The Zero Lower Bound and Endogenous Uncertainty

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ABSTRACT

This paper documents a strong negative correlation between various measures of macroeconomic uncertainty and real GDP growth since the onset of the Great Recession. Prior to that event the correlation was weak and in many cases not statistically less than zero, even when restricting the data sample to only include recessions. At the same time, many central banks reduced their policy rate to its zero lower bound (ZLB) and continued to target that rate more than six years later. We contend the constraint imposed by the ZLB contributed to the stronger negative correlation that emerged in late 2008. To test our theory, we use a model where the ZLB occasionally binds. The model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the policy rate is close to or at its ZLB. Our model is also consistent with the stronger relationships that emerged in the data between real GDP growth and both inflation and nominal interest rate uncertainty in late 2008.

Keywords: Monetary Policy; Uncertainty; Economic Activity, Zero Lower Bound, Survey Data

JEL Classifications: E32; E47; E58

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1 INTRODUCTION

There is significant interest in understanding the relationship between uncertainty and economic activity. Several papers find a negative relationship in the data using various measures of uncertainty. For example, Bloom (2009) shows that unexpected increases in uncertainty, given by stock price volatility, are associated with declines in industrial production. Bekaert et al. (2013), Bloom et al. (2014), and Pinter et al. (2013) find similar relationships in the data. While those papers focus on financial market volatility, Jurado et al. (2015) use broader proxies for uncertainty and find that spikes in uncertainty are more infrequent but more persistent and more negatively correlated with hours, employment, and industrial production than the previous literature has typically estimated. Leduc and Liu (2014) report that uncertainty shocks in the post-2008 period contributed to a much larger fraction of the observed unemployment fluctuations than in other recessions. They speculate that this finding is attributable to the zero lower bound (ZLB) constraint on monetary policy.

This paper documents that a strong negative correlation between uncertainty and real GDP growth only emerged since the Great Recession. We use forward-looking measures of uncertainty from survey and stock market data and estimates of realized volatility from a time-varying VAR with stochastic volatility. Before the Great Recession the correlation was weak and in many cases not statistically less than zero, even when restricting the data sample to only include quarters when the economy was in a recession. At the same time, many central banks reduced their policy rate to its ZLB and continued to target that rate more than six years later. We contend the constraint imposed by the ZLB contributed to the stronger negative correlation that emerged in late 2008.

To test our theory, we use a New Keynesian model that imposes a ZLB constraint on the short-term nominal interest rate. The model predicts an increase in output uncertainty near and at the ZLB. When the nominal interest rate is far from its ZLB, uncertainty surrounding output is nearly constant and low. Therefore, the model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the short-term nominal interest rate is close to or at its ZLB. We also find that it is consistent with the stronger relationships that emerged in the data between real GDP growth and both inflation and nominal interest rate uncertainty in late 2008.

There is an established literature that explores how uncertainty affects economic variables in structural models. This literature typically examines how endogenous variables respond to second moment shocks that exogenously increase uncertainty. While the economic effects depend on the shocks, these models all produce negative relationships between uncertainty and economic activity.

Rational expectations models also contain uncertainty that is endogenous. In these models, households make predictions about the future values of both exogenous and endogenous variables. They also make forecasts about the degree of uncertainty surrounding those predictions. The measure of uncertainty in our theoretical model is equivalent to those forecasts, which in a mathematical sense is the expected volatility of the forecast errors regarding future variables.

Our results are important for the growing literature that links uncertainty and economic activity because they show that for particular states of the world uncertainty responds to what is...
happening in the economy. An increase in uncertainty occurs as the short-term nominal interest rate approaches its ZLB due to the restriction the constraint places on the ability of the central bank to stabilize the economy. Output becomes more responsive to shocks that hit the economy and, therefore, the distribution of future realizations of output become more dispersed when compared to the same distributions that exist when the short-term nominal interest rate is far from its ZLB. This result increases the expected volatility of the output forecast errors (i.e., uncertainty rises near or at the ZLB). Of course, these results do not rule out that economic activity can respond to uncertainty. It merely shows that in at least one case, when the short-term nominal interest rate is near or at its ZLB, uncertainty is responding to an event that is endogenous to the state of the economy.

It is well-known in the literature that the ZLB constraint has an important effect on the economy. Gust et al. (2013) estimate a nonlinear New Keynesian model with a ZLB constraint to quantify how much of the recent decline in output was due to the binding constraint. They find the constraint accounts for about 20% of the drop in U.S. real GDP from 2008 to 2009 and, on average, it caused output to be 1% lower from 2009 to 2011 than it would have been without the constraint. Nakov (2008) finds the optimal discretionary monetary policy leads to a more negative output gap at the ZLB when households face uncertainty about the real interest rate than when they have perfect foresight. Nakata (2012) also studies the effects of uncertainty when the ZLB binds by varying the standard deviation of discount factor shocks. He finds higher uncertainty increases the slope of the policy function for output, meaning positive discount factor shocks lead to a larger reduction in output when the ZLB binds. Basu and Bundick (2014) show cost and demand uncertainty shocks cause business cycle fluctuations, which become more pronounced when the ZLB binds. Specifically, they find that a positive demand uncertainty shock causes output to decline by 0.2% when the ZLB does not bind and by 0.35% when it binds. Moreover, they calculate that demand uncertainty shocks can account for one-fourth of the drop in output in late 2008. These results show that the ZLB constraint increases the responsiveness of output to shocks, but we believe we are the first to explore what it means for expected second moments of endogenous variables.

Several other recent papers also study endogenous uncertainty but in different contexts. For example, Bachmann and Moscarini (2012) examine a model where uncertainty increases in recessions because it is less costly for firms to experiment with price changes to learn about their market power. Van Nieuwerburgh and Veldkamp (2006) argue that low production during a recession leads to noisy forecasts that impede learning and slow the recovery. In a related paper, Fajgelbaum et al. (2013) allow for self-reinforcing episodes of high uncertainty and low economic activity. In their model, firms learn about fundamentals by observing the investment activity of other firms. Investment is low in recessions and, since information flows slowly, uncertainty is high, which further discourages investment and causes an uncertainty trap. Gourio (2014) finds that the volatility of output is countercyclical because customers, suppliers, and workers expect larger losses when adverse shocks raise the probability that firms default. Navarro (2014) sets up a model where financial crises endogenously generate higher volatility. In our model, the constraint imposed by the ZLB reduces the effectiveness of monetary policy, which makes output more responsive to shocks that hit the economy and increases the expected forecast error volatility.

The rest of the paper is organized as follows. Section 2 describes our measures of uncertainty in the data and computes correlations between those measures and economic activity. Section 3 introduces our theoretical model, its calibration, and the solution method. Section 4 provides a theoretical explanation for the stronger negative correlations that emerged in late 2008. Section 5 compares the correlations in the model to equivalent correlations in the data. Section 6 concludes.
2 Relationship between Economic Activity and Uncertainty

This section introduces three forward-looking measures of macroeconomic uncertainty and shows how they are correlated with economic activity. We first compute the correlations using real GDP as our measure of economic activity and then compute equivalent correlations using industrial production. We also calculate similar correlations using a time-varying VAR with stochastic volatility.

2.1 Data Description

Figure 1 shows three alternative measures of economic uncertainty: (1) the Chicago Board Options Exchange S&P 100 Volatility Index (VXO), (2) the dispersion in large manufacturers’ forecasts of business activity from the Business Outlook Survey (BOS), and (3) the dispersion in forecasts of real GDP 1-quarter ahead from the Survey of Professional Forecasts (SPF). The shaded regions correspond to recessions, according to the National Bureau of Economic Research. We focus on these data series because they are forward-looking measures of uncertainty and are able to capture changes in people’s expectations over time, as opposed to predictions about future uncertainty that are based on statistical relationships (e.g., a GARCH model). We also believe macroeconomic uncertainty is an important factor that influences the behavior of all of these measures, even though they represent different segments of the economy.

The VXO measures the expected volatility in the S&P 100 stock market index over the next month at an annualized rate. For example, if the value on the vertical axis is $x\%$, then people expect there is a $68\%$ chance the S&P 100 index will change by $\pm x/\sqrt{12}\%$ over the next month. We average the daily series each quarter so it is consistent with the frequency of real GDP releases. The Business Outlook Survey (BOS), which is conducted monthly by the Federal Reserve Bank of Philadelphia, asks large manufacturing firms to forecast whether general business activity will increase, decrease, or remain unchanged over the next six months. Following Bachmann et al. (2013), the forecast dispersion (FD) in the firms’ survey responses in month $t$ is given by

$$\text{BOS FD}_t = \sqrt{\text{Frac}^+_t + \text{Frac}^-_t - (\text{Frac}^+_t - \text{Frac}^-_t)^2},$$

where an increase (decrease) in business activity is labeled as $+1$ ($-1$) and Frac$^+$ (Frac$^-$) is the fraction of firms who forecast that outcome. Thus, the BOS FD is the standard deviation of the
responses in each month. We average the monthly BOS FD series each quarter and then standardize the values so the vertical axis displays the number of standard deviations from the mean response. The Survey of Professional Forecasters (SPF), which is conducted quarterly by the Federal Reserve Bank of Philadelphia, asks individuals who regularly make forecasts as part of their jobs to predict macroeconomic aggregates for the next four quarters (e.g., real GDP, inflation, interest rates). We focus on the forecasts of real GDP, which are denoted by \( y \). The inter-quartile FD is given by

\[
\text{SPF FD}_t = 100 \times (\log(y_{75}^{t+1|t-1}) - \log(y_{25}^{t+1|t-1})).
\]

This value is the percent difference between the 75th and 25th percentiles of the quarter \( t \) forecasts of real GDP in quarter \( t + 1 \), given all observations in quarter \( t - 1 \) and earlier. We use the inter-quartile range, rather than more extreme percentiles, because on average only 41 firms complete the survey each quarter, which implies the tails of the distribution would have a very small sample.

