Income Dispersion, Asymmetric Information and Fluctuations in Market Efficiency

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April 6, 2006

Abstract

Recessions are times when markets function less efficiently. This phenomenon is usually the domain of theories that rely on changes in preferences (demand shocks) or constraints on price-setting (sticky prices). In our simple model of decentralized trade with asymmetric information, income dispersion measures uncertainty about buyer characteristics. Counter-cyclical income dispersion makes the asymmetric information friction stronger in recessions: optimal prices rise and trade volume falls. Unlike preference changes or price-setting constraints, income dispersion is observable. Using income dispersion estimates to quantify the model’s effect, we find that model prices and markups have properties similar to business cycle data. Productivity shocks are strongly amplified: output is more than three times as volatile and more than twice as persistent as our exogenous productivity process.

Recessions are times when resources are not put to their most productive uses. Goods take longer to sell (Bils and Kahn 2000), workers are unemployed, and prices do not fall enough to clear markets. Bils (1987) characterizes recessions as times when output prices do not fall as much as marginal cost. Firms’ markups are high, but sales volume is low. Booms are times when unit profits are lower, but sales volume, and thus total profits are high (Rotemberg and Woodford 1999). Recessions are not just declines in productivity, they are also times when markets allocate goods less efficiently: market frictions are counter-cyclical. But in standard business cycle models, markets are always perfectly efficient (Kydland and Prescott 1982).

Existing models of counter-cyclical market frictions rely primarily on changes in preferences (demand shocks), or constraints on price setting (sticky prices). Because preferences

*We thank George Alessandria, Mark Bils, Aubhik Khan, John Leahy, Nick Souleles, Harald Uhlig, Stijn Van Nieuwerburgh, Pierre-Olivier Weill, Michael Woodford and seminar participants at NYU, Philadelphia Fed and Rochester for helpful comments and conversations.
and pricing constraints are not observed, these theories cannot be easily quantified or tested. By contrast, this paper develops a mechanism that doesn’t involve changes in preferences, doesn’t involve any non-optimal prices, can be embedded in a standard business cycle model and can be calibrated to get a sense of its quantitative effect.

The model is based on a simple, but realistic environment. When a buyer enters a store, she sees a posted price for a good and can purchase the good or not. If the buyer values the good more than the seller does, efficiency dictates that trade should occur. However, the seller does not know each buyer’s income, and occasionally overestimates her willingness to pay. If the seller’s posted price is too high, an efficient transaction will not take place. Although this is an asymmetric information friction, similar in spirit to that of Myerson and Satterthwaite (1983), the key unknown variable (income) is measurable. This feature of the model makes it possible to calibrate the information friction and to quantify how much it changes over the business cycle. We assume that sellers know the distribution of buyers’ incomes at each date, but see each buyer as an independent draw from that distribution. Thus, when income dispersion is high, sellers’ uncertainty about each buyer’s willingness to pay will be high, and market efficiency will be low.

Tying market inefficiency to income dispersion is useful because we know how income dispersion varies over the business cycle: it rises in recessions. We take counter-cyclical income dispersion as given and ask how much inefficiency the mechanism produces and what its testable implications are. Surprisingly, this simple friction matches the sign and order of magnitude of several important moments in the data. The three main findings are (1) the effect of productivity shocks on output is amplified, (2) prices are counter-cyclical and smooth, and (3) markups are counter-cyclical and smooth.

In recessions, marginal costs are higher. Yet instead of trying to cut profit margins to compensate, sellers pursue high-margin/low-volume pricing strategies. They do this because uncertainty about a buyer’s type makes sales uncertain. In booms, the gains from trade are high and sellers are eager to ensure that trade occurs. In recessions, the gains from trade are lower and sellers are more willing to risk those small gains on some trades to make higher margins on others.

Counter-cyclical income dispersion reinforces this cyclical variation in market efficiency. One way of interpreting the counter-cyclical income dispersion effect is as a mechanism that
Figure 1: **Lowering price is more beneficial when dispersion is low.**
The shaded area represents the increase in the probability of trade from lowering the price, by an amount equal to the width of the shaded area. This higher probability, times the expected gains from trade, is the marginal benefit to reducing the price. Willingness to pay is based on agents’ income.

lowers the price elasticity of demand in recessions. When incomes are more dispersed, buyers’ willingness to pay is also more dispersed. If sellers were to reduce prices in recessions, they would attract few additional customers (as in the left panel of Figure 1). This low elasticity makes the marginal benefit of lowering prices smaller and induces firms to keep prices high. Therefore, when dispersion is high, prices stay high. By contrast, in booms dispersion is low, so if a seller lowers her price she attracts many additional customers (as in the right panel of Figure 1). So in booms, sellers keep prices low. Because dispersion increases in recessions, but is not as volatile as output, prices inherit the same properties. The time-varying elasticity is similar in spirit to a demand composition effect, as in Gali (1994). But because the elasticity in our model comes from an observable feature of aggregate data, it can be measured and tested.

To illustrate the workings of the model’s key mechanisms, sections 1 and 2 set up and analyze a static version of the model. The effect of changes in output and income dispersion are illustrated by deriving comparative statics. But the sign and magnitude of output and dispersion effects depends on parameter values. To obtain more precise predictions for the model, section 3 sets up a dynamic version of the price-setting model. A dynamic version of this price-setting model with asymmetric information about buyer income matches many of the features of recession that Bils identified. The stochastic process for aggregate productivity is taken from the estimates of King and Rebelo (1999), which is representative of the real business cycle literature. The individual income process is taken from the estimates of
Storesletten, Telmer, and Yaron (2004). By feeding in these stochastic processes for the state variables, we can determine how much effect our trading friction generates. We compare cyclical behavior of prices, sales volume, and markups to other benchmark models of trade and to business cycle data.

Microeconomic studies have documented the effect of increased income dispersion on sales. For example, Goldberg (1996) estimates that blacks’ valuations for new cars are more dispersed than whites’. She then collects data on the initial offer to blacks and whites by a car salesman. The initial offer price is higher, and the probability of sale lower, for the group with more dispersed values.

This explanation raises the obvious question: Why does income dispersion rise in recession? One explanation is that job destruction in recessions is responsible (Caballero and Hammour 1994). Rampini (2004) argues that entrepreneurs’ incentives change in recessions, making firm outcomes and owners’ incomes more risky. Cooley, Marimon, and Quadrini (2004) and Lustig and Van Nieuwerburgh (2005) argue that low collateral values inhibit risk-sharing in recessions. Although any one of these explanations could be merged with this model to produce a model whose only driving process is technology shocks, we opt to keep the model simple and focused. Therefore, we take income dispersion as given and explore its implications for market efficiency.

