 \).

When \( 1 - (1 + \alpha) \sqrt{2(1 - 2\omega)/\sqrt{2 - \alpha + 3\alpha^2}} > 0 \) and \( 1 - \left( \frac{2\sqrt{1 - \bar{b}}}{\sqrt{3}} \right) > 0 \), both \( \tilde{e}_a \) and \( \bar{e} \) are positive. In this case the contribution limit improves expected citizen welfare when
\[
-\frac{2(1 - \omega)^{2/3}}{3\sqrt{3}} > -\frac{2\alpha(1 - 2\omega)(1 + \alpha)\sqrt{2(2 - \alpha + 3\alpha^2)(1 - 2\omega)}}{(2 - \alpha + 3\alpha^2)^2}.
\]

Using Mathematica, I solve for the numerical values for which all three of these conditions as well as the priors on \( \alpha, \omega, \) and \( \bar{b} \) are simultaneously met. There exists cut off values \( \alpha', \omega'_{L}(\alpha), \) and \( \omega'_{H}(\alpha) \) such that contribution limit \( \bar{b} \leq \omega \) improves expected citizen welfare if and only if \( \alpha \in (\alpha', 1), \omega \in [\omega'_{L}(\alpha), \omega'_{H}(\alpha)) \), and \( \bar{b} \in \left[ \frac{1}{4}, \omega \right] \). The values \( \omega'_{L}(\alpha) \) and \( \omega'_{H}(\alpha) \) depend on the value of parameter \( \alpha \), and determining the values \( \alpha' \) and \( \omega'_{H}(\alpha) \) require the calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. \( \omega'_{H}(\alpha) \) is greater than \( \omega'_{L}(\alpha) \) only when \( \alpha \) is high enough, implying that \( \alpha' \) is approximately 0.750427.

This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any \( \alpha \), the range of \( \omega \) for which a limit can have a positive impact is even more restrictive. For any \( \alpha > \alpha', \omega'_{L}(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260199, and \( \omega'_{H}(\alpha) \) takes on values between \( \frac{1}{4} \) and 0.260751. At its maximum, the difference between \( \omega'_{H}(\alpha) \) and \( \omega'_{L}(\alpha) \) is approximately 0.00215, meaning that for any value \( \alpha \), only very specific values of \( \omega \) result in the contribution limit being beneficial. Furthermore, because \( \omega'_{H}(\alpha) \) is close to \( \frac{1}{4} \), even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected citizen welfare.

### 10.3 Proofs

**Proof (Lemma 1).** Follows immediately from analysis in paper. ■
Proof (Proposition 1). Follows immediately from analysis in paper and Lemma 1. ■

Proof (Proposition 2). Where \( W(p^*, p^o) = -\sum_{n=1}^N \gamma_n \times |p^* - p^o| \), let \( w_n = -\gamma_n \times |p^o_n - p^*_n| \) for each \( n \). When there are no contribution limits, the politician chooses \( p^*_n = p^o_n \) for all \( n \), and social welfare \( W = 0 \), and \( w_n = 0 \) for all \( n \).

Consider the case when there are contribution limits. The parameter \( M \) denotes the realized number of interest groups that contribute \( \bar{b} \) in equilibrium. The ex ante probability of any \( M \in \{0, 1, ..., 2N\} \) equals \( \varphi(M) = \frac{2N!}{M!(2N-M)!} F(\bar{e})^{2N-M} (1 - F(\bar{e}))^M \). Therefore, \( M > K \) with probability \( \sum_{m=K+1}^{2N} \varphi(m) \).