Since our measures of uncertainty represent different segments of the economy (i.e., the stock market, manufacturing, and output), it is not surprising that they have different properties. The VXO is the least noisy measure, and it is only modestly higher during the 1991 and 2001 recessions. The three most prominent spikes in the index correspond to Black Monday (1987), the Enron scandal (mid-to-late 2002) and the second Gulf War (early 2003), and the 2008 financial crisis.\(^3\) The SPF spikes around the time of Black Monday, after 9/11, and during the 2008 financial crisis. It also increases during the first Gulf War. The BOS rises right before the 1991 and 2001 recessions and during the Great Recession, which causes the largest spike in all three measures.

Sill (2012) computes a measure of uncertainty based on the standard deviation of the mean probability distribution assigned by the forecasters. Unfortunately, we could not use this measure because (1) the bins in the distribution changed in 1992Q1 and in 2009Q2, (2) the output variable was nominal GNP instead of real GDP before 1992Q1, and (3) individuals forecast the annual average GDP growth rate, which causes the information set to increase during the year. The last concern is the most problematic for our study because it restricts the sample to only one value per year, which leaves us with too small of a sample to calculate correlations. For those reasons, we use forecaster disagreement as a proxy for uncertainty, which is common in the literature. Moreover, several papers have shown that disagreement provides a good approximation for uncertainty [e.g., Bachmann et al. (2013), Bomberger (1996), Clements (2008), and Giordani and Soderlind (2003)].

2.2 Correlations Table 1 shows the correlations between the growth rate of economic activity (i.e., quarter-over-quarter log differences in either real GDP or industrial production) and our measures of uncertainty with different data samples.\(^4\) The top row is based on data before the Great Recession (1986Q1-2008Q2) and the second row uses data since the onset of the Great Recession (2008Q3-2014Q2). Our data series begins in 1986Q1 rather than an earlier date because (1) there were major changes in monetary policy beginning in the 1980s, and (2) the VXO is only available since 1986Q1 and we wanted to draw connections to this commonly used measure of uncertainty.\(^5\) We evaluate whether the correlations, \( \rho \), are statistically below zero using a one-tailed t-test.

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\(^3\)See Bloom (2009) for a more detailed list documenting spikes in realized and expected stock price volatility.

\(^4\)We also examined realized stock price volatility, but the results are similar to the VXO so they are not reported.

\(^5\)There is also evidence of a structural break at the start of the Great Moderation. The correlation between real GDP growth and the SPF FD before the Great Moderation (1968Q4-1985Q4) is \(-0.36\), and the correlation since the Great Moderation (1986Q1-2008Q2) is \(-0.09\), which are significantly different at a 5\% level. The correlations with realized stock price volatility in the two samples are also significantly different at a 5\% level. Although it is interesting to examine why the correlations changed during the Great Moderation period, we leave that exercise for future research.
All of the uncertainty measures are positively correlated, but the correlations in the pre-Great Recession sample are relatively weak ($\rho(\text{SPF FD, VXO}) = 0.51$, $\rho(\text{SPF FD, BOS}) = 0.18$, $\rho(\text{BOS, VXO}) = 0.22$). In the post-Great Recession sample, the correlations are much stronger ($\rho(\text{SPF FD, VXO}) = 0.71$, $\rho(\text{SPF FD, BOS}) = 0.38$, $\rho(\text{BOS, VXO}) = 0.60$), which indicates that there was something happening in the economy that affected all of the uncertainty measures.

**Real GDP Growth vs. Uncertainty** The first three columns of table 1 use real GDP as a measure of economic activity. The results indicate that a strong negative relationship between our uncertainty measures and real GDP growth only emerged in recent data. The correlations based on the pre-Great Recession sample are weak and they are not statistically less than zero when calculated with the VXO or SPF FD. Moreover, if we remove the quarters when the exogenous events identified in Bloom (2009) occurred in the pre-Great Recession sample, then none of the correlations are significant at a 5% level. In sharp contrast, all of the correlations based on data since the Great Recession are significant at a 1% level. We also use the Fisher z-transformation to test whether the correlations in these samples are statistically different. Those tests reveal that the correlations are different at a 1% level when calculated with the VXO and BOS and at a 5% level with the SPF. We obtain similar correlations between real GDP and the SPF FD over longer forecast horizons.

<table>
<thead>
<tr>
<th></th>
<th>Real GDP Growth vs. Uncertainty</th>
<th>IP Growth vs. Uncertainty</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>VXO</td>
<td>BOS FD</td>
</tr>
<tr>
<td>Pre-Great Recession</td>
<td>$-0.04$</td>
<td>$-0.20^{**}$</td>
</tr>
<tr>
<td>(1986Q1-2008Q2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-Great Recession</td>
<td>$-0.74^{***}$</td>
<td>$-0.70^{***}$</td>
</tr>
<tr>
<td>(2008Q3-2014Q2)</td>
<td></td>
<td></td>
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<tr>
<td>Past Recessions</td>
<td>$0.09^{†}$</td>
<td>$-0.16$</td>
</tr>
<tr>
<td>(1968Q4-2007Q3)</td>
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Table 1: Correlations between the growth rate of economic activity (quarter-over-quarter log differences) and various measures of macroeconomic uncertainty. A † † denotes a correlation calculated with realized stock price volatility as a measure of uncertainty. The values are statistically less than 0 at a ***1%, **5%, and *10% significance level.

The results in table 1 suggest that recessions may be a source of high uncertainty, but, perhaps counterintuitively, the bottom row shows there is little evidence for this relationship in the data. Both the SPF and BOS surveys began in 1968Q4. Between that date and the beginning of the Great Recession, the U.S. economy experienced 6 recessions, totaling 27 quarters. The correlations between real GDP growth and uncertainty in those quarters have roughly the same magnitude as the pre-Great Recession correlations. Moreover, none of the values are statistically less than zero, even at a 10% significance level. These results suggest there are unique features of the Great Recession that led to a strong negative relationship between uncertainty and real GDP growth.

**Industrial Production Growth vs. Uncertainty** We focus on real GDP, but empirical work has often used industrial production as a measure of economic activity [e.g., Bloom (2009), Bekaert et al. (2013), and Jurado et al. (2015)]. To compare our findings with the literature, the last three columns of table 1 use industrial production growth instead of real GDP growth to compute correlations with uncertainty. Similar to our results based on real GDP growth, the correlations between

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6The SPF computes growth rates of real GDP with the forecasts of real GDP in levels. The dispersion in the implied quarter-over-quarter growth rate of real GDP produces virtually identical correlations to those shown in table 1.
industrial production growth and our uncertainty measures are stronger in the post-Great Recession sample than the pre-Great Recession sample. The Fisher z-transformation test shows the correlations in the two samples are statistically different at a 1% level when calculated with the VXO and SPF FD and a 5% level with the BOS FD. Interestingly, the correlations with the SPF FD and BOS FD are stronger and statistically less than 0 in both the pre- and post-Great Recession samples.

We also calculate correlations between both of our measures of economic activity and the SPF forecast dispersion for industrial production (SPF IP FD). This measure of uncertainty is analogous to the SPF FD for real GDP. We find that all of the correlations are more negative in the post-Great Recession sample, but they are statistically less than zero in both samples. Furthermore, the SPF IP FD is more strongly correlated with industrial production growth than real GDP growth.

These findings suggest that there is something different about the manufacturing sector that sets it apart from the overall economy. In our paper, we are interested in the connection between uncertainty and real GDP growth. It is beyond the scope of this paper and our model to formally explore why the relationship is stronger with manufacturing related variables, but we speculate that the difference may have to do with the fact that manufacturing activity is more volatile and more sensitive to changes in business conditions than other segments of the economy. Given that the sectors of industrial production make up less than 20% of U.S. GDP in recent years, the results for industrial production are apparently not strong enough to drive the results for the whole economy.