1 Static model

The paper’s main results are about the dynamics of aggregate variables and their covariance with output. To illustrate why prices and the probability of trade have the dynamics they do, we start with a static version of the model.

Preferences and endowments There is a large but equal number of buyers and sellers. They have identical preferences over consumption $c$ and end-of-period money holdings $m'$:

$$U(c) + U(m') = \frac{(c + 1)^{1-\alpha} - 1}{1 - \alpha} + \frac{(m' + 1)^{1-\alpha} - 1}{1 - \alpha}, \quad \alpha > 0 \quad (1)$$

The additive form of constant relative risk aversion (CRRA) preferences over consumption and money $U(c) + U(m')$ makes sense if preferences over money are seen as representing
preferences over future consumption. What is non-standard about these preferences is the 
(+1) in each term. We ensure that each agent will have at least one unit of each good to 
keep the gains from trade finite. Acquiring none of a good yields zero utility \(U(0) = 0\), 
not negative infinite utility. If buyers had none of the sellers’ good or sellers had none of 
the buyers’ good and standard CRRA preferences, then marginal gains from trade would be 
infinite. Trading frictions only have an interesting role to play when gains from trade are 
positive, but finite.

We study an endowment economy. A high productivity \(z\) means that both buyers’ and 
sellers’ endowments are larger. Buyers, indexed by \(i\) are endowed with money \(m_i\). Their 
endowments are heterogeneous: \(\log(m_i) = \log(z) + \sigma \varepsilon_i\) where \(\varepsilon_i \sim N(0,1)\). We will refer 
to \(m_i\) as a buyer’s income. Sellers are identical and are endowed with \(Az\) units of the 
consumption good, \(A > 0\). Buyers and sellers both want balanced consumption bundles. An 
increase in \(z\) makes no-trade consumption bundles less balanced, increasing gains from trade.

Matching process Every buyer is matched with a seller and every seller is matched with 
a buyer. This assumption allows us to abstract from matching frictions and focus on the 
frictions that arise from asymmetric information. Matching is random: The probability of 
a seller meeting a buyer with a particular income depends only on the aggregate income 
distribution.

Bargaining and price-setting A seller posts a price \(p\), which is a quantity of money he 
will accept for \(\delta\) units of the consumption good. To keep choices simple, the size of trades \(\delta\) is 
given. Since sellers have an endowment of \(Az\), we naturally impose that sales must be smaller 
than the endowment, \(\delta < Az\). The price is a take-it-or-leave-it offer. Given the quoted price 
\(p\), a buyer decides whether to accept the offer, \(a_i = 1\), or not, \(a_i = 0\). If trade takes place, \(\delta\) 
units of the consumption good changes hands. We use this form of price-setting because it’s 
simple, realistic, and avoids the multiple equilibria problems of bargaining.

Information Everything is common knowledge, except buyers’ income, \(m_i\). The distribu-
tion of \(m_i\) is known, but the income of any particular buyer is not known. Therefore, sellers 
are uncertain about what the profit-maximizing price is, for each trade. This is the source 
of the market inefficiency. If sellers knew each buyer’s income, then they would extract all
the buyers’ surplus, but every efficient trade would be executed. Because of the information asymmetry, some Pareto-improving trades do not occur. This is similar in spirit to the efficiency impossibility result of Myerson and Satterthwaite (1983).

**Equilibrium** An equilibrium is an individual buyer acceptance rule $a(p, b_i)$, a probability of trade $\mathbb{E}\{a(p, b_i)\}$, and a price $p^*$ such that:

1. Taking as given the price $p$, the acceptance rule $a(p, m_i)$ maximizes the utility (1) of buyer $i$ subject to the constraints $m_i' = m_i - pa(p, m_i)$ and $c = a(p, m_i)\delta$.

2. Taking as given the probability of trade $\mathbb{E}\{a(p, m_i)\}$ and the distribution of buyers, sellers choose a price $p^*$ to maximize expected utility (1), subject to the constraints: $m_i' = pa(p, b_i)$ and $c = Az - a(p, b_i)\delta$.

The buyer’s acceptance rule is a simple cutoff strategy. Buyer $i$ maximizes utility by accepting all prices $p \leq \overline{p}_i$ where the cutoff price $\overline{p}_i$ leaves the buyer indifferent between accepting and not accepting the seller’s offer: $U(m_i - \overline{p}_i) + U(\delta) = U(m_i) + U(0)$. Since $U(0) = 0$ the cutoff price is:

$$\overline{p}(m_i) := m_i - U^{-1}(U(m_i) - U(\delta)).$$

This maximum price a buyer is willing to pay is monotonically increasing in her income. Therefore, for each price $p$ there exists a unique $\underline{m}(p)$ that represents the minimum income that a buyer must have to buy at price $p$.

Sellers choose price to maximize expected utility, taking buyers’ optimal strategy as given:

$$p^* \in \arg\max_{p \geq 0} \{\Pr (m_i \geq \underline{m}(p)) [U(Az - \delta) + U(p)] + \Pr (m_i < \underline{m}(p)) U(Az)\}. \tag{3}$$

Since the optimal price does not change when constants are added or subtracted, maximizing (3) is equivalent to maximizing the probability of trade times the utility gain from a trade, $\Pr [m_i \geq \underline{m}(p)] \Delta U(p)$, where the utility gain is $\Delta U(p) := U(p) + U(Az - \delta) - U(Az)$. The probability of trade is:

$$\Pr (m_i \geq \underline{m}(p)) = \Phi \left( \frac{1}{\sigma} \log \left( \frac{z}{\underline{m}(p)} \right) \right), \tag{4}$$
where $\Phi$ denotes the CDF of the standard normal distribution. The utility-maximizing price $p^*$ balances the gains from trade against probability of trade. More specifically, the first order condition tells us that the optimal price equates the price elasticity of utility gain, with the price elasticity of demand:

$$\frac{\partial \log(\Delta U(p))}{\partial \log(p)} = -\frac{\partial \log(\Pr (m_i \geq m(p)))}{\partial \log(p)}.$$

(5)

2 Analytic results with log utility ($\alpha \rightarrow 1$)

We use general CRRA utility for numerical analysis, but restricting preferences to the special case of log utility makes our mechanism transparent. We will explain our effects using log utility and then show how the same effects are operating with more general CRRA preferences in the next section.

