General Trading Costs in Pure Theory of International Trade

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ABSTRACT
We use the HOS model of international trade to find a link between trading costs and pattern of trade, not just its effect on volume of trade. We prove that higher trading costs in labor-abundant countries will restrict volume of world trade by working against factor endowment bias and conversely for the capital-abundant nation. Asymmetric trading cost between goods may have paradoxical output effects. Relatively capital-abundant country will be worse off with increasing trading cost at home and abroad, whereas labor-abundant country may gain from further increase in trading cost relative to autarky.

Keywords: International Trade, Factor-intensity, General equilibrium

(JEL CL. No: F1, D5)
SECTION 1

The purpose of this paper is to formalize the notion of general trading costs in pure theory of international trade. We start from the basic Heckscher-Ohlin-Samuelson (HOS) model of trade and explicitly bring in a trading sector which processes trading or transaction in the economy. The trading cost is modeled as the typical “iceberg” type of cost whereby a fraction of the value of output to be transacted is lost in the process if there are no traders in the system. We highlight the total value of trade that is transacted in an economy which, by definition, must include total value of production in exportables and demand for the import competing good. Resources are needed to carry out trading and the lost value of trade goes towards compensating the traders. The general equilibrium approach we adopt identifies traders with potential workers and therefore in equilibrium with perfect intersectoral labor mobility, trading remuneration must coincide with the wage. The economy then divided into two segments, one where production takes place by using capital and labor a la the standard 2x2 model and the other where trading takes place by using labor alone. Such a system is related to the hybrid structure accommodating the HOS and the specific-factor model as analyzed in earlier works by Jones and Marjit (1992), Marjit and Beladi (1996, 1999), Marjit (2005) drawing from a well known policy paper by Gruen and Corden (1970).

That trading costs adversely affect the volume of trade and limit the scope for international transaction have been amply demonstrated theoretically and empirically in several papers by Anderson (2000), Anderson and Wincoop (2004), Davis (1998), Feenstra (1998, 2004), Laussel and Riezman (2008), Bandopadhyay and Roy (2007), Bernard, Jensen and Schoot (2006), Limao and Venables (2001). The impact of

However, our main objective in this paper is to internalize the concept of trading cost in an otherwise simple model of trade and emphasize the fact that trading as a separate activity uses resources like production related activities. Therefore, such cost should affect the pattern of trade and relative prices in a systematic way. The main results we derive in this context are as follows:

(1) Trading costs tend to increase the relative price of the labor intensive good in autarky. Thus the volume of trade will be asymmetrically affected in a labor-abundant and capital-abundant country.

(2) Asymmetric product specific trading costs may have paradoxical output and relative price effects. For example larger trading costs for capital intensive good may actually increase the volume of production in capital intensive sector. However, the same for the labor intensive good must reduce production of the labor intensive good.

(3) In the post-trade situation a decline in the trading costs may reduce welfare in a labor abundant country whereas welfare must go up for the capital rich nation.

The model is developed in the next section. Section 3 discusses the relationship between trading costs and the pattern and volume of trade. Section 4 talks about the welfare impact. Last section concludes.

**SECTION 2**

**The Basic Model**

With this backdrop let us consider a world economy consisting of two economies: a home and a foreign economy. The variables of the foreign economy are denoted by
asterisk. Foreign economy is considered in order to gauge the difference in relative price of foreign with that of home when trading cost changes. Our main focus is on the home economy.

Home economy is considered to be a perfectly competitive one producing two tradeable goods; capital-intensive good $X$ and labor-intensive good $Y$. Traders are needed to complete the process of transaction from production to consumption. A part of the total labor force is absorbed in the production of $X$ and $Y$, and others get employment due to transaction activities. This transaction related intermediation gives rise to trading costs. $\alpha_X$ is the fraction of good $X$ and $\alpha_Y$ is the same for $Y$ that is lost due to trading cost. $[\alpha_X P_X + \alpha_Y P_Y]$ represents the maximum total value of the goods that can be spent on those who are actually involved in trading activities. Let $Z$ represent the sector and $L_Z$, the people who are exclusively engaged in such operations. These people are paid out of the difference between commodity price and material input cost of production. We assume competitive market for trading costs to be consistent with the otherwise standard specifications of the competitive general equilibrium model.

Foreign economy is characterized by similar variables but with an asterisk. Perfect competition prevails in all markets in both the countries and production functions for $X$ and $Y$ are assumed to exhibit constant returns to scale (CRS) and diminishing marginal productivity (DMP).