Time-varying VAR with Stochastic Volatility

To provide further evidence for how the correlations between real GDP growth and uncertainty changed in late 2008, this section computes correlations based on a measure of uncertainty that does not rely on stock market or survey data. Following Primiceri (2005), we use a time-varying VAR with stochastic volatility to estimate the volatility of real GDP growth each quarter and then show how those estimates are correlated with economic activity. One difference between Primiceri (2005) and our estimation is that we use real GDP growth instead of the unemployment rate so we can draw comparisons to equivalent statistics in our theoretical model. Another difference is we use data from 1958Q2 to 2014Q2, where the first ten years train the prior distributions of the parameters. The model, given by,

\[ y_t = B_{0,t} + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + A_t^{-1}\Sigma_t \varepsilon_t, \quad t = 1, \ldots, T, \]

has a 2-quarter lag, so our estimates are from 1968Q4 to 2014Q2, which is the same period as the survey data. \( y_t \) is a 3 × 1 vector that includes real GDP growth, the inflation rate, and the T-Bill rate, \( B_{0,t} \) is a 3 × 1 vector of time varying intercepts, \( B_{1,t} \) and \( B_{2,t} \) are 3 × 3 matrices of time varying coefficients, and \( \varepsilon_t \) is a normally distributed shock with an identity variance-covariance matrix. \( A_t \) and \( \Sigma_t \) are the result of a triangular reduction of the variance-covariance matrix, where

\[ A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix}, \quad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix}. \]

The model is estimated with Bayesian MCMC methods using code that accompanies Koop and Korobilis (2010). The code implements a correction to the algorithm outlined in Appendix A of Primiceri (2005), as explained by Del Negro and Primiceri (2015). The estimate for the volatility of real GDP growth, \( \sigma_{1,t} \), is a proxy for macroeconomic uncertainty. We calculate correlations between real GDP growth and its estimated volatility with both the pre- and post-Great Recession samples. Our estimates are based on 100,000 draws from the posterior probability distribution.
Figure 2 shows the distributions of the correlation, \( \rho \), between real GDP growth and its volatility. With the pre-Great Recession sample, the correlation exceeds zero in 2.67\% of draws and the median is \(-0.24\). In contrast, the correlation with the post-Great Recession sample exceeds zero in only 0.72\% of draws and the median is \(-0.53\), which is more than twice as strong. When we only include quarters when the economy was in a recession, the correlation exceeds zero in 7.62\% of draws and the median is less negative (\(-0.14\)). To give us a better sense for whether there is evidence that the correlations in the two samples are different, we plot the distribution of the post-Great recession correlations minus the pre-Great Recession correlations. A small fraction of positive values provides evidence for our previous finding that the correlation between real GDP growth and uncertainty has been more negative since the Great Recession. Our estimation reveals that the difference between the correlations is positive in only 6.99\% of the draws and the median difference is \(-0.29\). These results are stronger if we remove the influence of the Gulf Wars and 9/11, which are arguably exogenous events. In that case, the difference between the correlations is positive in only 5.20\% of draws and the median difference falls to \(-0.32\). These results provide support for our findings in table 1, even though they based on a different measure of uncertainty.

Correlations Outside the U.S. We also examine correlations in the Euro area and the U.K. The European Central Bank (ECB) has conducted a survey of professional forecasters since 1999Q1. It asks participants to forecast Euro area real GDP growth over various horizons. For example, the survey conducted in 1999Q1 requests forecasts for 1999Q3, given the last GDP release is from 1998Q3. Similar to the U.S. SPF, we calculate the forecast dispersion as \( \text{ECB SPF FD}_t = |\hat{y}_{t+2|t-2} - \hat{y}_{t+2|t-2}^{25}| \) , where \( \hat{y}_{t+2|t-2}^x \) is the \( x \)th percentile of the quarter \( t \) forecast of real GDP growth in quarter \( t + 2 \), given observations in quarter \( t - 2 \) and earlier. The correlation between Euro area real GDP growth and the ECB SPF FD with the pre-Great Recession sample (1999Q1-2008Q2) is
−0.32, while the correlation with the post-Great Recession sample (2008Q3-2014Q2) is −0.47.\(^7\)

The Bank of England also conducts a survey called the Survey of External Forecasters (BOE SEF), which has asked its participants to forecast real GDP growth since 1998Q1.\(^8\) Prior to 2006Q2, the survey asked for projections in quarter 4 of the survey year, quarter 4 1 year ahead, and the same quarter 2 years ahead. For example, the forecast dates in the 2006Q1 survey were 2006Q4, 2007Q4, and 2008Q1. Since 2006Q2, the survey has asked for projections for the same quarter 1, 2, and 3 years ahead. Unfortunately, we cannot calculate correlations with the pre-Great Recession sample because the forecast horizons change. With the post-Great Recession sample, the correlation between real GDP growth and the dispersion in forecasts 1 year ahead is −0.46, which is similar in magnitude to the correlations we computed with the U.S. and Euro area SPF.\(^9\)

Given the short sample of the survey data, we also computed correlations between real GDP growth and uncertainty using estimates of real GDP volatility from a time-varying VAR with stochastic volatility. Unfortunately, we cannot do the same exercise for the Euro area since data is only available from 1995Q1. For the U.K., we used the same data and sample period we used to estimate the model with U.S. data. With the pre-Great Recession sample, the correlation exceeds zero in 73% of draws and the median is 0.08. The post-Great Recession sample, however, is positive in only 0.05% of draws and the median is −0.67. Moreover, the post-Recession correlation exceeds the pre-Great Recession correlation in only 0.05% of draws and the median difference is −0.74. These results indicate a strong negative correlation also emerged in the U.K. in late 2008.

2.3 SUMMARY This section examined correlations between several proxies for uncertainty and two measures of economic activity. We found strong evidence that the correlations are stronger since the Great Recession. The correlations with real GDP before the Great Recession are weak, and often not statistically less than zero. The correlations with industrial production are negative in both periods but stronger since the Great Recession. Moreover, the correlations during other recessions are weak, which suggests there is something unique about the Great Recession period.

A major difference between the Great Recession and previous recessions is that the Fed has been constrained by the ZLB on the federal funds rate. Central banks typically conduct open market operations to stimulate demand during an economic downturn, but by 2008Q2 the Fed had cut the federal funds rate to 2% and economic conditions were sufficiently poor that a policy rate near zero was possible. In the following months, the Fed continued to reduce the federal funds rate and in 2008Q4 it hit its ZLB. More than six years after the ZLB was first hit, the Fed’s target interest rate remains near zero. The BOE and ECB faced constraints similar to the Fed during the Great Recession. The bank rate in the U.K. was reduced to 0.5% in 2009Q1. The Euro-zone deposit rate was cut from 3.25% in 2008Q4 to 0.25% by the end of 2009Q1, and was further reduced to 0% in 2012Q3. We contend that the binding constraint the ZLB placed on current and future monetary policy is an important factor that contributed to the negative correlation that emerged between real GDP growth and macroeconomic uncertainty since mid-2008. The rest of the paper provides a theoretical explanation for why the correlations are much stronger when the central bank is constrained and shows the model is capable of reproducing our empirical findings.

\(^7\)We obtain similar results when we use the industry survey in the European Commission’s Business and Consumer surveys. This survey asks manufacturers how they expect their production to develop over the next three months.

\(^8\)See Boero et al. (2008) for more information about the BOE SEF and how it compares to similar surveys.

\(^9\)There is also a survey of Japanese professional forecasters, but it began in mid-2004 and does not provide a large enough sample. See Komine et al. (2009) for details about the survey and analysis of the forecasters’ performance.
3 Theoretical Model and Measure of Uncertainty

This section lays out our theoretical model and describes how we measure uncertainty. The model includes a ZLB constraint that occasionally binds due to discount factor and technology shocks.

3.1 Model

There are three actors in the model: (1) a representative household that has access to a one-period nominal bond, (2) a representative firm that bundles a continuum of intermediate inputs to produce a final good, and (3) a central bank that sets the short-term nominal interest rate.

Households

A representative household chooses \( \{c_t, n_t, b_t\}_{t=0}^{\infty} \) to maximize expected lifetime utility, \( E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi \tilde{n}_t^{1+\eta}/(1 + \eta)] \), where \( \chi > 0 \), 1/\( \eta \) is the Frisch elasticity of labor supply, \( c \) is consumption, \( n \) is labor hours, \( b \) is the real value of a 1-period nominal bond, \( E_0 \) is an expectation operator conditional on information available in period 0, \( \tilde{\beta}_0 = 1 \), and \( \tilde{\beta}_t = \prod_{j=1}^{t} \beta_j \).