Substituting log utility $U(c) = \log(c+1)$ in equation (2) reveals that buyers purchase the consumption good when:

$$p \leq \frac{\delta}{1 + \delta}(m_i + 1).$$

(6)

This decision rule implies that the probability of trade taking place at price $p$ is:

$$\Pr \left( m_i \geq \frac{\delta + 1}{\delta}p - 1 \right) = \Phi \left( \frac{1}{\sigma} \log \left( \frac{z\delta}{(1 + \delta)p - \delta} \right) \right).$$

(7)

Effect of dispersion changes  If $\sigma = 0$, then there is no uncertainty about the buyer's willingness to pay, all buyers have $m_i = z$. In the absence of asymmetric information, the seller captures all gains from trade, by setting a take-it-or-leave-it price, equal to the buyer's valuation: $p^* = \delta(z + 1)/(1 + \delta)$. When uncertainty increases, sellers reduce their price to make sure that buyers will still trade with them. At this slightly reduced price, they still earn most of the gains from trade. With a large prize on the line, sellers are loathe to risk it by setting too high a price.

When uncertainty $\sigma$ is high, however, the logic of a seller changes. The seller still sets the optimal price $p^*$ by balancing the expected benefit of increased profits from raising the price against the expected lost probability of trade. But when buyers' willingness to pay is very dispersed, increasing the price results in a smaller decrease in the probability of trade. This
induces sellers to raise prices when \( \sigma \) becomes high. Figure 2 illustrates this non-monotonic effect of dispersion on price. Proposition 1 states the income dispersion-price relationship formally.

**Proposition 1.** For each \( z \), there exists a unique cutoff level of income dispersion \( \bar{\sigma}(z) > 0 \) such that if \( \sigma < \bar{\sigma}(z) \), then prices are decreasing in dispersion, \( \partial p^*/\partial \sigma < 0 \), and if \( \sigma > \bar{\sigma}(z) \), prices are increasing in dispersion, \( \partial p^*/\partial \sigma > 0 \).

![Figure 2: Optimal price \( p^* \) as function of dispersion \( \sigma \).](image)

Price is the seller’s posted price for \( \delta \) units of the good.

This result, proven in Appendix A, comes from applying the implicit function theorem to (5). The left side, the elasticity of the seller’s utility gain from trade, depends on price \( p \) but not on dispersion \( \sigma \). The non-monotonicity arises because when \( \sigma \) is low, the demand elasticity (right side of (5)) is very price-sensitive. Sellers compensate for a moderate increase in \( \sigma \) by reducing price to ensure that most trade still occurs. When dispersion is high, demand is less price sensitive and the utility gain term dominates pricing decisions. Since an increase in dispersion makes demand less elastic, sellers raise prices.

**Effect of productivity changes**  The effect of productivity changes is less obvious because productivity affects both the aggregate demand elasticity and the price elasticity of unit profits. Prices are non-monotonic in productivity; they can be pro- or counter-cyclical, depending on the mean level of productivity.

To simplify the discussion, we make a parametric assumption which will always be satisfied in our calibrated examples, which rules out a small number of ‘perverse’ outcomes, and which
makes it easier to exposit the mechanics of our model.

**Assumption 1.** The size of trades $\delta$ is sufficiently large, $\delta \geq \sqrt{1-\delta}$.

Prices are non-monotonic in output. When output is very low, sellers do not have many goods, even for their own consumption, so the opportunity cost of selling one is high. Since consumption goods are scarce, their price is high. As productivity $z$ rises, a seller gains more from trading and lowers price to make trade more likely. In this region, price is countercyclical. But when $z$ is sufficiently high, the probability of trade and the elasticity of demand are not very sensitive to price. Therefore, sellers focus on capturing gains from trade. To do this, sellers set $p^*$ high and approximately proportional to $z$, meaning price rises when the gains from trade rise. So, when the gains from trade and average probability of trade are sufficiently high, prices are pro-cyclical. Figure 3 illustrates these effects, which are summarized in the following proposition.

**Proposition 2.** For each $\sigma > 0$, there exists a unique cutoff level of productivity $\overline{z}(\sigma) \geq \delta/A$ such that if $z < \overline{z}(\sigma)$, then prices are decreasing in productivity, $\partial p^*/\partial z < 0$, but if productivity is sufficiently high, $z > \overline{z}(\sigma)$, then prices are increasing in productivity $\partial p^*/\partial z > 0$.

![Figure 3: Optimal price $p^*$ as function of productivity $z$. Prices fall, then rise, as productivity increases.](image)

The static effect of productivity on prices dictates the dynamic cyclical behavior of the probability of trade. Equation (7) shows that changes in productivity affect the probability of trade in two ways. First, an increase in productivity increases each buyer’s income $m_i$
and hence directly increases the probability of trade, \( \Pr(m_i \geq \underline{m}(p)) \). Second, an increase in productivity changes the price \( p^* \) and therefore the probability of trade. For low levels of productivity, the price \( p^* \) is decreasing. So in low-productivity times, the probability of trade is clearly pro-cyclical. But when productivity is higher, \( p^* \) is increasing in productivity; this indirect effect offsets the direct effect of higher average income. In the calibrated examples that follow, the price effect is always dominated by the direct effect of higher average income, making the probability of trade always pro-cyclical.

The static model delivers three insights that will be important in understanding why the dynamic calibrated model replicates features of the data. (1) The probability of trade is pro-cyclical. We motivated this exercise with facts suggesting that market exchange is less efficient in a recession. This is the result that tells us that our mechanism does deliver such an effect. (2) Prices are increasing in productivity \( z \) if the gains from trade are high, are insensitive for medium levels of the gains from trade, and are decreasing in productivity \( z \) for low levels of gains from trade. (3) More dispersion in buyer incomes causes prices to fall if dispersion is low, but causes prices to rise if dispersion is high. The second and third insights cannot be tested without determining the relevant range of gains from trade and income dispersion. Also, since changes in gains from trade and income dispersion are correlated, their combined effect depends on what their relative volatilities are. To resolve these questions and compare the model’s indirect implications to data, we turn to a dynamic calibrated model.