The symbols and basic equations are in consistence with Jones (1965). To build the system of equations, we use following notations:

$P_i$ = Price of $i^{th}$ good, $i = X, Y$

$w$ = Return to labour
\( r = \) Return to capital, \( K \)

\( a_{ij} = \) Technological co-efficient

\( \bar{K} = \) Total supply of capital

\( \bar{L} = \) Total supply of labour

\( L_z = \) Labor engaged in trading activities

Let us assume commodity \( Y \) as the numeraire and set \( P_x = P \).

Competitive price conditions are:

\[
\begin{align*}
wa_{x\alpha} + ra_{x\alpha} &= P(1 - \alpha_x) \\
wa_{y\alpha} + r a_{y\alpha} &= (1 - \alpha_y)
\end{align*}
\]

(1)

(2)

Full employment conditions are:

\[
\begin{align*}
a_{x\alpha}X + a_{y\alpha}Y &= K \\
a_{x\alpha}X + a_{y\alpha}Y &= \bar{L} - L_z
\end{align*}
\]

(3)

(4)

Had there been no sector doing trading intermediation in the RHS of equation (4) we could have only \( \bar{L} \).

Note that, trading cost is required not for production. Trading cost comes into the picture only when the produced goods are brought to the consumers. Here \( X \) is importable and \( Y \) is exportable. This means in the post trade situation the cost equation for the \( Z \) sector would be

\[
[\alpha_x P(X + M) + \alpha_y(Y)] = wL_z
\]

(5)

Any imported amount of \( X \), i.e. \( M \) and export of \( Y \) must be taken into account while calculating the total trading cost. Where, \( \alpha \in [0,1] \); a low \( \alpha \) will mean lower the degree of trading costs. We start from autarky by using (5). In that case (5) boils down to

\[
[\alpha_x P(X) + \alpha_y(Y)] = wL_z
\]

(5a)
We can close the model by incorporating a homothetic demand function. This is,
\[
\frac{X_D}{Y_D} = f(P), f'(P) < 0
\] (6)

Here \(X_D\) and \(Y_D\) signifies demand for respective commodities.

Factor endowments of labor and capital are constant at \(\bar{L}, \bar{K}\). With given prices and trading costs (\(P, \alpha_X\) and \(\alpha_Y\)) \(w\) and \(r\) can be determined from equation (1) and (2). Factor proportions in turn get determined from factor prices because of CRS assumption. Now, let us start from some \(L_z\) such that \((\bar{L} - L_Z) > 0\). Then, given \((w, r)\) and hence \(a_{ij}\) (\(a_{ij}\) is constant because of CRS) and with a given value of \(L_z\) we can solve for \(X\) and \(Y\) from equation (3) and (4). This completes the solution of the model.

Moreover, we can also solve for \(L_z\). With \(w\) determined the RHS of (5) would be linear in \(L_z\) with slope \(w\). Given \(P\) with an increase in \(L_z\) LHS of (5) must fall as labor resource is smaller in size now. This implies that new production equilibrium at the given price level would be on a lower production possibility frontier, yielding lower value of production. Thus LHS of (5) is negatively sloping in \(L_z\). Hence, we have figure -1 where \(L_{z0}\) is determined. Now with \(L_{z0}\) we can determine everything else in the system, in particular \(X\) and \(Y\) or \(\left(\frac{X}{Y}\right)\).

**SECTION 3.A**

With a rise in \(P\), \(w\) will fall and \(r\) will go up as per the Stolper-Samuelson theorem. Given \(L_Z\), this will make the labor constraint more and capital constraint less binding. Hence due to Rybczynski theorem \(X\) will go up and \(Y\) will go down.

Now, let us look at (5). RHS in figure-1 will rotate downward since \(w\) is lower and \(L_z\) is given. Note that due to the envelope property and also for the fact that trading
cost is the same for both sectors, change in \[ \alpha_x P(X) + \alpha_y (Y) \] will be approximated by \( dP.X \) which is greater than zero since \( P \) rises. Hence, the LHS in figure-1 will shift up. This is demonstrated in figure -2. Therefore \( L_Z \) will increase further curtailing \( Y \) and increasing \( X \). Thus a rise in \( P \) will raise \( \frac{X}{Y} \), the usual supply-side response. By using the homothetic demand function we can close the model and can determine the equilibrium value of \( P \). Figure-3 gives us the equilibrium autarkic price \( P_A \).