Following Eggertsson and Woodford (2003), \( \beta \) is a time-varying discount factor that follows

\[
\beta_t = \tilde{\beta} (\beta_{t-1}/\tilde{\beta})^{\rho_\beta} \exp(\varepsilon_t),
\]

where \( \tilde{\beta} \) is the steady-state discount factor, 0 \( \leq \rho_\beta < 1 \), and \( \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2) \). These choices are constrained by \( c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\tilde{\pi}_t + d_t \), where \( \tilde{\pi}_t = p_t/p_{t-1} \) is the gross inflation rate, \( w \) is the real wage rate, \( i \) is the gross nominal interest rate set by the central bank, and \( d \) is real dividends from intermediate firms. The optimality conditions to the household’s problem imply

\[
w_t = \chi \theta c_t, \\
1 = i_t E_t [\beta_{t+1}(c_t/c_{t+1})/\tilde{\pi}_{t+1}],
\]

Firms

The production sector consists of a continuum of monopolistically competitive intermediate firms and a final goods firm. Intermediate firm \( f \in [0, 1] \) produces a differentiated good, \( y_t(f) \), according to \( y_t(f) = z_t n_t(f) \), where \( n_t(f) \) is the amount of employment used by firm \( f \). \( z_t \) represents the level of technology, which is common across firms and evolves according to

\[
z_t = \tilde{z} (z_{t-1}/\tilde{z})^{\rho_z} \exp(v_t),
\]

where \( \tilde{z} \) is steady-state technology, 0 \( \leq \rho_z < 1 \), and \( v \sim \mathcal{N}(0, \sigma_v^2) \). Each intermediate firm chooses its labor supply to minimize its operating costs, \( w_t n_t(f) \), subject to its production function.

The representative final goods firm purchases \( y_t(f) \) units from each intermediate goods firm to produce the final good, \( y_t \equiv \left[ \int_0^1 y_t(f)^{(\theta-1)/\theta} df \right]^{\theta/(\theta-1)} \), according to a Dixit and Stiglitz (1977) aggregator, where \( \theta > 1 \) measures the elasticity of substitution between the intermediate goods.

The final goods firm maximizes profits to determine its demand function for intermediate good \( f \),

\[
y_t(f) = (p_t(f)/p_t^{\theta} y_t),
\]

where \( p_t = \left[ \int_0^1 p_t^{(1-\theta)} df \right]^{1/(1-\theta)} \) is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price, \( \text{adj}_t(f) \). Using the functional form in Ireland (1997), \( \text{adj}_t(f) = \varphi \left[ p_t(f)/(\tilde{\pi} p_{t-1}(f)) - 1 \right]^2 y_t/2 \), where \( \varphi \geq 0 \) scales the size of the adjustment cost and \( \tilde{\pi} \) is the steady-state gross inflation rate. Real dividends are then given by \( d_t(f) = (p_t(f)/p_t) y_t(f) - w_t n_t(f) - \text{adj}_t(f) \). Firm \( f \) chooses its price, \( p_t(f) \), to maximize the expected discounted present value of real dividends

\[
E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t (c_t/c_{t+1}) d_t(f).
\]

In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

\[
\varphi \left( \frac{\pi_t}{\tilde{\pi}} - 1 \right) \frac{\pi_t}{\tilde{\pi}} = (1 - \theta) + \theta (w_t/z_t) + \varphi E_t \left[ \beta_{t+1} \frac{c_t}{c_{t+1}} \left( \frac{\pi_{t+1}}{\tilde{\pi}} - 1 \right) \frac{\pi_{t+1} y_{t+1}}{y_t} \right].
\]

Without price adjustments costs (i.e., \( \varphi = 0 \)), the real marginal cost of producing a unit of output \((w_t/z_t)\) equals \( (\theta - 1)/\theta \), which is the inverse of a firm’s markup of price over marginal cost, \( \mu \).
Monetary Policy The central bank sets the gross nominal interest rate according to

\[ i_t = \max \{ i, \pi_t / \pi^* \} \frac{\phi_i}{\phi_y} (y_t / y^p_t)^{\phi_y} \exp(\nu_t) \],

where \( \phi_i \) and \( \phi_y \) are the policy responses to the inflation and output gaps, \( \pi^* \) is the inflation rate target, which equals the steady-state inflation rate, \( y^p_t = (\chi \mu)^{-1/(1+\eta)} z_t \) is the potential output target (i.e., the level of output when \( \varphi = 0 \)), and \( \nu \sim N(0, \sigma^2_\nu) \) is a monetary policy shock. This rule ensures that \( i_t \geq i \) so households do not expect an interest rate less than \( i \) in any future period.

Competitive Equilibrium The resource constraint is given by \( c_t = y_t - ad_{jit} = y^gdp_t \), where \( y^gdp_t \) includes the value added by intermediate firms, which is their output minus price adjustment costs. Thus, \( y^gdp_t \) represents real GDP in the model. A competitive equilibrium consists of sequences of quantities \( \{ c_t, n_t, b_t, y_t, y^p_t \}_{t=0}^\infty \), prices \( \{ w_t, i_t, \pi_t \}_{t=0}^\infty \), and exogenous variables \( \{ \beta_t, z_t \}_{t=0}^\infty \) that satisfy the household’s and firm’s optimality conditions, (2), (3), and (5), the production function, \( y_t = z_t n_t \), the monetary policy rule, (6), the solution for potential output, \( y^p_t = (\chi \mu)^{-1/(1+\eta)} z_t \), the stochastic processes, (1) and (4), the bond market clearing condition, \( b_t = 0 \), and the resource constraint, given initial conditions, \( \beta_0 \) and \( z_{-1} \), and sequences of exogenous shocks, \( \{ \varepsilon_t, \nu_t, \eta_t \}_{t=0}^\infty \).

3.2 Measure of Endogenous Uncertainty A recent and growing segment of the literature introduces stochastic volatility (SV) into dynamic stochastic general equilibrium models to study the effects of exogenous changes in uncertainty. Our work differs from these papers in that we focus on how uncertainty about future variables endogenously respond to the state of the economy. To illustrate our measure of endogenous uncertainty, though, it is useful to first describe how one measures uncertainty in a model with SV. As an example, suppose a model includes an exogenous random variable \( x \), such as technology or government spending, that evolves according to

\[ x_t = \rho_x x_{t-1} + \sigma_t \varepsilon_t \]

where \( 0 \leq \rho_x < 1 \) and \( \varepsilon \) is white noise. SV is introduced into the model by assuming the standard deviation of the shock is time-varying and follows an independent exogenous process specified by the modeler, which relaxes the common assumption of homoscedastic innovations. Given the linear process governing \( x \), the expected forecast error, \( FE_x \), equals

\[ E_t[FE_{x,t+1}] = E_t[x_{t+1} - E_t x_{t+1}] = 0. \]

Although the forecast error is mean zero, there is uncertainty about its future value. One measure of that uncertainty is the expected standard deviation of the forecast error for \( x \), which is given by

\[ \sqrt{E_t[FE^2_{x,t+1}]} = \sqrt{E_t[(x_{t+1} - E_t x_{t+1})^2]} = \sqrt{E_t[(x_{t+1} - \rho_x x_t)^2]} = \sqrt{E_t \sigma^2_{x,t+1}}. \]

Models that allow for SV in various shocks are able to match features of the data that models with homoscedastic errors can not match, but they do not explain why volatility changes over time because the uncertainty is exogenous. However, there is always uncertainty that is endogenous to the model (i.e., uncertainty due to the dispersion in the realizations of future variables instead of second moment shocks). We quantify the degree of endogenous uncertainty by following the logic of the SV literature. Specifically, the uncertainty surrounding \( y^gdp_t \), 1 quarter ahead, is given by

\[ \sigma_{y^{gdp},t} \equiv \sqrt{E_t[(y^gdp_{t+1} - E_t y^gdp_{t+1})^2]}, \]

which varies over time like it does with SV shocks, except the fluctuations are now endogenous responses to the state of the economy. One example of such a state is a binding ZLB constraint. We focus on the uncertainty surrounding real GDP, but we can also calculate this measure of uncertainty for other variables in the model, including the inflation and nominal interest rates.
3.3 Calibration We calibrate our model at a quarterly frequency to match moments in the data from 1986Q1 to 2014Q2. The parameters are summarized in Table 2. The steady-state discount factor, $\bar{\beta}$, is set to 0.9966, which equals the ratio of the average 3-month T-Bill rate to the average quarterly-over-quarter percentage change in the GDP implicit price deflator. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which is consistent with Peterman (2012). The leisure preference parameter, $\chi$, is calibrated so that steady-state labor equals $1/3$ of the available time. The elasticity of substitution between intermediate goods, $\theta$, is calibrated to 6, which corresponds to an average markup over marginal cost equal to 20%. The costly price adjustment parameter, $\varphi$, is set to 160, which matches the estimate in Ireland (2003). The lower bound on the gross nominal interest rate, $I$, is calibrated to 1.00023, which equals the average 3-month T-bill rate from 2009Q1 to 2014Q2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$\bar{\beta}$</td>
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</tr>
<tr>
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<td>3</td>
</tr>
<tr>
<td>$\theta$</td>
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</tr>
<tr>
<td>$\pi^*$</td>
<td>1.0055</td>
</tr>
<tr>
<td>$I$</td>
<td>1.00023</td>
</tr>
</tbody>
</table>

Table 2: Calibrated parameters.