For each \( m \in \{K+1, ..., 2N\} \), \( \varphi(m) > 0 \) since \( \bar{e} \in (0, 1) \) and \( F(\bar{e}) \in (0, 1) \) for any \( \bar{b} \in (0, B(1)) \). Thus, \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \), the politician is less than fully informed with positive probability. With probability \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \) there exists at least one interest group for which the politician knows \( e_i^o \in [\bar{e}, 1] \), but does not know \( e_i^0 \) when he chooses a policy profile. Without loss of generality assume this is group \((1, 1)\). The politician implements policy \( p_i^* = E \mu e_i^1 - \mu e_i^{-1} \), where \( E \mu e_i^1 = \int e f(e) de \). Given the continuous distribution of \( e \), both \( E \mu e_i^1 \neq e_i^1 \) and \( E \mu e_i^{-1} - \mu e_i^{-1} \neq e_i^{-1} - e_i^{-1} \) with probability one. Citizen welfare attributable to issue 1 is \( w_1 = -\gamma_n \times |E \mu e_i^1 - \mu e_i^{-1} - e_i^1 + e_i^{-1}| \). Since \( E \mu e_i^1 \neq E \mu e_i^{-1} \neq e_i^{-1} - e_i^{-1} \) with probability one, it follows that \( w_1 < 0 \). For all other issues \( m \in \{2, ..., N\} \), \( w_m \leq 0 \), and \( W(p^*, p^o) = \sum_{n=1}^N w_n < 0 \).

When there are contribution limits, \( W(p^*, p^o) < 0 \) with probability \( \sum_{m=K+1}^{2N} \varphi(m) > 0 \), and \( W(p^*, p^o) = 0 \) with probability \( 1 - \sum_{m=K+1}^{2N} \varphi(m) \). Therefore, expected citizen welfare is strictly negative and therefore strictly less than welfare when there are no contribution limits. ■

Proof (Lemma 3). See Section 7.2 for derivation of the equilibrium. Assume \( j = 1 \), and \( e = e_1^0 = e_n^{-1} \).

Consider three cases: \( e \in [0, \bar{e}_a) \), \( e \in [\bar{e}_a, \bar{e}_b) \), and \( e \in (\bar{e}_b, 1] \). If \( e \in [0, \bar{e}_a) \), then \( b_n^1 = b_n^{-1} = B_a(e) \), and \( p_n^* = e - e = 0 \). So, when \( e \in [0, \bar{e}_a) \), \( Ep_n^* = 0 \).

If \( e \in [\bar{e}_a, \bar{e}_b) \), then \( b_n^1 = b_n^{-1} = \omega \). Let \( \Theta_2 \) be the equilibrium probability an interest group that provides \( \omega \) receives access when \( b_n^1 = b_n^{-1} = \omega \). Therefore, when \( e \in [\bar{e}_a, \bar{e}_b) \), \( Ep_n^* = \Theta_2 e + (1 - \Theta_2) \bar{e}_b - \Theta_2 e - (1 - \Theta_2) \bar{e}_b = 0 \).

If \( e \in (\bar{e}_b, 1] \), then \( b_n^1 > \omega \) and \( b_n^{-1} = \omega \). Let \( \Theta_1 \) be the equilibrium probability an interest group that provides \( \omega \) receives access when \( b_n^1 > \omega \) and \( b_n^{-1} = \omega \). Therefore, when \( e \in (\bar{e}_b, 1] \), \( Ep_n^* = e - \Theta_1 e - (1 - \Theta_1) \bar{e}_b = (e - \bar{e}_b) (1 - \Theta_1) > 0 \).

For all values of \( e \), \( Ep_n^* = (1 - F(\bar{e}_b))(e - \bar{e}_b)(1 - \Theta_1) > 0 \). If \( j = -1 \), the same method shows that \( Ep_n^* < 0 \). ■

Proof (Proposition 3). See Section 7.2 for derivation of the equilibrium. Because \( B_F(e) = B_R(e) \) for any \( e \leq \bar{e}_b \), it follows that \( b_n^1 \) is independent of \( (n, j) \)'s wealth when \( e_i^0 \in [0, \bar{e}_b) \). Given \( b_n^1 \), the probability \( \Theta_{WL}(e \bar{b}^1, e_n^{-1}) \) is independent of \( (n, j) \)'s wealth. Therefore, if \( e_i^j \in [0, \bar{e}_b) \), then \( (n, j) \) faces the same
expected payoff $EU^j_n$ independent its wealth.

Alternatively, if $e \in (\bar{e}_b, 1]$, then $B_R(e) > B_P(e)$. Since $B_R(e^*_n) = \arg \max_b EU^j_n(b)$ and $B_P(e^*_n) = \omega$ is a feasible contribution for a rich group, it follows that $EU^j_n(B_R(e^*_n)) > EU^j_n(B_P(e^*_n))$. Therefore, when $(n, j)$ is rich, $EU^j_n$ is the same as if he was poor when $e^*_n \in [0, \bar{e}_b]$, and $EU^j_n$ is strictly greater than if he was poor when $e \in (\bar{e}_b, 1]$.

