College Attainment and the Changing Life Cycle Profile of Earnings

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Abstract

I document the life-cycle earnings profile for the 25-year-old college- and high school-educated white men in 1940, 1950, 1960 and 1970. I find that older cohorts have flatter average life-cycle earnings profile: The average annual real earnings of college-educated individuals from the 1940 cohort rose by a factor of 4, while those for the 1970 cohort rose by a factor of less than 2.5. Using a version of the Ben-Porath model, I propose an explanation based on the composition effect. In my model, all individuals have a high school diploma and are differentiated by their ability. They must decide whether to work or go to a four-year college. There is a threshold ability above which individuals choose to attend college and below which they work. As in the Ben-Porath model, life-cycle earnings profiles have a hump shape, and individuals with higher ability have steeper earnings profiles. All cohorts face the same ability distribution and an exogenous sequence of wage rate per unit of human capital that grows at a constant rate. That is, individuals from the 1970 cohort will face the same growth in wage rate as the 1940 cohort but start with a higher initial level. A higher initial level of wage rate increases college attainment implying that the average ability is lower for both college- and high school-educated individuals. Since lower ability individuals have less steep increment in their earnings, the average college life-cycle earnings profile for the 1970 cohort will be flatter than that of the 1940 cohort. A similar result holds for the average high school earnings profile. My model is able to quantitatively explain approximately 67 percent of the flattening in the average life-cycle earnings profile for college-educated individuals and about 35 percent of that for high school-educated individuals. The model also consistently predicts the unconditional earnings profile behavior across generations.
1 Introduction

Among the notable changes that affected the labor market since the 1940s, the behavior of the life-cycle earnings profiles of successive cohorts has received little attention, with the notable exception of Kambourov and Manovskii (2009). In this paper, I document that these profiles are noticeably flatter for recent cohorts than for older cohorts, and I propose a quantitative theory to account for this movement.

Figures 1 and 2 plot the average life-cycle earnings profile for the 1940, 1950, 1960, and 1970 cohorts for college- and high school-educated white men, respectively. These two earnings profiles are also referred to as the conditional earnings profiles in the paper. One observation from these two figures is that for each successive birth cohort, the earnings profiles are getting flatter and flatter. Take for instance, from Figure 1, the average annual real earnings of 25-year old college-educated individuals in 1940 increases 3.96 times by the time they are 55 years old. However, those for 25-year old college-educated individuals in 1970 increases only 2.19 times in the same length of time. A similar trend is observed for high school-educated individuals: 3.44 times for those in the 1940 cohort and 1.28 times for those in the 1970 cohort (see Figure 2 and Table 1). This paper aims to provide a quantitative explanation for this observed phenomenon.

I construct a model of education choice and human capital accumulation building upon the work of Ben-Porath (1967), Heckman et al. (1998) and Huggett et al. (2006). All individuals have a high school diploma and decide whether or not to go to college. Individuals are heterogeneous with respect to their ability to accumulate human capital across their life cycle. There are two technologies for human capital accumulation: on-the-job and college.

1 Refer to Appendix A for the construction of life-cycle earnings profile.
2 Ability has become a standard feature of human capital models and original work by Mincer (1958), Becker (1964) and Ben-Porath (1967) link human capital investment to life cycle earnings.
Individuals who devote more time and resources (hereafter referred to as goods) and those with higher abilities will accumulate more human capital. The optimal choice of schooling implies that there is a threshold ability above which individuals choose to attend college and below which they choose to work. As with the Ben-Porath model, my model delivers that all individuals have a hump-shaped life-cycle earnings profile and that individuals with higher ability accumulate human capital faster and, hence, have higher earnings. All cohorts face the same ability distribution and an exogenous wage rate per unit of human capital that grows at a constant rate. This means that all cohorts face the same growth in wage rate but recent cohorts start with a higher initial level. As the wage rate increases, more individuals attend college, i.e. the threshold ability is lower for recent cohorts. This implies that in recent cohorts, new college-educated individuals have lower ability than the college-educated individuals of older cohort. This selection mechanism reduces the overall earnings growth for college-educated individuals in recent cohort. A similar mechanism operates for high school-educated individuals: the remaining high school-educated individuals in recent cohorts are less able than in older cohorts on average. Since growth in earnings is smaller for individuals with lower abilities, this implies that the average life-cycle earnings profile for college-educated individuals from 1970 cohort is flatter than that for the 1940 cohort. This is the same for high school-educated individuals.

I calibrate the model parameters to moments characterizing the earnings profile of the 1940 cohort not conditioning on education. Using the calibrated parameters, I conduct the following experiment. I compute optimal decisions for a sequence of cohorts, starting with 1940 and ending with the 1970 cohort. Each cohort differs from its predecessor in only one dimension: the level of wage rate per unit of human capital that it faces at the beginning of its life. I report the earnings profiles of these cohorts and their education choices. I find that the earnings profiles flatten, between the 1940 and 1970 cohorts, in the model as in the data and that educational attainment rises in line with the data as well. Quantitatively,
my model is able to explain approximately 67 percent of the flattening for college-educated individuals, 35 percent of that for high school-educated individuals and 50 percent of that for all individuals.

The paper contributes to the literature in the following ways. First, using Census data, I am able to build the earnings profiles for earlier cohorts than previously examined in the literature. As a result, I was able complement some of the previous explanations by proposing a theory that is consistent with the flattening observed before the baby boom generation enters the labor market. Early evidence on the flattening are given by Welch (1979) and Berger (1985). They document flatter earnings profile at the time when the baby boom generation is entering the labor market and suggest that earnings profiles are flatter for cohorts that have a larger size. In my analysis, the 1970 cohort would correspond to the baby boom generation. However, as documented above, the flattening of life-cycle earnings profile is observed from successive cohorts starting from the 1940 cohort, where fertility is in fact decreasing in the birth years of the 1940, 1950, and 1960 cohorts. This suggest that the flattening has little connection with the cohort’s size.

Second, this paper shows that there is important differences between cohort-based and cross-sectional profiles. Thus, it is not always useful to make comparisons based on cross-sectional data when the cohort-specific element is nontrivial. Consider the following illustration: cross-sectional comparison of earning of a 55-year old college-educated individual to a 25-year old college-educated individual is what is traditionally calculated as the experience premium. However, since recent cohorts have a lower life-cycle earnings profile than older cohorts, the

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4 Kambourov and Manovskii (2009)’s results also arrive at the same conclusion.
calculated experience premium contains a cohort-specific element that we do not want as well as the accumulated experience element that we wanted. In the situation where the cohort-specific characteristic is not inconsequential, using cross-sectional data can be at best misleading.

Third, there is a large literature on earnings heterogeneity in macroeconomics. One dimension is in life cycle earnings. As documented above, I observe a systematic flattening in the life-cycle earnings profiles of successive cohorts starting from the 1940 cohort. The question is: What has changed in the economy to account for the observed phenomenon. For Kambourov and Manovskii (2009), this is due to recent cohorts losing occupation-specific human capital as they change jobs more often than their predecessors. In my paper, the explanation of this complex phenomenon is parsimonious: It is the changing composition of the labor force, as more individuals choose to attend college, that is causing the flattening. Thus, the flattening of life-cycle earnings profile can be understood as part of the broad literature on earnings.

The rest of the paper proceeds as follows. In section 2 I introduce the model. I calibrate the model in section 3 and presents results in section 4. In section 5 I discuss two exercises. I conclude in section 6.

2 Model

2.1 Environment

The economy is populated by overlapping generations of individuals. Each individual lives for $T$ periods and is ex-ante heterogeneous in terms of ability, $a \in R^+$. Ability denotes his capacity to accumulate human capital over his life cycle and is distributed according to a
time-invariant cumulative distribution function, \( A \). I assume that ability is observable for each individual before schooling and consumption decisions are made. Ability is immutable with education.