The inflation rate target, $\pi^*$, is calibrated to 1.0055 to match the average quarter-over-quarter percentage change in the GDP implicit price deflator. We set the monetary response to deviations from the inflation target, $\phi_\pi$, equal to 2 and the response to deviations from potential output, $\phi_y$, equal to 0.1. The persistence of the discount factor, $\rho_\beta$, equals 0.87 and the standard deviation of the shock, $\sigma_\varepsilon$, equals 0.002, which are close to the values estimated in Gust et al. (2013). We set the persistence of the technology process equal to $\rho_z = 0.9$ and the standard deviation of the shock equal to $\sigma_v = 0.002$. The standard deviation of the monetary policy shock, $\sigma_z$, is set to 0.00225.

In choosing the parameters of the three stochastic processes, our primary goal was to match the volatility of real GDP, since we focus on movements in its forecast error volatility in the model. In the data, the annualized standard deviations of quarter-over-quarter percent changes in real GDP, the GDP deflator inflation rate, and the 3-month T-Bill rate are 2.45%, 0.95%, and 2.45% per year, respectively. To determine how these values compare to the data, we ran 10,000 Monte Carlo simulations of the model that are each 114 quarters long (i.e., the same length as our data from 1986Q1 to 2014Q2). We then compute the median standard deviations of real GDP, the inflation rate, and the interest rate. Those values and their 90% credible sets equal 2.22% (1.78, 3.20), 0.94% (0.67, 1.41), and 1.98% (1.60, 2.45) annually. All three credible sets include the values in the data and the median standard deviations of real GDP and inflation are close to their historical averages.

The parameters of the stochastic processes, which affect the frequency and duration of ZLB events, are also chosen so our model is consistent with how long people expected the ZLB to bind in the U.S., rather than the actual ZLB duration. Figure 3 plots median forecasts of the T-Bill rate $q$ quarters ahead using data from the SPF (left panel) and Blue Chip (right panel) surveys. Specifically, we show the SPF (Blue Chip) forecasts made in the first quarter (February) from 2009 to 2013. Both surveys indicate that people did not initially expect the T-Bill rate to remain near zero for long periods. For example, in 2009 and 2010 forecasters in both surveys predicted the T-Bill rate would exceed 0.5% within 3 quarters, despite the severity of the recession. Swanson and Williams (2014) use options data to calculate the probability that the federal funds rate would be
below 0.5% for the next 5 quarters. That probability ranged from 20% to 45% between 2008Q4 and 2010Q2 and was typically below 60% until 2011Q3, meaning households placed greater weight on leaving the ZLB than staying at the ZLB. Only after the Fed communicated date-based forward guidance in August 2011 did forecasters expect the T-Bill rate to remain at zero for a longer period.

Figure 4 plots histograms of the durations of each ZLB event in our model. To compute these histograms, we initialize 10,000 simulations at two alternative notional interest rates (i.e., the policy rate the central bank would set if it was not constrained by the ZLB): $\tilde{i}^* = 0.9$ (left panel) and $\tilde{i}^* = -0.5$ (right panel). We then count the number of quarters in the first ZLB event for each simulation and report the frequency of the ZLB event durations across the 10,000 simulations. The unconditional average ZLB event is 2.32 quarters. When we condition on a notional interest rate that is consistent with estimates during and after the Great Recession, the average ZLB event rises to 3.34 quarters. Therefore, our calibration produces ZLB events with a similar average duration to what households expected before the Fed communicated date-based forward guidance to the public. It is also possible for the model to generate much longer ZLB events. For example, 4.25% (1.35%, 0.50%) of the ZLB events are greater than 8 quarters (12 quarters, 16 quarters).

### 3.4 Solution Method

We solve the model using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This method discretizes the state space and iteratively solves for the updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. Those techniques capture the expectational effects of going to and returning to the ZLB. See Richter et al. (2014) for a formal description of the algorithm.

Benhabib et al.’s (2001) finding that constrained models have two deterministic steady-state equilibria has generated considerable discussion in the literature about whether there are conditions
in which a unique MSV solution exists in stochastic models with a ZLB constraint. Specifically, they find two nominal interest rate/inflation rate pairs that satisfy the steady-state equilibrium system. In one steady state, the central bank meets its positive inflation target, whereas in the other steady state the economy experiences deflation. Richter and Throckmorton (2015) show that the numerical algorithm used in this paper converges to the inflationary equilibrium as long as there is a sufficient expectation of returning to a monetary policy rule that conforms to the Taylor principle. Our algorithm, however, never converges to the deflationary equilibrium. Although addressing questions related to the deflationary steady state is an interesting topic for future research, our analysis, like most macroeconomic research on the ZLB constraint, focuses on the solution centered around the inflationary steady state. For further information on this topic see Gavin et al. (2015).

4 Monetary Policy, the ZLB Constraint, and Uncertainty

We first build intuition on why the ZLB generates a strong negative correlation between real GDP growth and uncertainty in the model by fixing technology at steady state, so that ZLB events are endogenous only due to positive discount factor shocks. We also show how the central bank’s response to inflation affects this relationship. We then allow for stochastic changes in technology.

4.1 Model with Constant Technology We begin our theoretical analysis by holding technology constant (i.e., $z_t = \bar{z}$ for all $t$). In this case, the model contains only one state variable, $\beta_{-1}$, which is exogenous and ranges from \pm 1.6\% of its steady state. In all of our results, $\hat{y}$ denotes percent deviation from steady state output (i.e., $\hat{y}_t = 100(y_t - \bar{y})/\bar{y}$), $\hat{\sigma}_y$ denotes percent of steady state output (i.e., $\hat{\sigma}_{y,t} = 100(\sigma_{y,t}/\bar{y})$), and a tilde denotes a net interest rate (i.e., $\tilde{x} = 100(x - 1)$).

The top row of figure 5 plots the decision rules for real GDP, $\hat{y}^{gdp}$ (left panel), and real GDP uncertainty, $\hat{\sigma}_{gdp}$ (right panel), as a function of $\hat{\beta}_{-1}$ for three different values of the central bank’s response to the inflation gap: $\phi_\pi = 2$ (solid line), $\phi_\pi = 2.5$ (dashed line), and $\phi_\pi = 3$ (dash-dotted line). The shaded regions indicate where the ZLB binds, which depends on the value of $\phi_\pi$. When $\phi_\pi = 2$ ($\phi_\pi = 2.5$, $\phi_\pi = 3$), the ZLB binds in states where $\hat{\beta}_{-1} > 0.62$ ($\hat{\beta}_{-1} > 0.68$, $\hat{\beta}_{-1} > 0.75$). The bottom row plots the probability density function of future real GDP as a percent deviation from its mean, $100 \times (y_{t+1}/E_t[y_{t+1}] - 1)$. We display the density functions for three values of...
The initial notional interest rate: $\tilde{r}_0^* = 0.9\%$ (solid line), $\tilde{r}_0^* = 1.8\%$ (dashed line), $\tilde{r}_0^* = -0.5\%$ (dash-dotted line) when $\phi_\pi = 2$. Each notional interest rate is inversely related to the discount factor state ($\beta_{-1}$), which is shown in the legend. The density functions are informative because they illustrate why macroeconomic uncertainty changes across different states and parameters.

We begin by discussing how uncertainty changes across the discount factor states. The discount factor is a proxy for aggregate demand because it determines households’ degree of patience. When the discount factor is low (high), households are impatient (patient), and less (more) willing to postpone consumption. Firms respond to the higher (lower) demand by increasing (decreasing) their prices and output. Hence, the policy function for real GDP is downward sloping (top left panel). In discount factor states where the nominal interest rate is sufficiently far from its ZLB, the slope of the policy function for real GDP is virtually constant. Hence, the distribution of future real GDP values is independent of the state of the economy in this region of the state space. The probability density function in these states is also narrower than in states where the nominal interest
rate is close to or at its ZLB (bottom panel). Thus, real GDP uncertainty \( \hat{\sigma}_{y_{gdp}} \) is relatively lower and nearly constant in states where the central bank is unconstrained by the ZLB (top right panel).

As the economy enters states where the nominal interest rate is close to or at its ZLB, demand continues to decline and firms further reduce their prices. In states where the central bank is not constrained, it is able to respond to the lower inflation by cutting its policy rate to dampen the effects of the fall in demand. However, given a large enough decline in demand, the ZLB will bind and the central bank will be unable to further reduce its policy rate. Thus, the economy becomes more sensitive to further declines in demand, which leads to lower real GDP than if the central bank was unconstrained. A steeper policy function for real GDP widens the distribution of possible real GDP values next period and skews it toward output losses. For example, when \( \hat{\pi}_0^* = 0.9\% \), a ±1 standard deviation discount factor shock (i.e., ±0.2%) causes real GDP to move from its steady state by ±0.3%. When \( \hat{\pi}_0^* = -0.5\% \), the same change in the discount factor can cause real GDP to decrease by 0.7 percentage points or increase by 1.1 percentage points. The broader range of future real GDP values produces greater forecast error volatility and hence higher uncertainty.\(^\text{11}\)

Figure 5 demonstrates that real GDP uncertainty endogenously increases in our model when the nominal interest rate is near or at its ZLB. We recognize that uncertainty also spikes in the data when the ZLB was not a concern (e.g., 1987: Black Monday, 1990: first Gulf War, 2001: 9/11, mid-2002 to early-2003: Enron scandal and second Gulf War), but we do not view those episodes as a problem for our theory because they are due to events that are exogenous to our model.