3 Dynamic model

The dynamic model is simply a repeated static model where productivity \( z \) and income dispersion \( \sigma \) fluctuate. Time is discrete and infinite \( t = 0, 1, \ldots \). Productivity follows an AR(1) process:

\[
\log(z_t) = (1 - \rho) \log(\overline{z}) + \rho \log(z_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_\epsilon).
\] (8)

As before, an individual’s income is correlated with productivity, but has an idiosyncratic component as well: \( \log(m_{it}) = \log(z_t) + \varepsilon_{it} \).
3.1 Calibration

We study quarterly data. Because we want to use the exact income process estimated by Storesletten, Telmer, and Yaron (2004), we follow them in having idiosyncratic income with persistent and transitory components and the following parameters:

\[ \varepsilon_{it} = \xi_{it} + u_{it}, \quad u_{it} \sim \mathcal{N}(0, 0.065^2) \]  
\[ \xi_{it} = 0.988\xi_{it-1} + \eta_{it}, \quad \eta_{it} \sim \mathcal{N}(0, \sigma_{\eta t}^2), \]

and dispersion that increases when output is below average:

\[ \sigma_{\eta t} = \begin{cases} 
0.032 & \text{if } z_t \geq \overline{z}, \\
0.054 & \text{if } z_t < \overline{z}.
\end{cases} \]

We calibrate the productivity process \( z \) using the estimates of King and Rebelo (1999). The quarterly persistence of the Solow residual is \( \rho = 0.98 \), and the standard deviation of its innovation is \( \sigma_\varepsilon = 0.0072. \) The coefficient of relative risk aversion is set to \( \alpha = 1.5 \), a commonly used, conservative value. The size of a trade \( \delta = 0.8 \) is approximately the same size trade as in a competitive economy (see section 3.3).

The most difficult quantities to tie to data are average productivity \( \overline{z} \) and the scale factor \( A \). If endowments are too small, most trading pairs have no gains from trade. If they are too large, trading frictions disappear. Since we think of the US economy as a place where people normally produce specialized goods and therefore have moderate gains from trade, we calibrate the endowments to make the unconditional average probability of trade 0.65. Jointly with other parameters, \( A = \overline{z} = 1 \) achieves this balance. Model outcomes are sensitive to

\footnote{Our quarterly persistence and standard deviations are obtained from the annual estimates of Storesletten, Telmer, and Yaron (2004) as follows: \( \varphi = 0.952^{1/4} \), \( \sigma_R = 0.125Q \), \( \sigma_R = 0.211Q \), and \( \sigma_u = 0.255Q \) where \( Q := 1/(1 + \varphi + \varphi^2 + \varphi^3) = 0.2546. \) We are deviating from these results in one additional way: Our income shock changes variance, depending on whether productivity is above its mean or not. In the results of Storesletten, Telmer, and Yaron (2004), the variance of the income shock depends on whether output is above its mean. Because of the high correlation of productivity and output, these two processes are virtually indistinguishable. We make this change because conditioning the variance on output, an endogenous variable, is significantly more complicated.}

\footnote{These statistics refer to time series that are HP filtered with a smoothing parameter of 1600. All simulations in this paper begin by sampling the exogenous state variables for a ‘burn-in’ of 1000 quarters. This eliminates any dependence on arbitrary initial conditions. A cross-section of 2500 individuals is tracked for 200 quarters (the size of this panel corresponds to the dimensions in Storesletten, Telmer, and Yaron (2004)). Realizations of endogenous variables are then computed. The moments discussed in the text are averages over the results from 10000 runs of the simulation.}
these parameters. However, when we alter parameters in such a way to keep the probability of trade from being close to 0 or 1, our results are robust. It is crucial to choose parameters that make very low and very high trading frictions arise infrequently. But it doesn’t matter much what combinations of parameters we use to match that fact. After presenting our main results, we examine the behavior of the economy in a time when gains from trade are very low. We use those results to characterize a depression.

![Figure 4: Calibrated range of parameters.](image)

The direct effect of $z$ on $p^*$ is ambiguous, but the effect of $\sigma$ on $p^*$ is positive. In the left panel, the shaded part of the line is the 2 standard deviation range of productivity, based on estimates from King and Rebelo (1999). In the right panel, the 2 standard deviation range of income dispersion is indicated, based on the estimates of Storesletten, Telmer and Yaron (2004).

One of the reasons that it is important to calibrate the model is because the cyclical behavior of prices cannot be inferred from theory alone. Only when the relevant region of the parameter space is identified can the model’s predictions be compared to data. The left hand panel of Figure 4 is the same theory-based mapping between productivity and price as Figure 3, but with the calibrated 2 standard deviation interval of productivity around its mean highlighted. In this interval, prices react very little to changes in productivity but react strongly to changes in dispersion (right panel). Since the direct effect of productivity on prices is not very strong, the cyclical behavior of prices will be determined primarily by income dispersion. In this region, prices are monotonically increasing in income dispersion. Since income dispersion rises in recessions, this effect will tend to make prices counter-cyclical.

With our calibrated parameters, the probability of trade is given by a CDF which is steep for moderate levels of productivity. Small changes in productivity are associated with
relatively large changes in the probability of trade.

3.2 Defining GDP

We measure per-capita real GDP in this model by $y := Az \Pr(m_i \geq m(p))$. The idea is that in a frictionless model the probability of trade would equal one and GDP would be equal to the seller’s $Az$.

As discussed in section 4 below, it will turn out to be the case that our results are essentially identical if instead we measured GDP by the expected amount of consumption goods that a seller and buyer exchange in the market, $\Pr(m_i \geq m(p))\delta$. There are two ways to interpret this latter mapping. First, it could be that $Az$ is potential production of services. But the services are not actually rendered unless they are sold. Second, it could be that a seller actually produces $Az$ units of goods. But unless the goods are exchanged, they are not counted in GDP. They are unmeasured household production.

3.3 A competitive benchmark

A natural benchmark for comparisons is an economy with competitive markets. Suppose buyers and sellers have identical preferences, and face competitive prices. The only difference between this setting and our model is the agents’ budget constraints. Now each seller has the budget constraint $pc + m' \leq pAz$ while buyer $i$ has budget constraint $pc + m' \leq m_i$. The first order condition for all agents can be written $[(m' + 1)/(c + 1)]^\alpha = p$. After substituting $p$ into constraints, each seller demands $c_s = (pAz + 1 - p^{1/\alpha})/(p + p^{1/\alpha})$ and buyer $i$ demands $c_i = (m_i + 1 - p^{1/\alpha})/(p + p^{1/\alpha})$ units of consumption. Market clearing requires $c_s + \mathbb{E}\{c_i\} = Az$. This determines the competitive price:

$$p_c = \left(\frac{\mathbb{E}\{m_i\} + 2}{Az + 2}\right)^\alpha,$$

where $\mathbb{E}\{m\} = z \exp\left(\frac{1}{2} \sigma^2\right)$. 

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4 Quantitative results

In recessions, sellers pursue a low volume, high margin strategy. In booms, they earn lower profit per unit, but make higher total profits by selling more. This section quantifies these effects and compares their magnitudes to evidence from macroeconomic aggregates.