There are two levels of education: high school and four-year college. Time is discrete and indexed by \( t = 0, 1, \ldots, \infty \). Each individual is endowed with one unit of time per period. He enters with a high school education and chooses whether or not to go to college. If he chooses not to go to college, he enters the labor market and divides his time between work and human capital accumulation. If he decides to go to college, he will first spend \( s \) periods studying full time and will then go to work.

There is a perfect credit market in which each individual can borrow and save at a constant exogenous rate, \( r - 1 \). There is also no uncertainty and preferences are defined on consumption sequences only. The wage rate per unit of human capital is given exogenously and is assumed to admit a constant growth rate, \( g \). Since there are no borrowing constraint, uncertainty and leisure, the problem of maximizing lifetime utility is equivalent to maximizing lifetime earnings.

### 2.2 Technologies

Each individual with ability \( a \) enters the model with \( H^{hs} \) level of human capital. The ability distribution completely determines the distribution of initial human capital through the following equation:

\[
H^{hs}(a) = z_h a,
\]

where \( z_h \) is a productivity parameter common to all individuals.

There are two technologies for accumulating human capital: in college and on the job. The
college human capital accumulation function for each individual with ability $a$ is given by

$$H^{col}(a, e, H^{hs}) = (ze)^\eta(aH^{hs})^{1-\eta},$$

where $e$ represents present value expenditure towards the services affecting the quality of college education and $z$ represents the productivity parameter that is common to all individuals.

Each individual with ability $a$ has the following on-the-job human capital accumulation function.

$$h' = (1-\delta)h + H(a, n, h)$$

$$H(a, n, h) = a(nh)^\phi,$$

where $\phi \in (0,1)$, $\delta$ is the depreciation rate, $n \in [0,1]$ is the fraction of time committed towards accumulation of human capital on the job, $h$ is the accumulated human capital inherited from the last period and $h'$ is the accumulated human capital in this period.

In this model, human capital accumulated from education is an input to the production of on-the-job human capital, which is indirectly productive in the labor market. Consequently, there exists a tight link between ability and the level of human capital accumulated and ultimately the lifetime earnings of an individual. Thus, the ability of an individual is a representation of the capacity to both learn and earn.

Lastly, the wage rate per unit of human capital ($w$) is exogenous and assumes to grow at a constant rate $g$:

$$w_{t+1} = gw_t.$$

Similar to the Ben-Porath model, earnings inequality between and within education levels can be generated only be differences in the level of human capital and investment behavior.
(where both are functions of heterogenous ability). This is because both high- and low-skilled individuals command the same wage rate per unit of human capital.

2.3 Individual’s problem

Each individual enters the model with a high school diploma and chooses to have college education or not. He will choose the schooling level that gives him the highest net lifetime earnings. Once he enters the labor market, he cannot return to school. After a schooling decision is made, he will solve for the optimal sequence of time investment on-the-job, \( \{n_{\tau,j}(h)\}_{j=1}^T \), to maximize lifetime earnings.

I solve the problem backwards and in two steps. Since all individuals regardless of education level will ultimately enter into employment, in step one, given an arbitrary level of human capital, I can solve the on-the-job human capital accumulation problem. Using the optimal solution from the on-the-job problem, I proceed to step two to solve the schooling choice problem by choosing the level of education that maximizes an individual’s net lifetime earnings.

2.3.1 Human capital accumulation on the job

I formulate this part in the spirit of Ben-Porath (1967) and in the language of dynamic programming. For a given arbitrary level of human capital, each individual maximizes lifetime earning by choosing, in each period, the time he wants to spend accumulating human capital on the job and thereby determining the decision rules for \( h_{\tau,j}(h) \) and \( n_{\tau,j}(h) \) and value function, \( V_{\tau,j}(h) \). The value function, \( V_{\tau,j}(h) \), gives the maximum present value of lifetime earnings of an individual from cohort \( \tau \) at age \( j \).
The problem is

\[ V_{\tau,j}(h) = w_{\tau+j-1}(1-n) + \left( \frac{1}{r} \right) V_{\tau,j+1}(h') \]

subject to

\[ h' = (1 - \delta)h + H(a, n, h) \]

\[ n \in [0, 1] \]

\[ h \text{ given,} \]

where \(w_{\tau+j-1}\) is the exogenous wage rate per unit of human capital an individual from cohort \(\tau\) receives at age \(j\).

The on-the-job human accumulation problem for a college- and high school-educated individual differ in two aspects. The first is the age that the individual enters the labor market. Each high school-educated individual enters at age \(j = 1\) and each college-educated individual enters at age \(j = s+1\). For the college-educated individual, there are earnings forgone for the \(s\) periods they spend in college education. Each high school-educated individual from cohort \(\tau\) will enter labor market at age \(j = 1\) with \(w_\tau\). Recalling that there is no differentiation of skills through prices, consequently, \(s\) periods later, both the college- and high school-educated individuals at the age \(j = s + 1\) will face the same \(w_\tau g^s\). Second, for the earnings forgone because of college education, it is compensated through a college-educated individual’s higher initial level of on-the-job human capital. A high school-educated individual enters with level \(H^{hs}\) whereas a college-educated individual enters with a higher level \(H^{col}(a, e, H^{hs})\).

Solving the dynamic programming problem by value function iteration using terminal condition \(V_{\tau,T+1} = 0\), I obtain the sequence of value function at each age \(j\), \(\{V_{\tau,j}(h)\}_{j=1}^T\). It takes
the following form:

\[ V_{\tau,j}(h) = \alpha_{\tau,j} + \beta_{\tau,j} h \]

where

\[ \alpha_{\tau,j} = -w_{\tau+1} \left( \frac{a \phi \beta_{\tau,j+1}}{rw_{\tau+1}} \right)^{\frac{1}{1-\phi}} + \left( \frac{1}{r} \right) \alpha_{\tau,j+1} + \left( \frac{1}{r} \right) \beta_{\tau,j+1} a \left( \frac{a \phi \beta_{\tau,j+1}}{rw_{\tau+1}} \right)^{\frac{2}{1-\phi}} \]

\[ \beta_{\tau,j} = w_{\tau+1} + \left( \frac{1}{r} \right) (1-\delta) \beta_{\tau,j+1}. \]

Taking the first-order condition with respect to \( n \), the optimal rule for \( n \) is

\[ n_{\tau,j}(h) = \left[ \frac{a \phi \beta_{\tau,j+1}}{rw_{\tau+1}} \right]^{\frac{1}{1-\phi}} h^{-1}. \] \hspace{1cm} (1)

For interior solution, \( n_{\tau,j} \in [0, 1] \),

\[ 0 < \left[ \frac{a \phi \beta_{\tau,j+1}}{rw_{\tau+1}} \right]^{\frac{1}{1-\phi}} h^{-1} < 1. \]

The first inequality is automatically satisfied. The second inequality tells us that the condition for interior solution is

\[ h > A_{\tau,j}(a) = \left[ \frac{a \phi \beta_{\tau,j+1}}{rw_{\tau+1}} \right]^{\frac{1}{1-\phi}}. \]
The optimal decision rules are as follows:

\[
\begin{align*}
\tau,j(h) &= \begin{cases} 
    aA_{\tau,j}(a)^{\phi} + (1 - \delta)h, & \text{for } h > A_{\tau,j}(a) \\
    ah^\phi + (1 - \delta)h, & \text{for } h < A_{\tau,j}(a)
\end{cases} \\
\end{align*}
\]

\[
\begin{align*}
\tau,j(h) &= \begin{cases} 
    A_{\tau,j}(a)h^{-1}, & \text{for } h > A_{\tau,j}(a) \\
    1, & \text{for } h < A_{\tau,j}(a)
\end{cases} \\
\end{align*}
\]

### 2.3.2 Net lifetime earnings of high school-educated individual

Since each individual from cohort \(\tau\) enters the model with high school education at age \(j = 1\), I evaluate the value function at \(V_{\tau,1}(h)\). The first wage rate that each individual from cohort \(\tau\) faces as he enter the labor market is \(w_\tau\) with \(H^{hs}(a)\) level of human capital. The maximized value of net lifetime earning is then

\[
\tilde{V}_{\tau}^{hs}(a) = V_{\tau,1}(H^{hs}(a)) = \alpha_{\tau,1} + \beta_{\tau,1}H^{hs}.
\]

### 2.3.3 Net lifetime earnings of college-educated individual

Since each college-educated individual from cohort \(\tau\) with ability \(a\) will start work at age \(j = s + 1\), I evaluate the value function is at \(V_{\tau,s+1}(h)\). He will enter the labor market with human capital level \((ae)^{\eta}(aH^{hs})^{1-\eta}\), where \(e\) is endogenously determined.