We next consider how the inflation coefficient in the monetary policy rule affects endogenous uncertainty. We conduct this exercise for two reasons. First, it provides excellent intuition for the relationship between the slope of the policy function for real GDP and macroeconomic uncertainty. Second, it illustrates how the ability of the central bank to stabilize the economy affects macroeconomic uncertainty. When the central bank places more emphasis on inflation stability (i.e., a higher \( \phi_\pi \)), it affects the volatility of inflation and real GDP both away from and at the ZLB.

First consider the case where the nominal interest rate is far away from its ZLB. The top left panel of figure 5 shows that the slope of the policy functions flatten as \( \phi_\pi \) increases. That reflects the central bank’s success at stabilizing inflation around its target level. Real GDP becomes less responsive to discount factor shocks, and, as a result, the distribution of future real GDP becomes tighter around its expected value (i.e., uncertainty is lower when \( \phi_\pi \) is higher). We show this effect graphically in the top panel of figure 6, where we plot the actual density functions of future real GDP across three values of \( \phi_\pi \). For each \( \phi_\pi \) the discount factor equals its steady-state value.

Higher values of \( \phi_\pi \) have similar impacts on real GDP uncertainty when the ZLB constraint binds. Greater inflation stability means the ZLB binds at higher discount factors states. It also means that when the nominal interest rate is at its ZLB, households expect inflation will be relatively more stable when the nominal interest rate rises. Since households always expect the nominal interest rate to exit its ZLB, more stable inflation away from the ZLB leads to more stable inflation and real GDP at the ZLB. For example, if \( \hat{\pi}_0^* = -0.5\% \), then a ±1 standard deviation discount factor shock causes real GDP to range from −0.56% to −1.73% below its steady state when \( \phi_\pi = 2 \) and from −0.45% to −1.34% when \( \phi_\pi = 3 \). A flatter policy function for real GDP generates a narrower and more symmetric distribution for future real GDP (bottom panel), which suggests that one benefit of a higher \( \phi_\pi \) is that it alleviates uncertainty near and at the ZLB. Re-

\(^{11}\)Basu and Bundick (2015) compute impulse responses to an increase in the volatility of the discount factor. They show higher volatility arises endogenously at the ZLB through precautionary saving and contractionary bias channels.
Figure 6: Density functions of future real GDP with various monetary policy responses to inflation. The top (bottom) panel shows the density conditional on the steady-state discount factor (a discount factor where the ZLB constraint binds). The horizontal axes display future real GDP as a percent of its mean. The vertical axes show the density value.

Regardless of the value of $\phi_{\pi}$, however, uncertainty is unaffected by discount factor shocks when the nominal interest rate is far from its ZLB and increases sharply when it approaches and hits its ZLB.

Another way to compare the relationship between real GDP and uncertainty is with a generalized impulse response function (GIRF) of a shock to the discount factor following the procedure in Koop et al. (1996). The advantage of GIRFs is that they are based on an average of model simulations where the realization of shocks is consistent with households’ expectations over time. Figure 7 plots the responses to a 1 standard deviation positive discount factor shock at and away from the ZLB. The steady-state simulation (solid line) is initialized at the stochastic steady state. The deep recession simulation (dashed line) is initialized at a notional interest rate equal to $-0.5\%$.

The shock causes households to postpone consumption, which reduces real GDP and inflation. When the nominal interest rate is far from its ZLB, the drop in real GDP is damped by the monetary policy response. There is almost no change in uncertainty because households expect future shocks will have the same effect on real GDP regardless of the initial state of the economy. When the ZLB binds, however, the central bank cannot respond by lowering the nominal interest rate, which leads
to larger declines in real GDP. In this case, uncertainty sharply increases since households expect a wider range of future real GDP values when current and future monetary policy is constrained.

Next, we run Monte Carlo simulations of the model that are 114 quarters long so that each simulation is the same length as our data from 1986Q1 to 2014Q2. Figure 8 plots one of the simulations with a single ZLB event that occurs at the end of the simulation and lasts for 24 quarters (i.e., the number of quarters in our post-Great Recession data period). The top panel plots the paths of real GDP growth, $\Delta \log y^{gdp}$ (left axis, solid line), and real GDP uncertainty, $\hat{\sigma}_{y^{gdp}}$ (right axis, dashed line), which provides another way to visualize the correlation between real GDP growth and uncertainty. The bottom panel plots the paths of the nominal (solid line) and notional (dashed line) interest rates. The shaded region indicates periods when the ZLB binds. In that region, the notional rate is negative. Outside of the ZLB region, the nominal and notional rates are equal.

There are three key takeaways from this simulation. One, uncertainty surrounding future real GDP is state-dependent. When the ZLB does not bind, uncertainty is essentially constant, except in quarters when the nominal interest rate is near its ZLB. In those situations, the high probability of hitting the ZLB next period leads to persistently higher uncertainty. The closer the nominal interest rate is to the ZLB, the higher the uncertainty, which underscores the importance of expectational effects. When the ZLB binds, $\hat{\sigma}_{y^{gdp}}$ is as much as four times larger than its value outside the ZLB. The degree of uncertainty depends on the notional interest rate, which indicates how likely it is for the nominal interest rate to rise and exit the ZLB in the near-term. The smaller the notional interest rate, the less likely the nominal interest rate will exit the ZLB and the higher the uncertainty.

Two, there is a weak correlation between $\hat{y}^{gdp}$ and $\hat{\sigma}_{y^{gdp}}$ when the ZLB does not bind but a strong negative correlation between those variables when it does bind. The strength of those correlations depends on the likelihood of entering and staying at the ZLB. When the nominal interest rate is sufficiently far from its ZLB, the correlation is close to zero since uncertainty is
nearly constant. In periods when the nominal interest rate is near or at its ZLB due to positive discount factor shocks, real GDP is well below its steady state and uncertainty is relatively high.

Three, it is possible for uncertainty to decline while the ZLB binds, which implies that uncertainty is time-varying at the ZLB. This outcome occurs whenever the notional interest rate is below zero and there is a negative discount factor shock. A lower discount factor means the household is more optimistic about the future economy, which increases the expected nominal interest rate and reduces real GDP uncertainty. This feature of the model is important because macroeconomic uncertainty continued to fluctuate in the data even after many central banks reduced their policy rates to the ZLB in late 2008. In other words, our theory does not claim that uncertainty is always increasing when the ZLB binds, but rather that it is more strongly correlated with real GDP growth.

Our theoretical results are based on a model that does not consider the implications of unconventional monetary policies, such as quantitative easing or forward guidance. Modeling such policies is a difficult task and beyond the scope of this paper. We believe that such policies might have implications for the level of uncertainty predicted by the model. To the extent that those policies are successful, they will most likely reduce the expected volatility of future real GDP and therefore lower the level of uncertainty. However, we suspect that none of our qualitative findings (i.e., stronger negative correlations near and at the ZLB) would change. The reason is that unless these policies completely alleviate the constraint imposed by the ZLB, it will still be the case that real GDP is more responsive to shocks near and at the ZLB than it is away from the ZLB. As a
result, the correlations between uncertainty and real GDP growth will also be stronger at the ZLB.

4.2 MODEL WITH VARIABLE TECHNOLOGY Now suppose technology is time-varying according to (4). The model contains two state variables, $z_{-1}$ and $\beta_{-1}$, which range from $\pm 2.3$ and $\pm 1.6\%$ of their steady-state values, respectively. Figure 9 shows the policy functions for real GDP (left panel) and real GDP uncertainty (right panel). The shaded regions indicate where the ZLB binds, which represents $34.4\%$ of the state space. A low (high) level of technology increases (reduces) firms’ marginal cost of production. Firms respond by decreasing (increasing) their production and raising (reducing) their prices. The central bank responds by increasing (decreasing) its policy rate. Thus, the nominal interest rate hits its ZLB given a sufficiently high level of technology. In those states, the dynamics are similar to what occurs in high discount factor states—lower inflation raises the real interest rate, causing a sharp decline in real GDP and a large increase in uncertainty.

![Figure 9: Policy function for real GDP (left panel) and real GDP uncertainty (right panel). The horizontal (vertical) axes in these panels display technology (the discount factor), which is in percent deviations from steady state. The contours in the left panel display real GDP in percent deviations from its steady state. The contours in the right panel show real GDP uncertainty as a percent of steady-state output. The shaded regions indicate where the ZLB binds.](image)

The uncertainty surrounding future real GDP is mostly unaffected by the level of technology when the nominal interest rate is far from its ZLB. In that situation, uncertainty is stable and low, even when technology and the discount factor are both below their steady states. In contrast, when the nominal interest rate is close to or at its ZLB, regardless of whether it is due to unusually high technology or a high discount factor, forecast error volatility increases. As technology and the discount factor simultaneously increase and move away from their respective steady states, real GDP rapidly declines, which drives up uncertainty at the ZLB. Thus, variable technology represents another source of uncertainty but only when the central bank is constrained by the ZLB.