4.1 Amplification of GDP and re-calibration

We started with a standard calibration of a productivity process from King and Rebelo (1999) to see what our model would produce in comparison to other models that use the same technology shocks. However, the innovations to that shock process are chosen to let the real business cycle model match the volatility of output. There is no reason to think that this driving process is appropriate for our model. In fact, it produces more than three times too much volatility in output.

Result 1. Productivity shocks are amplified.

The relationship between output, productivity and income dispersion is illustrated in Figure 5. While a standard real business cycle model’s output volatility is roughly 15% higher than productivity volatility, this model’s output volatility is 368% higher.

![Figure 5: Productivity shocks are amplified (result 1)](image)

The three lines plot output log($y$), productivity log($z$), and income dispersion $\sigma$ as deviations from their population means. Data come from a simulation of the model with the productivity process re-calibrated to match the properties of output in the data.
Since we want the model outcomes to resemble the data, we recalibrate the productivity process to match the persistence and volatility of real GDP. The required autocorrelation $\rho$ falls from 0.98 to 0.40. Less persistence of productivity is needed because income dispersion is a highly persistent process. It is the source of long-run swings in output. The standard deviation of innovations to productivity falls by 80% from 0.0072, to 0.0014. Less volatile productivity is required because improvements in productivity are amplified by increases in the probability of trade. All the qualitative results that follow are true, whether we use the original, volatile productivity process, or the new, less-volatile one. We report results based on this new, lower-variance calibration, in order to facilitate comparisons between the model and data.

**Result 2. Prices are counter-cyclical and smooth.**

Figure 6 illustrates the time-series behavior of prices, output and income dispersion. The correlation of log output with log prices in this simulated model is $-0.94$. The standard deviation of log prices is 14% of the standard deviation of log output. By comparison, simulating the behavior of the competitive price $p_c$ [as defined in equation (12)] shows that prices in the competitive economy are smooth (standard deviation of 77% of the standard deviation of output) and less counter-cyclical (correlation with output is $-0.16$). GDP is smoother than, and almost perfectly correlated with productivity.$^3$

Another relevant benchmark is a model with perfect information. If a buyer’s type $m_i$ is perfectly observable, then a seller will set the price $p_i = \overline{p}(m_i)$ to extract all of the surplus [as in equation (2)]. With perfect information about buyers’ types, the correlation of log output and log prices $\mathbb{E}\{p_i\}$ becomes 0.41, and the standard deviation of log prices is 1.19 times the standard deviation of log output. Compared to this perfect information benchmark, the prices in our model are more counter-cyclical and less volatile.

Cyclical fluctuations in price $p$ capture terms of trade variation in an exchange between a buyer and a seller. For theoretical purposes, the nature of the buyers’ and sellers’ goods does not matter. But to compare model and data, we need to decide what kind of price will be the basis for comparison. We interpret the sellers’ good as a general purpose consumption good and interpret the buyers’ good as money. In a successful exchange, sellers hand over $\delta$.

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$^3$In a symmetric version of the competitive benchmark with $A = 1$ and $\sigma = 0$, price would be constant (independent of $z$). If $\sigma > 0$, as in our calibration, the competitive model delivers smooth, counter-cyclical prices. But that model does not deliver pro-cyclical market efficiency.
Figure 6: **Prices are counter-cyclical and smooth (result 2).**

The three lines plot output, log($y$), income dispersion, $\sigma$, and price, log($p$), from a simulation of the calibrated model. All series are shown as deviations from their population means.

Units of the consumption good for $p$ units of money. Therefore, the appropriate counterpart to our $p$ is a price index which measures the money cost of a bundle of goods. The price index we use is the implicit price deflator for total GDP, but our main findings would be unchanged if instead we used an implicit price deflator for aggregate consumption or if we used a fixed-basket consumer price index.

In quarterly macroeconomic aggregates (1947:1-1996:4), the correlation of the log GDP deflator with log GDP is $-0.54$. The standard deviation of the log GDP deflator is 0.55 times the log standard deviation of GDP (Stock and Watson 1999). While the model’s price correlation is too counter-cyclical at $-0.94$, it comes much closer to matching the data than the strong positive correlation in the perfect information model. The low standard deviation of the model’s prices is also closer to the data. But the model makes prices too smooth.

Another way to map the model’s price into the data is to think of price as the inverse of the wage, following Rotemberg and Woodford (1999). Appendix B shows how the model could be modified so that buyers supply labor to sellers, who own the production technology. In exchange for their labor, the buyers receive goods. The rate of exchange of goods for labor, the inverse of the real wage, behaves just like the price in this model. In the data, the inverse real wage and GDP have a correlation that is between $-0.21$ and $-0.54$ (Rotemberg and Woodford 1999). The model’s correlation falls within this range.
Figure 7: Market efficiency is pro-cyclical and volatile (Result 3). The two lines plot the probability of trade (the measure of market efficiency) and output, $\log(y)$, from a simulation of the calibrated model. Both series are shown as deviations from their population means.

The low volatility of prices in our model can be interpreted as a form of real price stickiness. In our model of decentralized trade, relatively efficient outcomes are only achieved if prices are sufficiently flexible. In our calibrated model, prices are inflexible and outcomes are inefficient. While traditional models of sticky prices have assumed exogenous constraints on price-setting or access to timely information, our real rigidity arises endogenously from observed features of macro data. In both types of models, the failure of sellers to lower prices makes recessions times when market frictions are strong.

Market efficiency One of the objectives of the paper was to construct an equilibrium model where preferences are fixed, prices are (constrained) optimal and yet market frictions are strong in recessions. Since almost every transaction is Pareto-improving, the probability of trade $\Pr(m_i \geq \overline{m}(p))$ measures market efficiency.

Result 3. Market efficiency is pro-cyclical and volatile.

Figure 7 illustrates the simulated time-series behavior of market efficiency. In contrast, the perfect information benchmark, or any model with a centralized market, predicts that trade always occurs, making its correlation and standard deviation zero.
Markups We have characterized recessions as times when firms pursue low-volume, high-marginal sales strategies. To measure this effect in our model, we define a markup to be the percentage difference between the price in the competitive market economy (12) and the price in our decentralized model. Since the competitive price $p_c$ is a price per unit and our model’s price is for $\delta$ units, we normalize our price by $\delta$ to make it comparable:

$$\text{markup} := \log \left( \frac{p^*}{p_c \delta} \right).$$

(13)

**Result 4.** Markups are counter-cyclical and smooth

Figure 8 illustrates the simulated time-series behavior of markups. The correlation of markups and log output in the simulated model is $-0.82$. The standard deviation of markups is 0.07 times the standard deviation of log output. In the perfect information model, the correlation with log output would be 0.88 while the standard deviation would be about 0.68 times that of output. In a perfectly competitive market, the markup is always zero, by definition.