The maximization problem is

\[
\tilde{V}_{\tau}^{col}(a) = \max_{e} \left\{ \left( \frac{1}{r} \right)^{s} \alpha_{\tau,s+1} + \left( \frac{1}{r} \right)^{s} \beta_{\tau,s+1} (ae)^{\eta} (aH^{hs})^{1-\eta} - e \right\}.
\]
Deriving the first-order condition with respect to $e$ gives the optimal expenditure for a college education and the level of human capital that a college-educated individual enters the labor market with:

$$
e_{\tau}^{col} = \left[ \left( \frac{1}{r} \right)^s \beta_{\tau,s+1}z^\eta \eta \right]^{\frac{1}{\eta}} aH^{hs}$$

$$h_{\tau}^{col} = \left[ \left( \frac{1}{r} \right)^s \beta_{\tau,s+1}z^\eta \eta \right]^{\frac{1}{\eta}} aH^{hs}.$$

The optimal net lifetime earnings of each college-educated individual is

$$\tilde{V}_{\tau}^{col}(a) = \left( \frac{1}{r} \right)^s \alpha_{\tau,j+1} + \left[ \left( \frac{1}{r} \right)^s \beta_{\tau,s+1}z^\eta \eta \right]^{\frac{1}{\eta}} \kappa aH^{hs},$$

where

$$\kappa = \eta^{\frac{\eta}{1-\eta}} - \eta^{\frac{1}{1-\eta}}.$$

### 2.4 Schooling decision

Each individual compares net lifetime earnings between a college education and a high school education and decides whether or not to attend college with the following decision rules:

$$\tilde{V}_{\tau}^{hs}(a) < \tilde{V}_{\tau}^{col}(a) \quad \text{ choose college}$$

$$\tilde{V}_{\tau}^{hs}(a) > \tilde{V}_{\tau}^{col}(a) \quad \text{ no college}.$$
In particular, the unique cohort-specific threshold $a^*_r$ is given by

$$\tilde{V}_{hs}^r(a) = \tilde{V}_{col}^r(a)$$

$$\alpha_{r,1} + \beta_{r,1}H^{hs} = \left(\frac{1}{r}\right)^s \alpha_{r,j+1} + \left[\left(\frac{1}{r}\right)^sz^n\beta_{r,s+1}\right]^{\frac{1}{1-\eta}} \kappa aH^{hs}.$$

### 2.5 Dynamics

The model is driven by an exogenous wage rate per unit of human capital. This section discusses the changes in net lifetime earnings when wage rate per unit of human capital changes. $\alpha_{r,j}$ and $\beta_{r,j}$ can be re-expressed as

$$\alpha_{r,j} = aw_r \sum_{t=j}^{T} \left(\frac{1}{r}\right)^{t-j} C \left(\frac{\zeta^{T-t} - 1}{\zeta - 1}\right) \left(\frac{1}{r}\right)^{t-j}$$

$$\beta_{r,j} = w_\tau g^{j-1} \left(\frac{\zeta^{T-j+1} - 1}{\zeta - 1}\right),$$

where

$$C = \left(\frac{g}{r}\right)^{1-\varphi} \left(\phi^{1-\varphi} - \phi^{1-\varphi}\right)$$

$$\zeta = \left(\frac{1}{r}\right) (1 - \delta) g.$$

Taking partial derivative with respect to $w_\tau$, it is fairly clear that both $\frac{\partial \tilde{V}_{hs}^r(a)}{\partial w_\tau} > 0$ and $\frac{\partial \tilde{V}_{col}^r(a)}{\partial w_\tau} > 0$.  

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3 Baseline Calibration

3.1 Strategy

In this section, I discuss my calibration strategy, which includes two steps. First, I assign some parameters values using prior information. Then, the remaining parameters are obtained by calibrating the model to earnings of the 1940 cohort. Recall that the aim of this paper is to explain the flattening of the two conditional life cycle earnings profiles.

In the baseline calibration exercise, I calibrate the model parameters to the unconditional earnings of the 1940 cohort\(^5\). In particular, the targets are (i) inverse of the coefficient of variation and (ii) the fraction of earnings at selected ages to the sum of mean earnings at ages 25, 35 45, and 55.

Using the model, the two conditional earnings profiles for the 1940 cohort are generated endogenously. So, the first objective is to test whether the model is able to generate the split between high school- and college-educated individuals correctly without using any information specific to each education group. Getting the conditional earnings profiles correct for the 1940 cohort will provide confidence to this calibration exercise. The model is then assessed based on how much flattening can be explained by the mechanism.

3.2 Details

One model period represents one calendar year. Each individual lives 42 model periods, \((T = 42)\), and enters the model at age 18 and exits at age 59. Retirement decision is not modeled in this paper. Upon entering the model, each individual chooses whether to enter

\(^5\)Appendix\([\text{C}]\) discusses how the model generates average unconditional earnings.
college or not. Those who do will spend four years in college \((s = 4)\) whereas each high school-educated individual will start employment immediately. The gross interest rate is \(r = 1.05\). The depreciation rate, \((\delta)\), taken from Huggett et al. (2006), is 0.0114.

The list of the remaining eight parameters is

\[
\theta = \{z, z_h, w_1, g, \phi, \eta, \mu, \sigma\},
\]

which consists of college human capital accumulation function productivity parameter \((z)\), productivity parameter from high school human capital accumulation function \((z_h)\), the initial wage rate per unit of human capital for the first model cohort \((w_\tau, \text{where } \tau = 1)\), growth rate \((g)\) in \(w_\tau\), on-the-job human capital accumulation function parameter \((\phi)\), college human capital production function parameter \((\eta)\), mean \((\mu)\) and standard deviation \((\sigma)\) of lognormal ability distribution. These eight parameters are going to be calibrated through the solving of nonlinear equations to minimize the distance between the selected data moments and their corresponding model-generated moments. I will discuss explicitly how this is done in later paragraphs. Refer to Table 3 for a quick summary.

Allowing the wage rate per unit of human capital to grow at a constant rate is somewhat of an extreme way of modeling. The observed wage rate in the data is equal to the wage rate per unit of human capital \((w)\) multiplied by the level of human capital; however, neither of is observed empirically. I let \(w\) grow at a constant rate \(g\) to impose discipline on the sequence of \(w\), where \(g\) is in turn disciplined by the data through calibration. This will allow the story to be illustrated by the changes in the human capital. This approach is not uncommon in human capital literature. Depending on the question of interest, authors either use this approach or an alternative approach, where the level of human capital is kept constant to allow changes in \(w\) to capture the story. The latter approach is usually used for skill-biased
technological change when the price of human capital, \( w \), is the focus.