Allowing for technology shocks increases the volatility of real GDP growth and the likelihood of spikes in uncertainty, since the shocks are an additional source of movements in the nominal interest rate. However, the simulation properties of the model with constant technology continue to hold—uncertainty is time varying and strongly correlated with real GDP near and at the ZLB,
although the correlations are slightly weaker because variable technology adds additional states of the economy where it is possible for real GDP to remain unchanged or even fall when uncertainty rises. One state where that occurs is along the contour where real GDP equals its steady state.

Figure 10: Generalized impulse responses to a 1 standard deviation positive technology shock at and away from the ZLB. The steady-state simulation (solid line) is initialized at the stochastic steady state. The deep recession simulation (dashed line) is initialized at a notional interest rate equal to $-0.5\%$. To compute the GIRFs, we first calculate the mean of 10,000 simulations of the model conditional on a random shock in the first quarter. We then calculate a second mean from another set of 10,000 simulations, but this time the random shock in the first quarter is replaced with a 1 standard deviation positive technology shock. The vertical axis is the percentage change in real GDP (or the difference in uncertainty) between the two means. The horizontal axes display the time period in quarters.

To obtain further insight for how technology shocks affect real GDP uncertainty and its relationship with real GDP, figure 10 plots the responses to a 1 standard deviation positive technology shock at and away from the ZLB. There are several interesting takeaways from this simulation. One, it demonstrates that technology shocks sharply increase real GDP uncertainty when the ZLB binds, similar to positive discount factor shocks. Two, a positive technology shock that occurs at the ZLB leads to a much smaller increase in real GDP than the same shock that occurs when the ZLB does not bind. Three, a positive technology shock generates a positive relationship between real GDP and real GDP uncertainty when the ZLB does not bind, whereas a positive discount factor shock (see figure 7) produces a negative relationship. Thus, the correlations between these variables will be weaker in periods when the ZLB does not bind than in the model with constant technology. The next section shows that feature of the variable technology model is necessary to match the correlations in the data prior to 2008. A positive technology shock also produces a positive relationship between real GDP and real GDP uncertainty at the ZLB, but the policy functions in the variable technology model are also steeper, which means the economy is more sensitive to discount factor shocks at the ZLB than in the model with constant technology. Thus, the correlations between these variables will be similar across the two models when the ZLB binds.

5 Model Predictions and the Data

This section compares correlations in the data to equivalent correlations in our theoretical model. We first outline our simulation procedure and then focus on the correlations between real GDP growth and real GDP uncertainty. We conclude by testing our model along two other dimensions.
5.1 Simulation Procedure  We perform Monte Carlo simulations of the model to obtain distributions of the correlations between macroeconomic variables and uncertainty. Similar to the analysis in section 2, we look at distributions conditional on a positive nominal interest rate and a rate close to or at its ZLB. Each simulation is initialized at the stochastic steady state and runs for 114 quarters, so the simulation length is the same as our data from 1986Q1 to 2014Q2. Unconditionally, ZLB events are infrequent in the model and can have a duration as short as one quarter. However, the model can also produce simulations with isolated ZLB events that have a much longer duration that is similar to the data. Thus, we specify that each simulation must satisfy two criteria: (1) it must have a single ZLB event, and (2) the ZLB event must have a duration of at least 16 quarters. Restricting ZLB events to that minimum duration yields an average ZLB event of 21.5 (20.7) quarters in simulations of the model with variable (constant) technology. Thus, these criteria allow us to summarize the predictions of the model with simulations that are characteristic of countries’ recent experiences at the ZLB. We first locate 10,000 simulations that satisfy these criteria, and then compare the median correlations away from and near the ZLB constraint to the correlations found in the data. We also briefly discuss properties of the conditional distributions.

<table>
<thead>
<tr>
<th>Sample</th>
<th>VXO</th>
<th>BOS FD</th>
<th>SPF FD</th>
<th>VAR</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Away from ZLB</td>
<td>−0.04</td>
<td>−0.20**</td>
<td>−0.09</td>
<td>−0.24++</td>
<td>−0.21++</td>
</tr>
<tr>
<td>Near the ZLB</td>
<td>−0.74***</td>
<td>−0.70***</td>
<td>−0.53***</td>
<td>−0.53++++</td>
<td>−0.42++++</td>
</tr>
</tbody>
</table>

Table 3: Comparison between correlations in the model and the data. The correlations in the model are between real GDP growth and the standard deviation of the real GDP forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and various measures of uncertainty. The breakpoint between the two samples in the model (\( \tilde{i}_{t} = 0.3725\% \)) matches the average T-Bill rate in 2008Q3, which is the breakpoint in the data. It is also when uncertainty first increases. The correlations in the data are significantly less than 0 at **1%, **5%, and *10% levels. The correlations in the model and VAR are positive with probabilities less than +++1%, +++5%, and +10%.

5.2 Real GDP Uncertainty  Table 3 compares the correlations in the model and the data. The correlations in the model are between quarter-over-quarter real GDP growth and the standard deviation of the real GDP forecast error 1-quarter ahead, which measures the uncertainty surrounding future real GDP. The “Away from ZLB” (“Near the ZLB”) correlations are based on the quarters in each simulation when \( \tilde{i} > 0.3725\% \) (\( \tilde{i} \leq 0.3725\% \)), where the cutoff is the value of the nominal interest rate in which real GDP uncertainty first increases. The correlations in the data are between real GDP growth and measures of uncertainty based on survey data, stock market data, and VAR estimates of output volatility. The “Away from ZLB” sample is from 1986Q1 to 2008Q2 and the “Near the ZLB” sample is from 2008Q3 to 2014Q2. The interest rate that separates the samples in the model equals the average T-Bill rate in 2008Q3, which is the breakpoint in the data.

There are several remarkable similarities between the correlations in the model and the data. One, the correlations with the “Near the ZLB” sample in both models are strongly negative and significantly less than zero, which is a robust feature of the data. Two, with the “Away from ZLB” sample, the model with constant technology predicts a weaker correlation than it does with the “Near the ZLB” sample, but it is stronger than the correlations with the SPF FD and VXO in the data. However, when the model includes technology shocks, its predictions better match the weak correlations in the data. Three, the correlations predicted by the model are closest in magnitude
to those based on the SPF FD, which is a forward-looking measure of uncertainty for the entire economy, instead of a measure that is based on stock market volatility (VXO), a particular sector of the economy (BOS FD), or realized output volatility (VAR). In both the model and the data with the SPF FD, the correlations in the “Near the ZLB” sample are strongly negative and significantly less than zero, whereas in the “Away from ZLB” sample the correlations are weak and not statistically significant. We recognize that other sources of endogenous or exogenous uncertainty may have contributed to the stronger negative correlations in the data since the onset of the Great Recession, but our findings provide strong evidence that the ZLB constraint is at least one important factor.

Figure 11: Distributions of the correlation, $\rho$, between real GDP growth and real GDP uncertainty.

The previous discussion focused on the median correlations from the model. To see the range of correlations that are possible, figure 11 plots the distributions of the correlations from all 10,000 simulations. The top (bottom) panel shows the correlations from the model with constant (variable) technology. The distributions from the model with variable technology are similar to the probability distributions from the time-varying VAR with stochastic volatility (figure 2). In the model with constant technology, the ZLB shifts the distribution to the left and skews the correlations towards more negative values. The “Away from ZLB” distributions, however, are also centered below zero.

Technology shocks change the “Away from ZLB” distributions. In that sample, the median estimate is less negative and far more simulations produce correlations that are greater than zero.
(11.5% compared to 4.3% in the model with constant technology). Similar to the VAR distributions, we can subtract the “Away from ZLB” distribution from the “Near the ZLB” distribution to determine the probability that the correlations are more negative when the nominal interest rate is near or at its ZLB. The difference between the two samples is positive in only 4.8% (3.1%) of the simulations of the model with constant (variable) technology, which provides more evidence that the ZLB was one major source of the stronger negative correlations that emerged in late 2008.

![Policy function graphs](image)

Figure 12: Policy function for inflation (top left panel), inflation uncertainty (top right panel), the nominal interest rate (bottom left panel), and nominal interest rate uncertainty (bottom right panel). The horizontal axes display the discount factor state in percent deviations from steady state. The vertical axes in the left panels display net rates. The vertical axis in the top (bottom) right panel shows the standard deviation of the forecast error for inflation (nominal interest rate) as a percent of steady-state inflation (interest rate). The shaded regions indicate where the ZLB binds.

### 5.3 Inflation and Interest Rate Uncertainty

Thus far we have focused on the correlation between real GDP growth and real GDP uncertainty. This section tests our model along different dimensions by looking at the correlations between real GDP growth and the uncertainty surrounding both inflation and the nominal interest rate. We begin by examining the properties of our theoretical model and then turn to the data using SPF forecasts of the inflation and T-Bill rates.