Bils (1987) measures firm markups by inferring firms’ marginal costs. He finds that markups increase an average of 4% in recession. To ask if our model comes close to matching this fact, we compute the average markup of firms for the bottom 14% of output realizations.
Table 1: Summary statistics for the model, benchmark models and the data. Standard deviation measures the variables standard deviation, relative to that of log output (real GDP). See text for a discussion of markups in the data.

The 14% corresponds to the fraction of quarters the postwar US economy has spent in an NBER-recession. We compare this recession markup to the average markup for the top 86% of output realizations. We find that markups are 3.5% higher in recessions. Table 1 summarizes the results from our model.

### 4.2 The role of counter-cyclical income dispersion

One thing these results do not tell us is how much of the model’s effect come from changes in income dispersion and how much could be generated by changing output with a constant amount of information asymmetry. To distinguish these two effects, we reran the simulated model, with one change: income dispersion was constant, equal to the average level of income dispersion in the full model. In the modified model prices become almost constant, with standard deviation and output correlation that are indistinguishable from zero. This reveals that all of the price movement in the model is coming from changes in income dispersion. The reason that prices do not move is illustrated in Figure 4. Output falls in a region where prices are flat. Changing parameters such as risk aversion, the mean level of output, or the variance of output shocks would make this result less extreme.

Constant dispersion lowers the volatility of the probability of trade and leaves it almost perfectly pro-cyclical. The reason is that higher productivity increases the gains from trade and the probability of trade. Since productivity is the only force left in the model and it moves both the probability of trade and GDP, the two variables become perfectly correlated. Because of the variation in the probability of trade, this model does still amplify the effect of
TFP shocks on output. But, by shutting down the effect of productivity on income dispersion, which in turn affects output, 2/3rds of the amplification is lost. These results suggest that it is not not the details of decentralized trade, but rather counter-cyclical income dispersion that drives our results.

4.3 Welfare costs of asymmetric information

If small information frictions cause such large disruptions to macroeconomic aggregates, should agents acquire information to reduce the friction? The answer to this question depends on the welfare costs of asymmetric information, particularly for the seller. In this setting, it is the seller who faces the incomplete information problem. Sellers are only likely to try to differentiate customer types if the cost of the information friction is higher than the cost of acquiring information.

The cost of information asymmetry The comparison we do is the following. We compute expected utility in our calibrated model. Then, we compute expected utility in a model where buyers’ incomes have the same distribution as in our model, but their income is public information. The expected utility of a seller in our model is 0.664. When sellers can extract the entire surplus from trade without the risk of setting price too high, they are better off; their expected utility increases to 0.737, an 11% increase. Expressed differently, sellers would be willing to sacrifice 34% of their profits, or 34% of their goods production, to achieve full information.

Conversely, buyers suffer when their types are known. Their utility falls from 0.674 in the asymmetric information model to 0.590 in the full-information model, a decline of 12%.

This is not a realistic comparison because no seller could ever know every buyers’ willingness to pay with certainty. What they can do is gather information or use pricing techniques to reduce the uncertainty about buyers’ incomes. Therefore, we estimate the benefit to reducing, but not eliminating, uncertainty about buyers’ incomes. To model the partial resolution of uncertainty, think of the individual-specific component of income as having two pieces: $\sigma_t \varepsilon_{it} = \sqrt{1-\gamma} \sigma_t e_{it} + \sqrt{\gamma} \sigma_t \tilde{e}_{it}$, where $e_{it}, \tilde{e}_{it} \sim N(0,1)$. The seller observes the first component $e_{it}$ of the individual-specific income, but does not know the second component $\tilde{e}_{it}$. Then the degree of asymmetric information is indexed by $\gamma$. As $\gamma \to 0$, all income is observable
and the information asymmetry disappears. As $\gamma \to 1$, this converges to our original model, where all income heterogeneity is unobserved.

To eliminate 10% of the uncertainty about buyer characteristics, sellers would be willing to forfeit 0.7% of their profits. To eliminate 50% of their uncertainty, sellers would forfeit 4.8% of their profits, and 90% elimination of uncertainty is worth 16% of profits. Thus, while the 34% sellers would be willing to pay to eliminate all uncertainty is large, most of that benefit is associated with completely, rather than partially resolving uncertainty. Since complete resolution of uncertainty is likely impossible, we are left with a modest willingness to pay for information on buyer characteristics. We conclude that our information friction could well persist, even if sellers have access to costly technologies for discriminating between buyers. Unless those technologies are cheap and effective, they may not be worthwhile to implement.

Although the potential gains to eliminating uncertainty are high for the seller, most of this gain is a distributional effect. The total welfare cost of asymmetric information is only 2.7% of expected utility. While this is small, it is an order of magnitude larger than the standard cost of a real business cycle.

**The cost of time-variation in income dispersion** Since the paper is about the effects of time-variation in income dispersion, we can ask what the cost of that variation is. While the information friction itself has moderate welfare effects, its time-variation does not. In a model with constant income dispersion (set equal to its unconditional mean), both buyers’ and sellers’ welfare changes by less than 0.1%. This is in line with standard business cycle welfare results.

### 4.4 Economic behavior in a depression

Since the model’s relationships between output, price and dispersion are nonlinear, the qualitative effect of a big shock can be quite different from a small one. We now illustrate the workings of the model far away from its calibrated parameter values.

How do model statistics change if the gains from trade become very low? To answer this question, we subject our economy to a depression-sized shock to productivity. Cole and Ohanian (2004) report that US real gross national product per adult fell 13% below
trend in 1930 and reached a trough of 39% below trend in 1933. To incorporate a shock of this magnitude into our model, we run simulations for many periods but then introduce a productivity shock that reduces GDP 39% below its long-run level. Following this shock, productivity follows the stochastic process given in (8). Figure 9 shows a typical realization.

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\textbf{Figure 9: \textit{Economic behavior in a depression.}}

A depression-sized shock to productivity at date $t = 0$.

During and in the immediate aftermath of a depression, productivity, not income dispersion is most important driving force, in the model. The cyclical properties of market efficiency do not change: The correlation of the probability of trade with output is the same as in the benchmark model. Prices are still negatively correlated with productivity, and therefore output: correlation of $-0.71$ instead of the $-0.94$ in normal times. Although prices are less counter-cyclical, markups are more counter-cyclical; their correlation with output is $-0.91$ instead of $-0.82$. These changes in covariances arise because of the nonlinear relationship between price and output. When output is very low, prices are very high and fall steeply as output rises. (See Figure 3.) For low enough levels of output, this relationship swamps the effects of income dispersion. Yet, the basic effects of the model persist.