Let us introduce a foreign economy, represented by ‘*’. Say both domestic and foreign economies are similar in technology and preference. But the difference lies in factor endowments. Let the foreign economy be capital abundant. Hence, \((K/L)^* > (K/L)\). When both the nations are symmetrically affected by trading costs, according to HOS prediction, for a given \( P \), \((X/Y)^* > (X/Y)\). This implies, \( P_A^* < P_A \) (‘A’ denotes autarkic situation). It is apparent that greater is the difference between \((K/L)^*\) and \((K/L)\) and hence \((P_A - P_A^*)\), bigger will be the volume of trade or the size of so called “trade triangle”.

Here it is worth mentioning that as far as the domestic production, domestic exports and domestic imports are concerned, intermediation is done only by domestic labor.

**SECTION 3.B**

**Symmetric change in domestic trading costs**

Suppose there is a change in trading costs in the home country owing to some reason. Say both \( \alpha_x \) and \( \alpha_y \) rise. Therefore, both \((1-\alpha_x)\) and \((1-\alpha_y)\) fall in the home, the labor-abundant country. Note that from (1) and (2) given \( P \) there will be symmetric response in both the price equations, \( \hat{w} = \hat{r} < 0 \) \( [\hat{^*}] \) denotes proportional
change as in Jones (1965)]. Hence, \( \left( \frac{w}{r} \right) \) does not change. However, there are two effects on LHS in (5). Given \( f(P(X+M)+Y) \), an increase in trading cost has increased LHS. But as \( w \) and \( r \) fall, value of national income goes down. Hence given \( \alpha_x \) and \( \alpha_y \), LHS should go down. The negative effect will vanish if we start from zero trading costs. To keep things simple we shall assume that initially \( \alpha_x = \alpha_y = 0 \). Then the RHS falls at a given \( Lz \) as \( w \) falls. Therefore, \( Lz \) must increase lowering \( Y \) and increasing \( X \). Subsequently a symmetric increase in \( \alpha_x \) and \( \alpha_y \) will lead to an increase in \( Lz \) and an increase in \( \left( \frac{X}{Y} \right) \).

This will reduce the gap between \( \left( \frac{X}{Y} \right)^* \) and \( \left( \frac{X}{Y} \right) \) for any given \( P \). The autarkic price gap \( (P_A - P_{A^*}) \) will also shrink and so will be the volume of trade. This is clearly demonstrated in figure-3.

Now, the degree of effective capital abundance in the labor-abundant country should be measured as \( \left( \frac{K}{L-L_z} \right) \). Therefore \( \left( \frac{K}{L} \right) < \left( \frac{K}{L-L_z} \right) \) and, \( \left( \frac{K}{L} \right) < \left( \frac{K}{L-L_z} \right) \).\( ^* \).

Therefore as both \( \alpha_x \) and \( \alpha_y \) rise in a labor abundant country, less labor is available for production related activities cutting back production of labor intensive good. It is also to be noted that there is no presumption as to which sector is more distorted by trading cost with \( \alpha_x \) and \( \alpha_y \) being the same. But as trading intermediation requires only labor, the labor-abundant country suffers in terms of the good over which it has comparative advantage. The message is that people, who could otherwise be involved in producing \( Y \), are being engaged in intermediation activities. Therefore, the trading related
transaction cost induced bias goes against the factor-endowment bias for a relatively labor-abundant country. Due to the same reason for a capital-abundant country’s natural endowment bias is further strengthened by trading cost. Precisely that is why and how the relative price and volume of trade gets asymmetrically affected for labor-rich and capital-rich countries.

Equation (5) provides with the following expression

\[ \hat{L}_z = (\hat{\alpha}_X + \hat{\alpha}_Y) V \alpha_x - \hat{w} \]  \hspace{1cm} (7)

Here \( V \alpha_x = \frac{P \alpha_x (X + M)}{P \alpha_x (X + M) + \alpha_y Y} \) and \( V \alpha_y = \frac{\alpha_y Y}{P \alpha_x (X + M) + \alpha_y Y} \).

\( V \alpha_x \) and \( V \alpha_y \) are essentially the value share of trading cost in X and Y, respectively with respect to total trading cost. A closer look reveals that these are nothing but the share of X and Y that requires trading intermediation. Note that this includes both consumption and production and \( V \alpha_x + V \alpha_y = 1 \).