The top panels of figure 12 show the policy functions for inflation (left panel) and inflation uncertainty (right panel). The bottom panels show the policy functions for the nominal interest
rate (left panel) and interest rate uncertainty (right panel). For simplicity, technology is fixed at its steady state. The shaded regions indicate where the ZLB binds. The intuition for these results follows from our discussion about real GDP. When the discount factor is low, households are impatient and would like to increase current consumption. Firms respond to the higher demand by increasing their price level, which raises inflation and causes the central bank to increase its policy rate. As the discount factor declines and households’ willingness to postpone consumption increases, firms cut their prices. When the central bank is unconstrained, it will reduce its policy rate to dampen the effects of the fall in demand. Given a sufficiently large decline in demand, the ZLB will bind and the central bank will be unable to respond to adverse shocks. As a consequence, positive discount factor shocks that occur at the ZLB cause larger declines in inflation (i.e., a steeper policy function). The more weight households place on those outcomes in the future, the greater the dispersion in future inflation outcomes and the higher the level of inflation uncertainty.

Interest rate uncertainty, in contrast, is lower at the ZLB due to the constraint faced by the central bank. Any negative demand shock that occurs at the ZLB will not affect the nominal interest rate. It is also possible that positive demand shocks will not trigger a rate increase if the notional interest rate is sufficiently negative. Therefore, households place less weight on a positive nominal interest rate in discount factor states where the ZLB binds (i.e., the dispersion in the future policy rate declines), and the uncertainty surrounding the interest rate converges toward zero.

<table>
<thead>
<tr>
<th>Sample</th>
<th>SPF IPD FD</th>
<th>Data SPF CPI FD</th>
<th>VAR</th>
<th>Model $z = \bar{z}$</th>
<th>$z \sim \text{AR}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Away from ZLB</td>
<td>-0.20**</td>
<td>-0.27***</td>
<td>-0.16++</td>
<td>-0.24++++</td>
<td>-0.17+++</td>
</tr>
<tr>
<td>Near the ZLB</td>
<td>-0.44**</td>
<td>-0.61***</td>
<td>-0.35+</td>
<td>-0.42+++</td>
<td>-0.40+++</td>
</tr>
</tbody>
</table>

Table 4: Comparison between correlations in the model and the data. The correlations in the model are between real GDP growth and the standard deviation of the inflation forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and either the dispersion in forecasts of the inflation rate from the SPF or realized inflation volatility from the VAR. The correlations in the data are significantly less than 0 at ***1%, **5%, and *10% levels. The correlations in the model and VAR are positive with probabilities less than +++1%, ++5%, and +10%.

Table 4 compares correlations in our model to analogous correlations in the data. The correlations in the model are between real GDP growth and the standard deviation of the inflation forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and the dispersion in forecasts of the inflation rate from the SPF. We use two measures of forecast dispersion: (1) the percent difference between the 75th and 25th percentiles of the individual forecasts of the GDP implicit price deflator (SPF IPD FD) and (2) the difference between the 75th and 25th percentiles of the individual forecasts of the CPI inflation rate (SPF CPI FD). We also compute correlations using estimates of realized inflation volatility from our time-varying VAR with stochastic volatility.

A much stronger correlation between real GDP growth and inflation uncertainty emerged in late 2008, similar to the correlations with real GDP uncertainty. All three measures of inflation uncertainty indicate that the correlations with real GDP growth were at least twice as strong from 2008Q3 to 2014Q2 as they were from 1986Q1 to 2008Q2. A $z$-transformation test shows the correlations with the SPF IPD FD are significantly different only at a 15% level, but the correlations

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$^{12}$We would prefer to use forecasts of core CPI inflation, but those were not included in the SPF until 2007Q1.
with the SPF CPI FD are significantly different at a 5% level. Moreover, if we extend the sample back to 1968Q4, the correlations with the SPF IPD FD are significantly different at a 5% level.

Both models have the same key feature as the data—the median correlation between real GDP growth and inflation uncertainty is much stronger when the nominal interest rate is close to or at its ZLB. Moreover, the difference between the correlations in the “Away from ZLB” sample and the “Near the ZLB” sample in the model with constant (variable) technology is positive in only 5.9% (4.0%) of the simulations, which provides further evidence that the correlations are more negative in the “Away from ZLB” sample. Neither model generates correlations in the “Near the ZLB” sample as negative as the correlations with the SPF CPI FD, but that is likely because they do not include an oil sector. The model with constant technology over-predicts the correlation with the SPF IPD FD and the VAR in the “Away from ZLB” sample, but with variable technology the correlation is between those values in the data. These results show that our model matches features of the data along another dimension. They also provide evidence that the ZLB contributed to the stronger negative correlations between real GDP growth and inflation uncertainty since late 2008.

Figure 13 shows data on real GDP growth (solid line, left axis) and interest rate uncertainty (dashed line, right axis), which equals the difference between the 75th and 25th percentiles of the individual forecasts of the 3-month T-Bill rate one quarter ahead (SPF T-Bill FD). The “Post-Great Recession” sample is indicated by the shaded region. The predictions in the model match two key features of the data. One, uncertainty about the T-Bill rate converges toward zero as the economy moves into the Post-Great Recession sample. Forecasters are aware that the short-term interest rate is constrained by the ZLB, which reduces their disagreement. In the model, interest rate uncertainty approaches zero as the probability of exiting the ZLB declines. In other words, the longer households expect to remain at the ZLB, the more certain they are that the nominal interest rate will remain near zero in the future. Two, real GDP growth and T-Bill uncertainty are positively correlated in the Post-Great Recession sample. In the model, a drop in demand that causes the ZLB to bind decreases real GDP and simultaneously reduces uncertainty about the nominal interest rate. These results demonstrate that the model is consistent with the data along a third key dimension.
6 Conclusion

This paper documents a strong negative correlation between macroeconomic uncertainty and real GDP growth since mid-2008. Prior to that time, the correlation between those variables was weak and in many cases not statistically less than zero, even when restricting the data sample to only include recessions. Why did the Great Recession lead to a stronger negative correlation compared to previous recessions? One answer is the ZLB on the short-term nominal interest rate. During the Great Recession many central banks sharply reduced their policy rates and effectively hit the ZLB for the first time in their history. We contend that central banks’ inability to further reduce their policy rates in response to adverse economic conditions contributed to the stronger negative correlation between real GDP uncertainty and real GDP growth that emerged in late 2008.

To test our theory, we use a model where the policy rate occasionally hits its ZLB due to technology and discount factor shocks. The correlations between uncertainty and real GDP are weak when the policy rate is far from its ZLB, but strongly negative near and at the ZLB. This result occurs in the model because real GDP becomes more responsive to shocks that hit the economy when the central bank is constrained by the ZLB, which increases the dispersion of future real GDP. As a result, our model has the same key feature as the data—prior to the Great Recession, which is when the policy rate was far from its ZLB, the correlations were weak, but they are strongly negative since the policy rate approached its ZLB at the onset of the Great Recession. Our model is also consistent with the stronger relationships that emerged between real GDP growth and both inflation and interest rate uncertainty. While it may be the case that the ZLB is not the only factor causing the stronger correlations, our results provide strong evidence that it is an important factor.

References


PLANTE, RICHTER, AND THROCKMORTON: THE ZLB AND ENDOGENOUS UNCERTAINTY


A Data Sources


**Euro Area Real GDP**: 2010 Chained linked volumes, 12 countries, seasonally adjusted and adjusted data by working days. Source: Eurostat, Euro-Indicators Database (EUROIND), National Accounts: ESA 2010, Main GDP Aggregates, GDP and Main Components Table.


**U.K. 3-Month Treasury Rate**: Quarterly averages of monthly values. Source: Organisation for Economic Co-operation and Development, Main Economic Indicators Database. Available on the Federal Reserve Economic Database with Series ID: IR3TTS01GBM156N.

**U.S. VXO**: Expected volatility in the S&P 100 over the next 30 days at an annualized rate. We calculate a quarterly average of the daily observations. Source: Chicago Board Options Exchange, VIX Historical Price Data (“old methodology”).

**U.S. BOS**: Future general activity; percent of firms forecast a decrease (GAFDSA), an increase (GAFISA) and no change (GAFNSA). Source: Federal Reserve Bank of Philadelphia, Business Outlook Survey, revised monthly data.

**U.S. SPF**: Cross-sectional forecasts dispersion in Real GDP, Industrial Production, the GDP Price Index, and the 3-month T-Bill Rate. Source: Federal Reserve Bank of Philadelphia, Survey of Professional Forecasters, Dispersion measure D3 (D1 for the T-Bill Rate).

**ECB SPF**: Individual forecasts of future quarterly real GDP growth rates (year on year percentage change of real GDP). Source: ECB Survey of Professional Forecasters individual data.

**BOE SEF**: Individual forecasts of future quarterly real GDP growth rates (year on year percentage change of real GDP). Source: Bank of England (individual data not available online).