Production function parameters such as \( \phi \) and \( \eta \) govern the shape and the increase in the earnings profile. Proper parameterization ensures that the earnings profile behave regularly. The parameters \( \phi \) and \( \eta \) in theory are between zero and one. However, the Ben-Porath model generates earnings to infinity very easily, meaning that the model is extremely sensitive to changes in these two parameters. Consequently, the combination of values these two parameters can take in reality is much smaller. For example, the parameter \( \phi \) is surveyed by Browning et al. (1999) to take values between 0.5 to almost 1. Under this range of values, the parameter \( \eta \) cannot exceed 0.55 or earnings can go to infinity. The productivity parameter \( z \) has almost the same effect as \( \phi \) and \( \eta \) except that it affects the college earnings profile only. It, however, has a nontrivial role in making sure that when plotted across different levels of ability, lifetime earnings of college-educated individuals cuts high school lifetime earnings from below. This guarantees that lifetime earnings of high-school individuals are higher than that of college-educated individuals when ability is low and vice versa when ability is high. Productivity parameter \( z_h \) affects the initial level of human capital coming from high school education. Therefore it has a level effect on earnings. A high \( z_h \) limits the growth of earnings simply because earnings start from a higher level. This parameter is useful in controlling the extent of growth in the earnings. The parameter \( w_\tau \) has the same function. The parameter \( z_h \) also has a direct significance in ensuring that there are no corner solutions in the choice of \( n_{\tau,j}(h) \). I want to avoid corner solutions because when \( n_{\tau,j} = 1 \), the individual is accumulating human capital full time. This corresponds to a semblance of schooling that is not quite defined in the context of the model. The growth rate, \( g \), determines the extent of a leftward shift in lifetime earnings when plotted across different levels of ability. A higher \( g \) induces a greater shift. Together \( \eta, \phi, z, z_h, w_1 \) and \( g \) determine the sequence of the cohort-specific ability thresholds.
The parameters $\mu$ and $\sigma$ characterize the lognormal ability distribution. These parameters govern the range and frequency of ability levels that enter the earning functions but do not alter ability thresholds. The separate determination of thresholds and distribution creates the need to properly specify the location parameter to guarantee a sensible fit between the two. This separation makes matching targets singularly difficult. In particular, I cannot be sure of the direction of change in the simulated ratios when I change these parameters. Below is an illustration of this problem.

For example, a higher $z$ indicates a higher lifetime earnings for any college-educated individual. However, a higher $z$ can result in lower average lifetime earnings for college-educated individuals for this reason: From the determination of threshold ability, a higher $z$ results in a faster increase in lifetime earnings when plotted against levels of ability. This means that, given no changes in the lifetime earnings of high-school individuals, the threshold is lower for a higher $z$ compared to a lower one. So, although individually, higher $z$ results in higher earnings, collectively, average earnings can be lower through lower conditional average ability. Since ability enters the model in a nontrivial manner, the effect of an increase in $z$ on the average earnings profile is unknown.

Using prior information, I calibrate the eight remaining parameters. The chosen targets are as follows:

1. Inverse of the coefficient of variation at age 35, \( \left( \frac{\mu_{35}}{\sigma_{35}} \right) \)

2. Inverse of the coefficient of variation at age 45, \( \left( \frac{\mu_{45}}{\sigma_{45}} \right) \)

3. Inverse of the coefficient of variation at age 55, \( \left( \frac{\mu_{55}}{\sigma_{55}} \right) \)

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6It is worth noting that even though parameters $\eta$ and $z$ are college specific, they can indirectly affect the earnings of high school-educated individuals through the support of the ability distribution. This is because as the threshold ability changes, the parameters $\mu$ and $\sigma$ will need to accommodate this change.

7$\mu_{35}$ is the normalized mean earnings at age 35.

This is mathematically equal to $\frac{E_{35}}{E_{25}}$, where $E_j$ is the mean earnings of all individuals at age $j$. 

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4. Fraction of average earnings at age 25 to sum of mean earnings at ages 25, 35, and 45, 
   \( \left( \Omega_{25,35,45}^{25} \right) \)

5. Fraction of average earnings at age 35 to sum of average earnings at ages 35, 45, and 55, \( \left( \Omega_{35,45,55}^{35} \right) \)

6. Fraction of average earnings at age 45 to sum of average earnings at ages 25, 45, and 55, \( \left( \Omega_{25,45,55}^{45} \right) \)

7. Fraction of average earnings at age 55 to sum of average earnings at ages 25, 35, and 55, \( \left( \Omega_{25,35,55}^{25} \right) \)

8. The 90/10 ratio at age 25

Since model units are different from data units, the chosen targets are all unit free. The aim is to minimize the distance between ratios produced by the Census data and the model-simulated data. Therefore, a measure of distance is built using both the simulated data and the Census data. Below is a system of nonlinear equations in eight unknowns. For a given wage rate per unit of human capital sequence, the parameters are calibrated such that the ratio between observed data and their model counterpart is close to 1: \( F(\theta) = 1. \)

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\(^8\)Mathematically this is \( \frac{E_{25}}{E_{25} + E_{35} + E_{45}} \), where \( E_j \) is the average earnings of all individuals at age \( j \).
\[ F(\theta) = \begin{cases} 
2.914 / \frac{\mu_{35}}{\sigma_{35}} \\
2.927 / \frac{\mu_{45}}{\sigma_{45}} \\
2.824 / \frac{\mu_{55}}{\sigma_{55}} \\
0.166 / \Omega_{25,35,45}^{25} \\
0.231 / \Omega_{35,45,55}^{35} \\
0.401 / \Omega_{25,45,55}^{45} \\
0.544 / \Omega_{25,35,55}^{55} \\
5.357 / \frac{E_{90}^{25}}{E_{25}^{25}} 
\end{cases} \]

The results of the calibration and the corresponding calibrated parameters are presented in Tables 3 and 4. Once calibrated parameters are obtained, they are substituted into the model to simulate earnings data.

4 Results

This section discusses the results from the baseline calibration. Recall that the main objective is to explain the flattening of the conditional life-cycle earnings profile. The model is calibrated to unconditional earnings and conditional earnings profiles are generated endogenously. In the following paragraphs, I discuss the main results.
4.1 Unconditional flattening of life-cycle earnings profile

Figure 8 compares the life-cycle earnings profile of all individuals from the model-simulated data (hereafter, the model) and the Census data (hereafter, the data). The dotted lines represent the data and the solid lines represent the model. The magenta and blue lines represent the 1940 and 1970 cohorts, respectively. For the 1940 cohort, the model tracks data very well. The precise magnitudes are in Table 5 in the “ALL” column. The model is able to explain half of the flattening in the unconditional life-cycle earnings profile. At first glance, the model performs very well. The model’s unconditional earnings profile for the 1940 cohort is very close to the data and the mechanism generates a decent amount of flattening. However, what is the significance of the flattening in the unconditional earnings profile generated by the model?

The significance of the flattening in the unconditional earnings profile generated by the model is as follows. It is difficult to verify a decrease in the average abilities of college- and high school-educated individuals since data on innate ability is not available. However, we can indirectly verify the decrease in ability using theoretical implication of the model. If the model is generating flatter earnings profiles for both college- and high school-educated individuals in the recent cohorts, the composition effect implies that the unconditional earnings profiles for recent cohorts should also be flatter. In this case, the model predicts flatter unconditional earning profile across cohorts. This is consistent with the behavior of the unconditional earnings profile from data.