Using the elasticity of demand and setting \( \alpha_x = \alpha_y = \alpha \) one can easily arrive at the following results.

\[ \hat{P} = (-\frac{1}{1 - \alpha}) \left[ (-\hat{L} + \hat{K}) - \lambda_{Lz} \left( \frac{1}{1 - \alpha} \right) \right] \]  \hspace{1cm} (8)

\[ \Delta \hat{P} = (-\frac{1}{1 - \alpha}) \left[ \lambda_{Lz} \Delta d \alpha \right] \]  \hspace{1cm} (9)

Here both \( |\alpha|, |\theta| < 0 \) because commodity X is capital intensive.
Thus the following proposition is immediate,

**PROPPOSITION I**: An increase in trading costs tends to make the labor intensive good dearer in autarky. This in turn will reduce the volume of trade in a labor-abundant country but will enhance the same in capital-abundant country.

QED

_Proof:_ See appendix A for detailed mathematical proof.

**SECTION 3.C**

Asymmetric change in domestic trading costs

We can have some interesting outcome if trading costs do not change symmetrically. Two interesting papers in this connection deserve to be mentioned. One is by Chakrabarti (2004) and the other is by Bernard, Jensen and Schott (2006). There may be two different cases in our model: one is when trading cost increases in capital-intensive goods and the other when labor-intensive goods are disturbed by greater trading costs.

Say trading cost increases in $X$ while that of $Y$ remains constant. From (1) RHS goes down as $\hat{A}_X > 0$. This leads to an increase in $w$ and a fall in $r$ since $X$ is capital-intensive. Given $Lz$, capital constraint will be more and labor constraint will be less binding. Therefore, production of $Y$ will increase and that of $X$ will fall following the standard Rybczynski effect. For a given $Lz$, RHS of (5) increases as $w$ goes up. LHS of (5) also increases as $\alpha_X$ goes up. Hence the effect on $Lz$ is uncertain. It essentially depends on the relative strength of these two changes. However, it is very much possible that $Lz$ may in fact go up. In that case following the Rybczynski argument production of $Y$ shrinks and that of $X$ increases.
When trading cost increases only in X, for a given P and given trading cost for Y equation (7) comes down to

\[ \hat{L}_Z = \hat{\alpha}_X \left[ V \alpha_x + \frac{\theta_{xy}}{|\theta| \left( 1 - \alpha_x \right)} \right] \]  

(10)

Therefore the following results are apparent.

\[ \hat{X} = (-) \frac{\lambda_{ll} \lambda_{ky}}{|\lambda|} \left[ \hat{\alpha}_X \left( V \alpha_x + \frac{\theta_{xy}}{|\theta| \left( 1 - \alpha_x \right)} \right) \right] \]  

(11)

\[ \hat{Y} = \frac{\lambda_{ll} \lambda_{ky}}{|\lambda|} \left[ \hat{\alpha}_X \left( V \alpha_x + \frac{\theta_{xy}}{|\theta| \left( 1 - \alpha_x \right)} \right) \right] \]  

(12)

\[ \hat{X} > 0 \iff \left| V \alpha_x \right| > \left| \frac{\theta_{xy}}{|\theta| \left( 1 - \alpha_x \right)} \right|. \]  

Under the same condition \( \hat{Y} < 0 \). This condition is most likely to hold true. However, we can’t ignore the other possibility when \( L_z \) would, in fact, fall due to an increase in \( \alpha_x \).

The intuitive explanation is very simple. \( \theta_{xy} \) should be relatively small as Y is labor-intensive. And also the \( \alpha_x \) may be tiny. If the volume of consumption of X is sufficiently large, \( V \alpha_x \) must not be insignificant. Sufficiently large consumption of X implies that if trading cost goes up in X, it will require a major chunk of labor to take care of this trading cost related intermediations. This will almost certainly more than offset the dampening effect on \( L_z \) caused by a higher \( w \) which is captured by \( \frac{\theta_{xy}}{|\theta| \left( 1 - \alpha_x \right)} \).