Proof (Proposition 4). Notation is consistent with Section 7.2. Let $\bar{\Theta}_2$ be the equilibrium probability that $(n, j)$ receives access if $b^j_n = b^{-j}_n = \bar{b}$, and let $\bar{\Theta}_1$ be the equilibrium probability that $(n, j)$ receives access if $b^j_n = \bar{b}$ and $b^{-j}_n < \bar{b}$. Therefore,

$$EP^*_n = \int_0^{\bar{e}} \int_0^{\bar{e}} \left( e - e \right) de^{-1} de + \int_0^{\bar{e}} \int_0^{\bar{e}} \left( e - \bar{\Theta}_1 e^{-1} - (1 - \bar{\Theta}_1) \int_0^{\bar{e}} f(e) de \right) de^{-1} de$$

$$+ \int_0^{\bar{e}} \int_0^{\bar{e}} \left( \bar{\Theta}_1 e + (1 - \bar{\Theta}_1) \int_0^{\bar{e}} f(e) de - e^{-1} \right) de^{-1} de$$

$$+ \int_0^{\bar{e}} \int_0^{\bar{e}} \left( \bar{\Theta}_2 e + (1 - \bar{\Theta}_2) \int_0^{\bar{e}} f(e) de - \bar{\Theta}_2 e^{-1} - (1 - \bar{\Theta}_2) \int_0^{\bar{e}} f(e) de \right) de^{-1} de$$

which reduces to $EP^*_n = 0$.

Rich and poor groups both contribute to $B_{CL}$. Because a group’s contribution is independent of wealth for any $e$, it follows that the realization of $\Theta_{CL}$ is independent of wealth. Therefore $EU^j_n$ is independent of wealth.

Proof (Lemma 4). Follows immediately from analysis in the paper, and that when $f(e) > 0$ for all $e \in [0, 1],$

$$\bar{e} = 2v \int_0^1 f(e) \left( 1 - e + \int_0^e F(y) dy \right) de > 0$$

Proof (Proposition 6). It is sufficient to prove that $\bar{b} = B(1)$ results in at least as high of expected citizen welfare than any other $\bar{b} > B(1)$, including $\bar{b} = \infty$ (no limit). Consider any contribution limit $\bar{b} \geq B(1)$. If the politician sells policy, then citizen welfare is $-\gamma$, and the politician’s payoff is $\nu \rho \bar{b} - \gamma$. If the politician sells access, then expected citizen welfare is $0$ since the politician can choose $p^* = p^o$, and the politician’s expected payoff is $2\nu \rho \int_0^1 f(e) \int_0^e (1 - F(y)) dy de$ which equals his expected total contributions when he sells access. For any $\bar{b} \geq B(1)$, expected citizen welfare from selling policy and expected citizen welfare from selling access are independent of $barb$. Therefore, $\bar{b}$ only impacts citizen welfare through its affect on the probability the politician sells access rather than policy.
The politician chooses to sell access rather than policy when
\[ 2v\rho \int_0^1 f(e) \int_0^e (1 - F(y)) \, dy \, de \geq v\rho \bar{b} - \gamma. \]
This simplifies to
\[ \frac{\gamma}{\rho} \geq v \left( \bar{b} - \int_0^1 f(e) \int_0^e (1 - F(y)) \, dy \, de \right). \] (8)

Define \( \tilde{z}' \) equal to the right hand side of Eq. 8, and let \( \tilde{z}'(\bar{b}) \) denote the value of \( \tilde{z}' \) when the contribution limit is \( \bar{b} \). \( \tilde{z}' \) is strictly increasing in limit \( \bar{b} \). Let the function \( H \) represent the ex ante distribution of \( \frac{\gamma}{\rho} \). Therefore, expected citizen welfare from contribution limit \( \bar{b} \) is \( EW(\bar{b}) = -\gamma H(\tilde{z}'(\bar{b})) \). \( \frac{\partial EW(\bar{b})}{\partial \bar{b}} < 0 \) for all \( \bar{b} \geq B(1) \). Therefore, \( \bar{b} = B(1) \) results in strictly higher expected citizen welfare than any other \( \bar{b} > B(1) \).

References