9Note that from Census data, I calculate the average annual real earnings for all individuals and not annual real earnings of the average individual. This two items are different in the model as ability enters in a non-linear manner and cannot be factored out. Therefore, even though, in the model, the average individual stays the same across cohorts, average annual real earnings is changing due to the increase in college attainment over cohorts.
4.2 Flattening of the conditional life-cycle earnings profiles

Figures 9 and 10 show the earnings profile for college- and high school-educated individuals, respectively. The solid lines represent the model and the dotted lines represent the data. The magenta and blue represent the 1940 and 1970 cohorts, respectively. Remember that part of the exercise is to test if the model is able to correctly generate the split between college and high school-educated individuals. At first glance, the model is doing reasonably well with one exception.

For the data, life-cycle earnings profile for college-educated individuals for the 1940 cohort has a strong increase from ages 35 to 45; giving the profile a somewhat convex shape before dipping at age 55. The model is not able to reproduce this result because of model limitation. The model is exogenously driven by a sequence of wage rate per unit of human capital that grows at a constant rate. Therefore, the shape of the earnings profile is crucially dictated by changes in the human capital investment decision: Individuals choose to accumulate more human capital when young as they have the rest of their life to recoup the investment. Following the same logic, as they age, they choose to accumulate less and less human capital. Therefore, the corresponding earnings profile is such that earnings growth is the fastest when an individual is young; as the individual ages, his earnings increases at a decreasing rate. Thus, the model is not able to generate an earnings profile where the increase in earnings between ages 35-45 is faster than that between ages 25-35.

Both simulated life-cycle earnings profiles are consistently concave. This is one of the artifacts of the Ben-Porath model. This shows that the depreciation rate (\( \delta \)) of 0.0114 is able to produce a decrease in average earnings at the end of the working life cycle as documented in Figure 1 from Huggett et al. (2006).
4.2.1 Performance of the mechanism

In this paragraph, I discuss the performance of the mechanism. First, the model is able to produce flattening of the life-cycle earnings profiles for both college- and high school-educated individuals but of different magnitudes. The model is able to account for a large proportion of flattening in the life-cycle earnings profile for college-educated individuals (see Figure 9), but only one-third of that for high school-educated individuals (see Figure 10). The first two rows of Table 5 report for the data and the model life-cycle earnings profile at ages 25 and 55 for high school-educated, college-educated and all individuals. The first line provides the results for the 1940 cohort and the second for the 1970 cohort.

The third line provides a measure of the extent of the flattening, which is useful for determining which education group has the most and least flattening. Based on the data, the average annual earnings of high school-educated individuals in the 1940 cohort increases 3.439 times by the time he is 55 years old; that for the 1970 cohort is 1.284 over the same length of time. The number 2.679 is obtained by dividing 3.439 by 1.284. I construct this for the rest of the columns. For the data (see columns three to five), the greatest flattening occurs in the life-cycle earnings profile for high school-educated individuals, the result for all individuals falls between the other two. For the model, however (see columns six to eight), the greatest flattening occurs in the life-cycle earnings profile for college-educated individuals, with, the result for all individuals again falling between the other two. Accordingly, the model cannot reproduce the fact that the greatest flattening is observed for high school-educated individuals; however through composition effect, flattening for all individuals is consistently between that for the high school- and college-educated individuals. In section 4.4, I discuss why the model generates more flattening for college-educated individuals than high school-educated individuals.
Table 6 notes the proportion of the data explained by the model. Columns two and three calculate for the data and the model, respectively, the change in the lifetime increment of 1970 cohort from 1940 cohort. The change is consistently smaller for the model than the data, which means that the model is not able to explain fully the flattening observed in the data. Taking the change produced by the model (0.219) divided by the change observed in data (0.626) for high school-educated individuals, I arrive at 0.349. Therefore, the model is able to explain 35 percent of the flattening observed in the data for high school-educated individuals; 67 percent of that for college-educated individuals and 50 percent for all individuals.

4.3 College attainment

Figure 7 shows the college attainment generated by the model (red line) and that in the data for 35 year olds (dotted black line).\textsuperscript{10} I choose 35 year olds because at this age, educational attainment would be stabilized. Recall that the model has not been calibrated to match college attainment for 1940 cohort, and yet, since this is a story of changing composition, it is critical that the model gets the right number of people into college for exercise to have validity.

Based on Figure 7, it appears that the model is doing well. The model is able to generate increasing college attainment over time. It slightly under-predicts the data for the 1940 cohort and over-predicts for the 1970 cohort; however, on average, the model is able to correctly predict college attainment. In Section 3, I mentioned that the model generates infinite earnings easily, which means the model is extremely sensitive to small changes in parameters. Because of this sensitivity, the model easily overselects the number of people who go to college; making it difficult to correctly predict college attainment.

\textsuperscript{10}I use the 30 to 39-year-old group from the Census data to allow for more data points.
4.4 Why the model induces greater flattening in the college life-cycle earnings profile?

The model is not able to replicate that there is greater flattening in the life-cycle earnings profile for high school-educated individuals than college-educated individuals. In the following paragraphs, I discuss why this is so.

From the model, the two cohorts are differ in two aspects: (i) their initial wage rate per unit of human capital (\(w_r\)), which is higher for recent cohort, and (ii) their conditional average ability, which is lower for the recent cohort, where (ii) is caused by the changing composition of the labor force induced by increasing college attainment. Here, I remove the composition effect by keeping ability levels constant across cohorts.

In Figures 11, I plot the life-cycle earnings profile for an individual with ability level, \(a\), such that he will have only high school education in both 1940 and 1970 cohort. I do the same for a college-educated individual in Figure 12. The ability level chosen for the high-school and college-educated individual is \(a = 0.02\) and \(a = 0.04\), respectively. The magenta color represent the 1940 cohort and blue represents the 1970 cohort.

From Figure 11, the life-cycle earnings profile does not change whether he is from the 1940 or the 1970 cohort. Examination of Equation 1 shows why this is so. Equation 1 reduces to Equation 2 after some algebra. For the high school-educated individual’s optimal time spent in human capital accumulation on the job, \(n\), to be cohort specific, it can only come from the level of initial human capital he has when he enters the labor market. However, since \(H^{hs} = z_ha\), the sequence of optimal time spent in human capital accumulation on the job for a high school-educated individual is only age dependent but not cohort specific. Since \(n\) is only age dependent, the optimal sequence of accumulated human capital on the job will also
not be cohort specific. The only cohort specific element in Equation \(3\) \(w_{\tau+j-1}\) factors out after normalization. Consequently, the life-cycle earnings profile for a high school-educated individual with ability level \(a = 0.02\) does not vary across cohort:

\[
\begin{align*}
n_{\tau,j}(h) &= \left[ \frac{a \phi \beta_{\tau,j+1}}{r w_{\tau+j-1}} \right]^{\frac{1}{1-\phi}} h^{-1} \\
&= \left( \frac{a \phi g}{r} \right)^{\frac{1}{1-\phi}} \left( \frac{\zeta^{T-j} - 1}{\zeta - 1} \right)^{\frac{1}{1-\phi}} h^{-1},
\end{align*}
\]

where

\[
\zeta = \left( \frac{1}{r} \right) (1 - \delta) g.
\]

\[
Earnings^{hs}_{\tau,j} = w_{\tau+j-1} \cdot h_j \cdot (1 - n_j)
\]

In contrast, the college-educated individual enters the labor market with a human capital level of \(H^{col}_{\tau} (a, e^{col}_{\tau}, H^{hs})\) where the optimum expenditure on college education, \(e^{col}_{\tau}\), is cohort specific. Therefore, applying the same argument as before (see Equation \(4\)), life-cycle earnings profile for a college-educated individual with ability level \(a = 0.04\) flattens across cohorts. (Figure 12)

\[
Earnings^{col}_{\tau,j} = w_{\tau+j-1} \cdot h_{\tau,j} \cdot (1 - n_{\tau,j})
\]

Hence, the model not only has a composition effect, it also has a level effect from \(w_{\tau}\). The level effect is explained as follows. Individuals will be willing to spend more on college education, \((e^{col}_{\tau})\), because they know they will face a higher earnings in the future. The connection comes from the fact that human capital accumulated in college is increasing in \(e^{col}_{\tau}\). As the flattening of high school life-cycle earnings profile in the model is only affect by
composition effect and college life-cycle earnings profile is affected by both the composition and level effects. Because of this difference, the model generates a more flattening in the life-cycle earnings profile of college-educated individuals than that of high school-educated individuals.