On the other extreme trading cost may increase only in Y. From (2) RHS goes down as \( \hat{\alpha}_y > 0 \). This reduces \( w \) and increases \( r \) since Y is labor-intensive. This in turn,
for any given $L_z$, lead to an increase in $X$ and a fall in $Y$. For a given $L_z$, RHS of (5) goes down as $w$ falls. LHS of (5) also increases as $\alpha_r$ goes up. Hence $L_z$ increases unambiguously. Then following Rybczynski theorem $Y$ production should fall and that of $X$ should increase. Under these circumstances equation (7) can be modified as

$$L_z = \hat{\alpha}_y \left\{ V \alpha_y - \frac{\theta_{xy}}{\theta} \frac{\alpha_y}{1 - \alpha_y} \right\}$$

(13)

Thus

$$\hat{X} = (-) \frac{\lambda_{xy} \lambda_{xy}}{\lambda} \left[ \hat{\alpha}_y \left\{ V \alpha_y - \frac{\theta_{xy}}{\theta} \frac{\alpha_y}{1 - \alpha_y} \right\} \right]$$

(14)

$$\hat{Y} = \frac{\lambda_{xx} \lambda_{xy}}{\lambda} \left[ \hat{\alpha}_y \left\{ V \alpha_y - \frac{\theta_{xy}}{\theta} \frac{\alpha_y}{1 - \alpha_y} \right\} \right]$$

(15)

The RHS of equation (15) is always negative whereas the value of $\hat{X}$ is unambiguously positive.

Hence we can write down the following proposition:

**PROPOSITION II**: Larger trading costs for capital intensive good may raise the volume of production of capital intensive good whereas the same for the labor intensive good unequivocally reduces the production of the labor intensive good.

QED

*Proof: See appendix A for detailed mathematical proof.

**SECTION 4**

So far we have not explicitly stated the welfare consequences of introducing trading costs in the standard general equilibrium model. Having a leakage in the form of trading costs related transaction activity entails inefficiency of some sort. Trading costs,
in fact, act as a tax on the labor-intensive sector. In the first best situation the economy should have produced more of the labor-intensive good. If the labor-abundant country wishes to engage in trade, prevalence of trading costs will restrict volume of trade and therefore the extent of the gains from trade will be affected. Thus the welfare loss is reinforced. Higher (lower) trading costs in a labor–abundant country will be harmful to the capital-abundant country since higher output of capital intensive good will depress (increase) world price of that good, causing a terms of trade loss(gain) for the capital-abundant country. Thus a reduction in trading costs will unequivocally raise the welfare of capital-rich nations. Interestingly once engaged in trade, the labor-abundant economy may gain (lose) from higher (lower) trading costs, through an improvement (deterioration) in the terms of trade. Then, we may have a case where the labor-abundant country in the post-trade situation can even gain (lose) from higher (lower) trading costs with a strong enough terms of trade effect. This is evident from the following expression for change in welfare.

\[
\frac{d\Omega}{d\alpha} = \left( -\frac{dP}{d\alpha} M(1-\alpha) + \left( \frac{dY}{d\alpha} + P \frac{dX}{d\alpha} \right) + \alpha P \frac{dM}{d\alpha} + PM \right)
\]

Since \( M = X_D - X \) and \( M = M(\Omega, P) \)

\[
\frac{d\Omega}{d\alpha} = \frac{1}{1-\alpha \beta_X} \left[ \frac{dP}{d\alpha} \left( -M(1-\alpha) + \alpha P \frac{\partial M}{\partial P} \right) + \left( \frac{dY}{d\alpha} + P \frac{dX}{d\alpha} \right) + PM \right]
\]

where, \( \beta_X = P \frac{\partial M}{\partial \Omega} \) or marginal propensity to import. Note that \( \frac{\partial M}{\partial P} \) is nothing but the substitution effect.
**PROPOSITION III:** A capital-abundant country’s welfare must increase with a reduction in the trading costs in the post-trade situation whereas the labor-abundant nation may experience a reduction in its welfare.

QED

**Proof:** Appendix B provides the detailed calculation.

**SECTION 5**

**Conclusion**

The purpose of this paper is to model general trading cost within a simple general equilibrium framework and then explain the relationship between international trade and trading costs. We argue that the standard HOS framework provides some insights regarding such a relationship. Trading is a labor-intensive activity. Hence, as more labor is attracted to this sector, labor-intensive traded good suffers, so does the volume of trade for the labor-abundant economy.