One feature of the model that does change dramatically is the persistence of output. It returns back to slightly below its normal level quickly, then takes a very long time to return all the way to trend. The reason for this pattern lies in the behavior of income dispersion. When the depression shock arrives, income innovations become volatile and the dispersion of income grows. But this new level of dispersion is not more than in a regular recession because
5 Conclusion

This paper presents a simple model that delivers intuitive relationships between output and market frictions and is based on observable variables. Instead of unobserved changes in preferences or price setting constraints, we wrote down a model where income dispersion determined sellers’ uncertainty about buyers’ willingness to pay. Because the key market friction is tied to a measurable variable, the theory offers quantitative predictions for how much effect this friction can generate. A simple friction, like not knowing how much a buyer earns, can generate smooth but counter-cyclical prices, a volatile and pro-cyclical probability of trade, and smooth counter-cyclical markups, all of the right orders of magnitude. Since counter-cyclical income dispersion can be generated by shocks to the real economy, this paper brings facts typically thought to be the domain of demand-based analysis into the realm of supply-driven models.

Booms are times when gains from trade are high and consumers are more homogenous in their willingness to pay for goods. Firms set prices to sell many units of their good. This makes their total profits high, even through per-unit profits are low. Downturns are times when gains from trade are low and the willingness to pay of any consumer is more uncertain. In these times, firms set prices to earn high profits per sale, lowering the probability of trade. They do this because the increase in buyers’ dispersion reduces the elasticity of demand for the good in recessions. Lowering price attracts fewer additional customers. So, firms optimally keep prices high.

Can our mechanism be reconciled with the long run decline in business cycle volatility and the contrasting increase in idiosyncratic volatility? We take income dispersion as given and so have nothing to say about why income dispersion might rise. But an augmented version of our model could explain why increased income dispersion might cause the decline in macro volatility. Firms can develop better tools for identifying customer characteristics:
on-line sales, customer discount cards, mid-week sales, student and senior discounts, all allow firms to target prices to a lower-variance distribution of customers. As consumers become more heterogeneous, these kinds of tools become more valuable. If they can be adopted for a cost, market-segmenting technologies will become more widespread, dampening the effect of cyclical changes in income dispersion. Our income dispersion mechanism, with a choice over market-segmenting technology, could reconcile less macro volatility with more micro volatility.

Extending the model with more intertemporal linkages could also allow it to explain cyclical movements in inventories. The counter-cyclical movements in inventory-sales ratios and the acyclical rate of stockouts has posed a puzzle to economists (Bils 2005). One way that these two facts can be reconciled is if there are more temporary or targeted discounts in recessions. Temporarily reducing prices to attract low-income customers is a likely cause of a stockout. Perhaps recessions are a time when trade is less likely but is also highly volatile. In future research we plan to use supermarket scanner price data to investigate this possibility.

Appendix A: The pricing function with log utility

With log utility, the gains from trade can be written \( \Delta(p, z) := \log[(p+1)(Az-\delta+1)/(Az+1)] \) while the probability of trade can be written \( \mu(p, z, \sigma) := \Phi[\log(kz/(p-k))/\sigma] \) where \( k := \delta/(1+\delta) \). With this notation, the first order condition for a maximum can be written:

\[
\frac{\partial \Delta(p, z)}{\partial p} p \frac{\Delta(p, z)}{\Delta(p, z)} = -\frac{\partial \mu(p, z, \sigma)}{\partial p} \frac{p}{\mu(p, z, \sigma)}.
\]

This implicitly determines a pricing function \( p(z, \sigma) \).

Proof of Proposition 1

Applying the implicit function theorem to (14) and using the second order condition for a maximum shows that a necessary and sufficient condition for \( \partial p(z, \sigma)/\partial \sigma > 0 \) is:

\[
\frac{\partial^2 \mu(p, z, \sigma)}{\partial \sigma \partial p} \mu(p, z, \sigma) > \frac{\partial \mu(p, z, \sigma)}{\partial \sigma} \frac{\partial \mu(p, z, \sigma)}{\partial p}.
\]

Computing the relevant derivatives and simplifying shows that this is true if and only if:

\[
\left(\frac{1}{\sigma} \log \left(\frac{p-k}{kz}\right)\right)^2 - \frac{1}{\sigma} \log \left(\frac{p-k}{kz}\right) H \left(\frac{1}{\sigma} \log \left(\frac{p-k}{kz}\right)\right) < 1,
\]

where \( H(w) := \phi(w)/(1 - \Phi(w)) > 0 \) denotes the standard normal hazard function for \( w \in \mathbb{R} \). This satisfies \( H(w) > w, H'(w) > 0, H(-\infty) = 0 \) and \( H(w)/w \to 1 \) as \( w \to \infty \). Let
\[ \varphi(w) := w[w - H(w)]. \] With a simple change of variables (16) can be written \[ \varphi(w) < 1. \] Due to the properties of a standard normal hazard function, it is straightforward to show that \( \varphi(w) \) is strictly decreasing with \( \varphi(0) = 0 \) and a unique \( \omega := -\varphi^{-1}(1) > 0 \) independent of \( z \) and \( \sigma \) such that \( \varphi(-\omega) = 1 \). We can therefore say that (16) is satisfied if and only if:

\[
p(z, \sigma) > k + k \sigma \exp(-\omega \sigma). \tag{17}
\]

Now let the right side of (17) be \( F(\sigma) := k + k \sigma \exp(-\omega \sigma) \) which is strictly decreasing in \( \sigma \) and also satisfies \( F(0) = k + k \sigma = p(z, 0) \). Therefore, as \( \sigma \to 0 \), we have \( p(z, 0) \to F(0) \) and so the necessary and sufficient condition (17) gives \( \lim_{\sigma \to 0} \{ \partial p(z, \sigma)/\partial \sigma \} \leq 0 \). Therefore, for each \( z \) there exists a unique \( \bar{\sigma}(z) > 0 \) such that \( p(z, \sigma) < F(\sigma) \) for all \( \sigma < \bar{\sigma}(z) \), \( p(z, \bar{\sigma}(z)) = F(\sigma(z)) \) and \( p(z, \sigma) > F(\sigma) \) for all \( \sigma > \bar{\sigma}(z) \).