5 Discussion

In this section, I discuss two other cases apart from the baseline case. In Case 1, I calibrate the model to the \textit{conditional} life-cycle earnings profiles of the 1940 cohort. In Case 2, I allow the growth rate of the wage rate per unit of human capital ($g$) to be differentiated by education, i.e., $g^{hs}$ and $g^{col}$.

5.1 Case 1: Calibrating to the conditional life-cycle earnings profile

In this exercise, I calibrate the model to the conditional life-cycle earnings profiles for the 1940 cohort and examine the amount of flattening produced. I am also interested in how well the model tracks data in the unconditional life-cycle earnings profile of the 1940 cohort and how much flattening is produced. This exercise reverses the experiment discussed in the baseline calibration. In a way, this is a simpler exercise because instead of relying on the model to endogenously produce the split between education groups, this exercise directly fixes the model to the conditional earnings profiles of the 1940 cohort and evaluates extend of the flattening produced by the mechanism. This exercise also serves to check the baseline calibration. The question I am interested in answering here is whether having a different calibration strategy for the model produces significantly different results.
Table 7 reports the values of the calibrated parameters. Since the targets in Case 1 are different from those in the baseline calibration, it is unsurprising that values of the parameters are different. However, the values are not vastly dissimilar to the ones in Table 3.

5.1.1 Flattening of the conditional life-cycle earnings profiles

Figure 13 shows for college-educated individuals and Figure 14 for the high school-educated individuals the life-cycle earnings profile produced by model (solid lines) and data (dotted lines). Table 9 reports the values from these figures. The magenta and blue lines represent the 1940 and 1970 cohorts, respectively. Remember that this exercise calibrates the model to the conditional earnings profiles of the 1940 cohort then lets the mechanism run to produce the average earnings profile for the subsequent cohorts. Therefore, it is not surprising that the simulated life-cycle earnings profiles for the 1940 cohort closely match those produced by the data. Again, however, the simulated profile is not able to match the increase in average college earnings at age 35 very well. Otherwise, the model is pretty good at matching all of the other points on the life-cycle earnings profile.

Table 10 reports the proportion of the data explained by the model. It explains 36 percent of the flattening observed in the data for high school-educated individuals and 67 percent of that for college-educated individuals. The numbers are 35 percent and 67 percent, respectively in the baseline.

5.1.2 Unconditional flattening of life-cycle earnings profile

Figure 5 shows the results for the unconditional life-cycle earnings profile. For the 1940 cohort, the model tracks the data very well. (See the “ALL” column in Table 9.) Recall
that I did not target any unconditional moments in this calibration. Given this fact, the model performs extremely well and gives confidence in the validity of this calibration. The model is able to explain 47 percent (50 percent in the baseline) of the flattening in the unconditional life-cycle earnings profile. This number again lies between those for college- and high school-educated individuals.

5.1.3 College attainment

Figure 16 shows college attainment generated by the model (solid red line) compared with the data (dotted black line). Recall that the model is not calibrated to the fraction of individuals with college degree for the 1940 cohort. Thus, the model predicts the right proportion of college-educated individuals in the 1940 cohort extremely well. The mechanism also reasonably predicts to the proportion of college-educated individuals for the 1970 cohort: 0.32 in the data and 0.361 (0.364 in baseline) in the model.

Overall, the results for the baseline and Case 1 is not significantly different. This is good because even though the choice of experiment taxes the model to different extents, the model does not produce totally different results which, would raise question about the validity of the baseline calibration if it did.

5.2 Case 2: Allowing for $g$ to be differentiated by education

In Case 2, I explore the possibility that human capital produced by high school- and college-educated individuals are non-substitutable. To achieve this, I let the growth rate of wage rate per unit of human capital for high school- and college-educated individuals to grow at $g^{hs}$ and $g^{col}$, respectively.
Since Case 2 is really baseline case with one additional parameter, I employ the calibration strategy of the baseline in this exercise. In addition, I use the proportion of college-educated individuals at age 35 in the 1940 cohort to pin down the prices of human capital.

The results of this exercise are shown in Figures 17, 18, 19, and 20 and reported in Tables 11, 12, 13, and 14.

The results of Case 2 do not warrant extensive discussion here because allowing for different growth rates introduces little change in the values of the parameters. Also, the flattening observed in both the conditional and the unconditional life-cycle earnings profiles change only slightly. However, the take away point is this: Calibrating to earnings of the 1940 cohort tells us that calibrated value for $g^{hs}$ is lower than $g^{col}$. I am interested to know if the difference is economically significant. I took an average of $g^{hs}$ and $g^{col}$ and rerun the model. Figures 21, 22, 23, and 24 show the earnings profiles plotted with the average growth rate. The earnings profiles do not change drastically. The proportion of data that is explained by the model is 45 percent, 31 percent and 62 percent for all, high school- and college-educated individuals, respectively. (For Case 2, the numbers are 47 percent, 35 percent and 64 percent.) The only difference is that college attainment is consistently underpredicted for almost all cohorts. Overall, I think this does not give support to the empirical observation by Goldin and Katz (2008) that skill biased technical change (SBTC) starts as early as 1940.

6 Conclusion

I begin the paper by documenting the flattening of the life-cycle earnings profile across cohorts. Using empirical observations: increase in average real annual earnings over time and increasing education attainment, I build a mechanism to explain the observed flattening
of earnings profile. The model is able to replicate the trend observed in the data as well as produce the correct movements for conditional average abilities. In terms of the flattening of the life-cycle earnings profile, the model also performs well. In the baseline exercise, it explains 67 percent of the flattening for college-educated individuals, around 35 percent of that for high school-educated individuals and half of the flattening for all individuals. The model is also able to deliver the concavity in the life-cycle earnings profile for both education groups and consistently predicts the correct proportion of college-educated individuals. The baseline experiment is: I calibrate the model to the unconditional life-cycle earnings profile for the 1940 and leave the model to determine endogenously the conditional life-cycle earnings profile. I find that the model-simulated conditional life-cycle earnings profile is very close to that of the data for the 1940 cohort. This result gives confidence in the validity of the baseline calibration exercise. It is, however, a model artifact that more flattening is observed in the simulate profile for college-educated individuals than high school-educated individuals.

In Case 1, I take a the direct route and calibrate the model to the conditional life-cycle earnings profiles. In Case 2, I differentiate the growth rate per unit of human capital based on education. Both calibration exercises produce a decent about of flattening, although, not significantly different from the baseline case. The take away point in Case 2 is: Although calibration results give a higher price of human capital for college-educated individuals, this difference is not significant enough to cause major changes in the earnings profiles. Therefore, this does not give support to the empirical observation by [Goldin and Katz (2008)] that skill biased technical change starts as early as 1940s.