**Footnote**

1. A considerable part of trading cost may be bureaucratic corruption and rent seeking. There is a large number of papers that deal with these issues.
2. Note that this is not Rybczynski effect. Since available productive labor resource shrinks, PPF moves down.
3. Interested readers may look into Jones, R. W (1965) for more detailed analysis and mathematical calculations.
4. Initial trading cost may not be necessarily 0. Without losing the essence of the model we can think of any positive value of $\alpha_x$ and $\alpha_y$ to start with. In that case the value of $\hat{P}$ would be (assume that $\alpha_x = \alpha_y = \alpha$)

$$\hat{P} = (-) \frac{1}{|\lambda|} \left[ (\hat{L} - \hat{K}) - \lambda_{LZ} \left( \frac{1}{1 - \alpha} \right) \hat{\alpha} \right]$$

$$\sigma_{\theta} - \frac{1}{|\lambda|} \lambda_{LZ} \left[ V \alpha - \frac{\theta_{xy}}{\theta} \right]$$
and that of $\Delta \hat{P}$ would be

$$\Delta \hat{P} = (-) \frac{1}{|\alpha|} \left[ \hat{L} - \hat{K} \right] \left( \frac{1}{1 - \alpha} \right) \Delta \alpha$$

If we start from zero trading cost, $\alpha = 0$ and $\hat{\alpha}$ would be $d\alpha$. One can check that this will provide us with the same result.

**Appendix A**

Differentiating and manipulating equation (1) and (2) we get,

$$\hat{w} = \frac{\theta_{XY}}{1 - \alpha} \left( \hat{P} - \hat{P} \alpha_\alpha \hat{\alpha}_\alpha \right) + \frac{\theta_{LX}}{1 - \alpha} \alpha_\alpha \hat{\alpha}_\alpha$$

(1.A)

$$\hat{r} = (-) \frac{\theta_{XY}}{1 - \alpha} \left( \hat{P} - \hat{P} \alpha_\alpha \hat{\alpha}_\alpha \right) + \frac{\theta_{LX}}{1 - \alpha} \alpha_\alpha \hat{\alpha}_\alpha$$

(2.A)

Where, $|\theta| = (\theta_{XY} - \theta_{KX}) = (\theta_{LX} - \theta_{LY}) < 0$. And, $\theta_i \Rightarrow$ value share of $j^\text{th}$ factor in $i^\text{th}$ commodity, $j = L$ and $K$, and $i = X$ and $Y$.

Therefore, $(\hat{w} - \hat{r}) = \frac{1}{|\theta|} \left[ \hat{P} - \alpha_\alpha \frac{\alpha_\alpha}{1 - \alpha} + \hat{\alpha}_\alpha \frac{\alpha_\alpha}{1 - \alpha} \right]$ (3.A)

Differentiating equation (3) and (4) and manipulating them one gets,

$$\hat{X} = \frac{\hat{L} \lambda_{XY} - \hat{L} z \lambda_{LX} \lambda_{KY} - \hat{K} \lambda_{LY}}{|\hat{\lambda}|}$$

(4.A)

$$\hat{Y} = \frac{\hat{K} \lambda_{LY} - \hat{L} \lambda_{KY} + \hat{L} z \lambda_{LX} \lambda_{KX}}{|\hat{\lambda}|}$$

(5.A)

Where, $|\hat{\lambda}| = (\lambda_{KY} - \theta_{LX}) = (\theta_{LX} - \theta_{KX}) < 0$. And, $\lambda_{ji} \Rightarrow$ share of $j^\text{th}$ factor in $i^\text{th}$ commodity, $j = L$ and $K$, and $i = X$ and $Y$. 
Hence, \( \left( \hat{X} - \hat{Y} \right) = \frac{\hat{L} - \hat{K} - \hat{L}Z \lambda_{LZ}}{\lambda} \) \hfill (6.A)

From equation (5),
\[
\hat{L}_Z = \left( \hat{\alpha}_X + \hat{\lambda} \right) V \hat{\alpha}_X + \hat{\alpha}_Y V \hat{\alpha}_Y - \hat{\nu}
\]  \hfill (7.A)

Here, \( V \hat{\alpha}_X \) and \( V \hat{\alpha}_Y \) represent share of trading costs in X and Y respectively.