**Proof of Proposition 2**

Applying the implicit function theorem to (14) and using the second order condition for a maximum shows that a necessary and sufficient condition for \( \partial p(z, \sigma)/\partial z > 0 \) is:

\[
\frac{\partial^2 \mu(p, z, \sigma)}{\partial z \partial p} \frac{\partial \mu(p, z, \sigma)}{\partial p} z > \frac{\partial \mu(p, z, \sigma)}{\partial z} \frac{z}{\mu(p, z, \sigma)} - \frac{\partial \Delta(z, p)}{\partial z} \frac{z}{\Delta(z, p)}. \tag{18}
\]

Computing the relevant derivatives and simplifying shows that this is true if and only if:

\[
\frac{1}{\sigma} \log \left( \frac{p - k}{k} \right) < H \left( \frac{1}{\sigma} \log \left( \frac{p - k}{kz} \right) \right) \left[ 1 - \frac{p + 1}{p - k} G(z) \right], \tag{19}
\]

where \( H(w) := \phi(w)/(1 - \Phi(w)) > 0 \) again denotes the standard normal hazard function for \( w \in \mathbb{R} \) and where:

\[
G(z) := \frac{\delta A z}{(Az - \delta + 1)(Az + 1)} \in (0, 1). \tag{20}
\]

\( G(z) \) is maximized at \( z^* = \sqrt{1 - \delta}/A \) with \( G'(z) < 0 \) for all \( z > z^* \). Assumption 1 then implies \( z > z^* \) and hence \( G'(z) < 0 \). Now define \( \psi(w) := w/H(w) \), which has the properties \( \psi'(w) > 0, \psi(-\infty) = -\infty, \psi(0) = 0, \) and \( \psi(\infty) = 1 \). We can write the necessary and sufficient condition as:

\[
\psi \left( \frac{1}{\sigma} \log \left( \frac{p - k}{k} \right) \right) < 1 - \frac{p + 1}{p - k} G(z). \tag{21}
\]

Notice that the left hand side of (21) is strictly increasing in \( p \) taking on the value \( -\infty \) at \( p = k \) and approaching 1 from below as \( p \to \infty \). Similarly, the right hand side is strictly increasing in \( p \) also taking on the value \( -\infty \) at \( p = k \) but approaching 1 - \( G(z) < 1 \) from below as \( p \to \infty \). By the intermediate value theorem there exists a unique \( \bar{\sigma}(z, \sigma) \) such that the left hand side is less than the right hand side if and only if \( p(z, \sigma) < \bar{\sigma}(z, \sigma) \) (we let \( \bar{\sigma}(z, \sigma) = 0 \) in the non-generic case where the left hand side is always greater than the right). Hence we can write the necessary and sufficient condition as:

\[
\frac{\partial p(z, \sigma)}{\partial z} > 0 \iff p(z, \sigma) < \bar{\sigma}(z, \sigma). \tag{22}
\]

Another application of the implicit function theorem shows that the cutoff function \( \bar{\sigma}(z, \sigma) \) is increasing in \( z \), \( \partial \bar{\sigma}(z, \sigma)/\partial z > 0 \) with boundaries \( \bar{\sigma}(\delta/A, \sigma) \geq k \) and \( \bar{\sigma}(\infty, \sigma) = \infty \). Hence for each \( \sigma > 0 \) there exists a \( \bar{\sigma}(\sigma) \geq \delta/A > 0 \) such that \( \partial p(z, \sigma)/\partial z > 0 \) if and only if \( z > \bar{\sigma}(\sigma) \).
Appendix B: An alternative model with labor

In this appendix we develop an alternative formulation of our model that shows how our mechanism generates procyclical but smooth fluctuations in real wages.

There is a large but equal number of workers and producers. They have identical preferences over consumption $c$ and leisure $\ell$ given by $U(c) + U(\ell)$ where $U(x) = [(x + 1)^{1-\alpha} - 1]/(1 - \alpha)$ for $x = c, \ell$. Producers are identical and have a technology for turning effective labor time into consumption goods: one unit of time produces $A$ units of goods. Workers have heterogeneous endowments of effective labor time. Worker $i$ has effective time endowment $z_i$ where $\log(z_i) = \log(z) + \sigma \varepsilon_i$ and $\varepsilon_i \sim N(0, 1)$. Every worker is matched with a producer. Since $z_i$ is not observable to producers ex ante, employment contracts are time-contingent. Producers post a wage that is a take-it-or-leave-it offer of $w$ goods in return for $\delta$ units of time. Worker $i$ accepts a wage of $w$ if and only if:

$$U(w) + U(z_i - \delta) \geq U(0) + U(z_i)$$

(23)

This implicitly defines a reservation wage $\overline{w}(z_i)$ that represents the lowest wage a worker with effective time $z_i$ will accept.

If a wage offer is accepted, the producer has utility $U(Az\delta - w) + U(1)$. If the wage is not accepted, the producer has $U(0) + U(1)$. (We assume that producers inelastically supply one unit of labor). Producers do not know the type $z_i$ of the worker they are matched with, but they do know the distribution of $z_i$ they are sampling from. Production takes place if and only if a wage $w \geq \overline{w}(z_i)$ is offered. Therefore, producers choose a wage $w^*$ that solves:

$$w^* \in \arg \max_{w \geq 0} \left\{ \Pr \left( w \geq \overline{w}(z_i) \right) U(Az\delta - w) \right\}.$$

(24)

We solve this problem numerically and calibrate it as in Section 3. The only difference is that we now need to set $A = 1.37$ to achieve an average probability of trade of about 0.65. As shown in Figure 10, the wage-setting model looks like the mirror-image of our benchmark price-setting model (Figure 4). For our calibration, the sensitivity of the wage to aggregate productivity is ambiguous but a rise in income dispersion (as in a recession) unambiguously reduces the wage while a fall in income dispersion (as in a boom) increases the wage.

With the small productivity shocks described in Section 4.1, the models correlation of log real wages and log output is 0.91: Real wages are procyclical. We also find that the standard deviation of log real wages relative to log output is only 0.23: Wages are smooth. The other implications of this wage-setting model are essentially the same as in our benchmark. For example, the probability of trade is procyclical and about 0.62 times as volatile as log output. The same productivity shocks that let the price-setting model match the empirical volatility and persistence of output enable the wage-setting model to match those moments as well. The model standard deviation of log output is about 0.013 (against 0.016 in the data) with a first order autocorrelation of 0.83 (against 0.80 in the data). In short, the wage-setting model delivers essentially the same empirical implications as our benchmark price-setting model.
Figure 10: **Calibrated parameter range in the labor model.**

The direct effect of $z$ on $w^*$ is ambiguous, but the effect of $\sigma$ on $w^*$ is negative

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**References**