Overall, I would argue that this Ben-Porath type model has performed well in many aspects, albeit some its parsimonious outlook.
Appendices

A Construction of life-cycle earnings profile from Census data

The data comes from the IPUMS-USA. In particular, I use Census data from 1940 to 2000. I look at only employed white males with the following education levels: (i) high school diploma, (ii) four-year college degree, and (iii) all individuals i.e. individuals with high school diploma and individuals with four-year college degree. The earnings profiles from (i) and (ii) are also referred to as the conditional earnings profiles, while that from (iii) is known as the unconditional earnings profile.\footnote{I select only employed white males because labor hours for white male individuals do not change considerably on the intensive margin, focusing on them reduces unnecessary complications in the data. In particular, I do not want the analysis to be convoluted by factors such as increase in labor hours on the extensive margin due to increases in the female labor-force-participation rate.}

I construct life-cycle earnings profiles for synthetic cohorts according to education level. I calculate the average annual real earning, in 2000 dollars, of college-educated individuals who are 25 years old in 1940, 1950, 1960 and 1970. I denote them as the 1940, 1950, 1960, and 1970 cohorts, respectively.\footnote{From the Census data, I group 20-29, 30-39 year old etc. I use these groups to allow for more data points, but refer to the groups as 25 year olds, 35 year olds, etc.} I recalculate the average earnings of each cohort every 10 years over 30 years to create a sequence of average earnings for college-educated individuals across the life cycle (see Figure 3). I repeat the same exercise for high school-educated individuals (see Figure 4) and all individuals (see Figure 6). Figures 1, 2, and 5 are obtained by normalizing each sequence by its first observation. A slower rise in average earnings is shown by a flatter slope and vice versa. That is, the flattening of life cycle earnings over cohorts indicates that earlier cohorts have greater increments in their life-cycle earnings profile than
later cohorts.

B Data

The main source of data is US census data from the IPUMS-USA. I used 1 percent sample for all years 1940 to 2000 except for 1970, for which I used 1970 Form 1 State sample.

The income variable is INCWAGE. It reports each respondent’s total pre-tax annual wage and salary income. INCWAGE includes wages, salaries, commissions, cash, bonuses, tips and other monetary income from an employer. Payments-in-kind or reimbursements for business expenses are not included. Since INCWAGE is expressed in nominal terms, it needs to be adjusted for comparisons over time.

The education variable is EDUCD (detailed version). EDUCD denotes a respondent’s highest educational attainment. This is denoted either as the highest year of school completed or highest degree earned. Classifications have evolved over time. For comparability, for 1940-1980, all respondents are classified according to the highest year of school completed. From 1990 onwards, respondents who have completed high school are classified according to highest degree earned and high school drop outs are classified according to highest year of school completed. A college degree is differentiated by assigning each degree the number of years it typically takes to achieve the degree: 2 years of college for associate’s degrees; 4 years of college for bachelor’s degrees; and 5+ years of college for graduate and professional degrees.

Since I am looking at earnings of employed white males, I use RACED (=100), SEX (= 1), EMPSTATD (= 10) and CLASSWRKD (≥ 20, < 29) to filter out the subsample needed. I use AGE to create the age intervals 20-29, 30-39, 40-49 and 50-59 to obtain the four data points for each cohort.
For occupation, I use OCC1950. Classification is according to the 1950 Census Bureau occupational classification system for occupational data, which uses a three-digit code to sort respondents. 1940 and pre-1940 data are reclassified to the 1950 classification system for comparability. I group respondents into occupation groups with broad categories 000, 100, 200, 300, 400, 500, 600, 700, 810 and 910.

C The unconditional earnings profile constructed in the model

The unconditional average earnings is calculated from

\[ E_{\tau,j} = p_{\tau} E_{\tau,j}^{col} + (1 - p_{\tau}) E_{\tau,j}^{hs}. \]

where \( E_{\tau,j} \), \( E_{\tau,j}^{col} \) and \( E_{\tau,j}^{hs} \) is the average annual earnings for all, college- and high school-educated individuals for cohort \( \tau \) and age \( j \), respectively. And \( p \) is the proportion of individuals with college degree from cohort \( \tau \). The earnings profile for each cohort is then calculated by normalizing by earnings at age 25 (\( E_{\tau,25} \)).
7 Figures and tables

Figure 1: Life-cycle earnings profiles of college-educated workers by cohort and normalized to 1 at age 25

Source: Census data

Figure 2: Life-cycle earnings profiles of high school-educated workers by cohort and normalized to 1 at age 25

Source: Census data
Figure 3: Average annual real earnings for college-educated workers by cohort

Source: Census data

Figure 4: Average annual real earnings for high school-educated workers by cohort

Source: Census data
Figure 5: Life-cycle earnings profiles of high-school and college-educated workers by cohort and normalized to 1 at age 25

Source: Census data

Figure 6: Average annual real earnings for high-school and college-educated workers by cohort

Source: Census data
Figure 7: Proportion of college-educated workers by cohort at age 35 – Model vs. Data (Baseline)

Source: Census data and author

Figure 8: Life-cycle earnings profiles of high school- and college-educated workers by cohort – Model vs. Data (Baseline)

Source: Author and Census data
Figure 9: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Baseline)

Source: Author and Census data

Figure 10: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Baseline)

Source: Author and Census data
Figure 11: Life-cycle earnings profiles of a high school-educated worker with ability=0.02 in the 1940 and 1970 cohorts (Baseline)

Source: Author

Figure 12: Life-cycle earnings profiles of a college-educated worker with ability=0.04 in the 1940 and 1970 cohorts (Baseline)

Source: Author
Figure 13: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 1)

Source: Author and Census data

Figure 14: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 1)

Source: Author and Census data
Figure 15: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 1)

Source: Author and Census data

Figure 16: Proportion of college-educated workers by cohort at age 35 – Model vs. Data (Case 1)

Source: Census data and author
Figure 17: Proportion of college-educated workers by cohort at age 35 – Model vs. Data (Case 2)

Source: Census data and author

Figure 18: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 2)

Source: Author and Census data
Figure 19: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 2)

Source: Author and Census data

Figure 20: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 2)

Source: Author and Census data
Figure 21: Proportion of college-educated workers by cohort at age 35 – Model vs. Data (Case 2, average ‘g’)

Source: Census data and author

Figure 22: Life-cycle earnings profiles of high-school and college-educated workers by cohort – Model vs. Data (Case 2, average ‘g’)

Source: Author and Census data
Figure 23: Life-cycle earnings profiles of college-educated workers by cohort – Model vs. Data (Case 2, average ‘g’)

Source: Author and Census data

Figure 24: Life-cycle earnings profiles of high school-educated workers by cohort – Model vs. Data (Case 2, average ‘g’)

Source: Author and Census data
Figure 25: Historical average of SAT scores

Source: College Board; National Center for Education Statistics
Table 1: Earnings profiles according to education

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<tr>
<td>1940</td>
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Source: Census data

Table 2: Present value of sum of average earnings at ages 25, 35, 45, and 55 for college and high school

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Table 3: Calibrated parameters (Baseline)

Highlighted parameters are calibrated

Source: Author

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Table 4: Calibration (Baseline)

Source: Census data and author
### Table 5: Earnings profiles – Model vs. Data (Baseline)

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</tr>
<tr>
<td><strong>Cohorts</strong></td>
<td><strong>HS</strong></td>
</tr>
<tr>
<td>1940</td>
<td>1.000</td>
</tr>
<tr>
<td>1970</td>
<td>1.000</td>
</tr>
<tr>
<td>1970</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Source: Census data and author

### Table 6: Proportion of data explained (Baseline)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school</td>
<td>0.626</td>
<td>0.219</td>
</tr>
<tr>
<td>College</td>
<td>0.426</td>
<td>0.283</td>
</tr>
<tr>
<td>All</td>
<td>0.521</td>
<td>0.258</td>
</tr>
</tbody>
</table>