Using homothetic demand and balanced trade condition we have,
\[
\hat{\rho} = \left( - \frac{1}{\hat{\alpha}_X} \right) \left[ \frac{\hat{L} - \hat{K} - \hat{L}Z \lambda_{LZ}}{\hat{\lambda}} \right] \hfill (8.A)
\]

where, \( \sigma_0 \) implies demand elasticity

When trading costs change symmetrically across sectors \( \alpha_X = \alpha_Y = \alpha \) equation (7.A) turns out to be
\[
\hat{L}_Z = \left( \hat{\alpha} + \hat{\lambda} \right) V \hat{\alpha}_X - \hat{\rho} \frac{\alpha}{1 - \alpha} \hat{\alpha} = 1 \hfill (9.A)
\]

Thus, \( \hat{\rho} = \frac{1}{\hat{\lambda}} \left[ \lambda_{LZ} \left( \frac{1}{1 - \alpha} \right) \hat{\alpha} \right] \) and \( \Delta \hat{\rho} = \frac{1}{\hat{\lambda}} \left[ \lambda_{LZ} \left( \frac{1}{1 - \alpha} \right) \Delta \hat{\alpha} \right] \) \hfill (10.A)

- This proves proposition I.

When trading costs change asymmetrically - there may be two cases: (a) \( \hat{\alpha}_X > 0 = \hat{\alpha}_Y \)

and (b) \( \hat{\alpha}_Y > 0 = \hat{\alpha}_X \). Substituting these conditions in the above equations one can easily arrive at the proposition what we have written in the text.

- Hence proposition II is proved.
Appendix B

The utility function is \( U = U(X_D, Y_D) \) \(^{(1.B)}\)

Differentiating above equation we get, \( d\Omega = dY_D + PdX_D \) \(^{(2.B)}\)

\( d\Omega \) denotes the change in real income or welfare in \( Y \) units.

We also know that the budget constraint is,

\[
Y_D + PX_D = wL + rK = w(L - Lz) + rK + wLz
\]

\[
Y_D + PX_D = (1 - \alpha)(PX + Y) + \alpha[P(X + M) + Y]
\]

\[
Y_D + PX_D = (PX + Y + \alpha PM)
\] \(^{(3.B)}\)

Therefore,

\[
\frac{d\Omega}{d\alpha} = (-) \frac{dP}{d\alpha} M(1 - \alpha) + \left( \frac{dY + PdX}{d\alpha} \right) + \alpha P \frac{dM}{d\alpha} + PM
\]

\(^{(4.B)}\)

Since \( M = X_D - X \) and \( M = M(\Omega, P) \).

\[
\frac{d\Omega}{d\alpha} (1 - \alpha \beta_X) = \left[ \frac{dP}{d\alpha} \left( - M(1 - \alpha) + \alpha P \frac{\partial M}{\partial P} \right) + \left( \frac{dY}{d\alpha} + P \frac{dX}{d\alpha} \right) + PM \right]
\]

\[
\frac{d\Omega}{d\alpha} = \frac{1}{1 - \alpha \beta_X} \left[ \frac{dP}{d\alpha} \left( - M(1 - \alpha) + \alpha P \frac{\partial M}{\partial P} \right) + \left( \frac{dY}{d\alpha} + P \frac{dX}{d\alpha} \right) + PM \right]
\] \(^{(5.B)}\)

where, \( \beta_X = P \frac{\partial M}{\partial \Omega} \).

Note that \( \frac{\partial M}{\partial P} \) signifies normal substitution effect and \( \beta_X \) is the marginal propensity to import.
We know that $\frac{dP}{d\alpha} < 0$, $\frac{\partial M}{\partial P} < 0$ because of negativity of substitution effect and $(\frac{dY}{d\alpha} + P \frac{dX}{d\alpha})$ is also negative as a rise in trading cost leads to lowering the value of total production for a given $P$. However, $(1 - \alpha \beta x) > 0$ since $0 < \alpha, \beta x < 1$. Therefore, if $\alpha$ falls, change in welfare would go in favor of a capital-rich nation as substitution effect is very unlikely to offset all other positive effects. Whereas for a labor-rich country welfare implication is ambiguous. It may fall if terms of trade effect is relatively weaker.

References


\[ \alpha_{x} P(X+M) + \alpha_{y}(Y) \], wL_{Z}

Figure -1
$[\alpha_x \, P(X+M) + \alpha_y \, (Y)] \, wL_Z$
Figure-3

\[ P = \frac{P_X}{P_Y} \]

\[ \frac{X}{Y} \quad \left( \frac{X}{Y} \right)^T \quad \left( \frac{X}{Y} \right)^* \]

\[ P_A \]

\[ P_{A*} \]

\[ P_{A*} \]

\[ \frac{X_D}{Y_D} \]

\[ O \quad X/Y \]