Source: Census data and author

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>number of periods</td>
<td>42</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
<td>1.05</td>
</tr>
<tr>
<td>$s$</td>
<td>time spent in college</td>
<td>4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.0114</td>
</tr>
<tr>
<td>$\phi$</td>
<td>on-the-job human capital accumulation function parameter</td>
<td>0.6424</td>
</tr>
<tr>
<td>$\eta$</td>
<td>college human capital accumulation function parameter</td>
<td>0.4944</td>
</tr>
<tr>
<td>$z$</td>
<td>college human capital accumulation function productivity parameter</td>
<td>3.3665</td>
</tr>
<tr>
<td>$z_h$</td>
<td>productivity parameter common to all workers</td>
<td>1.0175</td>
</tr>
<tr>
<td>$w_1$</td>
<td>initial wage rate per unit of human capital for first model cohort</td>
<td>3.9184</td>
</tr>
<tr>
<td>$g$</td>
<td>growth rate of $w_r$</td>
<td>1.0063</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mean of lognormal ability distribution</td>
<td>-3.7718</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation of lognormal ability distribution</td>
<td>0.3992</td>
</tr>
</tbody>
</table>

Table 7: Calibrated parameters (Case 1)
highlighted parameters are calibrated

Source: Author

<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
<th>Data Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_h^{25}$</td>
<td>0.164</td>
<td>0.169</td>
<td>1.032</td>
</tr>
<tr>
<td>$\Omega_h^{35}$</td>
<td>0.245</td>
<td>0.234</td>
<td>0.955</td>
</tr>
<tr>
<td>$\Omega_h^{45}$</td>
<td>0.405</td>
<td>0.401</td>
<td>0.990</td>
</tr>
<tr>
<td>$\Omega_h^{55}$</td>
<td>0.526</td>
<td>0.538</td>
<td>1.022</td>
</tr>
<tr>
<td>$\Omega_c^{25}$</td>
<td>0.154</td>
<td>0.166</td>
<td>1.077</td>
</tr>
<tr>
<td>$\Omega_c^{35}$</td>
<td>0.242</td>
<td>0.199</td>
<td>0.821</td>
</tr>
<tr>
<td>$\Omega_c^{45}$</td>
<td>0.408</td>
<td>0.396</td>
<td>0.971</td>
</tr>
<tr>
<td>$\Omega_c^{55}$</td>
<td>0.536</td>
<td>0.587</td>
<td>1.094</td>
</tr>
</tbody>
</table>

Table 8: Calibration (Case 1)

Source: Census data and author
<table>
<thead>
<tr>
<th>Cohorts</th>
<th>Age 25</th>
<th>Age 55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>COL</td>
</tr>
<tr>
<td>1940</td>
<td>1</td>
<td>3.439</td>
</tr>
<tr>
<td>1970</td>
<td>1</td>
<td>1.284</td>
</tr>
<tr>
<td>1940-1970</td>
<td>1</td>
<td>2.679</td>
</tr>
</tbody>
</table>

Table 9: Earnings profiles – Model vs. Data (Case 1)

Source: Census data and author

<table>
<thead>
<tr>
<th></th>
<th>(\frac{X_{1970} - X_{1940}}{X_{1940}})</th>
<th>(\frac{X_{1970} - X_{1940}}{X_{1940}})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>High school</td>
<td>0.626</td>
<td>0.223</td>
</tr>
<tr>
<td>College</td>
<td>0.426</td>
<td>0.286</td>
</tr>
<tr>
<td>All</td>
<td>0.521</td>
<td>0.245</td>
</tr>
</tbody>
</table>

Table 10: Proportion of data explained (Case 1)

Source: Census data and author

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>number of periods</td>
<td>42</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
<td>1.05</td>
</tr>
<tr>
<td>$s$</td>
<td>time spent in college</td>
<td>4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.0114</td>
</tr>
<tr>
<td>$\phi$</td>
<td>on-the-job human capital accumulation function parameter</td>
<td>0.6475</td>
</tr>
<tr>
<td>$\eta$</td>
<td>college human capital accumulation function parameter</td>
<td>0.4895</td>
</tr>
<tr>
<td>$z$</td>
<td>college human capital accumulation function productivity parameter</td>
<td>3.4450</td>
</tr>
<tr>
<td>$z_h$</td>
<td>productivity parameter common to all workers</td>
<td>1.0020</td>
</tr>
<tr>
<td>$w_1$</td>
<td>initial wage rate per unit of human capital</td>
<td>4.0290</td>
</tr>
<tr>
<td>$g_{col}$</td>
<td>growth rate of $w_r$ for college</td>
<td>1.0054</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mean of lognormal ability distribution</td>
<td>-3.6950</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation of lognormal ability distribution</td>
<td>0.3520</td>
</tr>
<tr>
<td>$g_{hs}$</td>
<td>growth rate of $w_r$ for high school</td>
<td>1.0048</td>
</tr>
</tbody>
</table>

Table 11: Calibrated parameters (Case 2)

highlighted parameters are calibrated
Source: Author
<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
<th>( \frac{Data}{Model} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{35} )</td>
<td>3.127</td>
<td>2.914</td>
<td>0.932</td>
</tr>
<tr>
<td>( \mu_{45} )</td>
<td>2.678</td>
<td>2.927</td>
<td>1.093</td>
</tr>
<tr>
<td>( \mu_{55} )</td>
<td>2.560</td>
<td>2.824</td>
<td>1.103</td>
</tr>
<tr>
<td>( \sigma_{35} )</td>
<td>3.127</td>
<td>2.914</td>
<td>0.932</td>
</tr>
<tr>
<td>( \sigma_{45} )</td>
<td>2.678</td>
<td>2.927</td>
<td>1.093</td>
</tr>
<tr>
<td>( \sigma_{55} )</td>
<td>2.560</td>
<td>2.824</td>
<td>1.103</td>
</tr>
<tr>
<td>( \Omega^{25}_{25,35,45} )</td>
<td>0.157</td>
<td>0.166</td>
<td>1.057</td>
</tr>
<tr>
<td>( \Omega^{35}_{35,45,55} )</td>
<td>0.245</td>
<td>0.231</td>
<td>0.941</td>
</tr>
<tr>
<td>( \Omega^{45}_{25,45,55} )</td>
<td>0.409</td>
<td>0.401</td>
<td>0.980</td>
</tr>
<tr>
<td>( \Omega^{55}_{25,35,55} )</td>
<td>0.529</td>
<td>0.544</td>
<td>1.029</td>
</tr>
<tr>
<td>( \frac{E^{90}<em>{25}}{E^{90}</em>{25}} )</td>
<td>5.418</td>
<td>5.357</td>
<td>0.989</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>0.170</td>
<td>0.173</td>
<td>1.020</td>
</tr>
</tbody>
</table>

Table 12: Calibration (Case 2)

Source: Census data and author

<table>
<thead>
<tr>
<th>Age 25</th>
<th>Age 55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td>HS</td>
</tr>
<tr>
<td>Cohorts</td>
<td></td>
</tr>
<tr>
<td>1940</td>
<td>1</td>
</tr>
<tr>
<td>1970</td>
<td>1</td>
</tr>
<tr>
<td>1940-1970</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 13: Earnings profiles – Model vs. Data (Case 2)

Source: Census data and author
Table 14: Proportion of data explained (Case 2)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{X_{1970} - X_{1940}}{X_{1940}}$</th>
<th>$\frac{X_{1970} - X_{1940}}{X_{1940}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>High school</td>
<td>0.626</td>
<td>0.219</td>
</tr>
<tr>
<td>College</td>
<td>0.426</td>
<td>0.274</td>
</tr>
<tr>
<td>All</td>
<td>0.521</td>
<td>0.244</td>
</tr>
</tbody>
</table>

Source: Census data and author

Note: Columns 2 and 3 show the change in lifetime increment of 1970 cohort from 1940 cohort. Column 4 shows the proportion of data explained by the model.
References


Kambourov, Gueorgui and Iourii Manovskii, “Accounting for the Change in Life-Cycle Profile of Earnings,” Manuscript 2009.


