Non Competitive Factor Prices and the Dynamics of Labor Share

Sekyu Choi
University of Pennsylvania

José-Víctor Ríos-Rull *
University of Minnesota,
University of Pennsylvania,
Federal Reserve Bank of Minneapolis
CAERP, CEPR, NBER

February 2008

Abstract

In this paper we explore the extent to which a family of real business cycles models where wages are not set competitively tailored to replicate cyclical facts about the labor market, is capable of generating the observed dynamics of labor share as described in Ríos-Rull and Santaeulalia-Llopis (2007). We build upon Merz (1995), Andolfatto (1996), Cheron and Langot (2004), among others, who analyze models where wages are determined via Nash bargaining, employment lags productivity, and labor share falls with productivity innovations. While these models account for various business cycles properties, they fail in replicating the dynamic empirical response of the labor share to technological shocks. We provide some explanations for these results and point out directions for future research.

*Email: sechoi@econ.upenn.edu, vr0j@umn.edu. We are grateful for comments and suggestions from seminar participants at the University of Pennsylvania, the 2007 AEA Meetings in Chicago, IL., the University of Minnesota, Toulouse, Simon Fraser University, ... All errors are our own.
1 Introduction

Most business cycle research is performed using the notion that factor shares of income (capital and labor) are constant. This is usually achieved by assuming a Cobb-Douglas production function and marginal productivities as factor prices. We can trace this assumption choice from a generalized belief in Kaldor (1961)’s stylized facts about growth. However, these facts represent (at best) gross generalizations of reality and fail to hold under rigorous statistical scrutiny. As noted by Ríos-Rull and Santaeulalia-Llopis (2007), factor shares of income are not constant. They identify an *overshooting* property: the impulse response of bivariate vector autoregressions between the Solow residual and the labor share displays the latter variable falling after a contemporaneous positive shock in technology, but then labor share experiences a long-lasting and persistent increase in its value, peaking five years later at a level larger (in absolute terms) than the initial drop. When looked in detail the dynamic behavior of labor share is strongly influenced by the lagged and strong behavior of employment and by the persistent response of output.

In this paper we explore whether breaking one of the two assumptions that lead to constant factor shares, namely competitive factor pricing induces dynamics of the labor share like those observed. Specifically, we explore the extent to which a family of real business cycles models where wages are not set competitively tailored to replicate cyclical facts about the labor market, is capable of generating the observed dynamics of labor share as described in Ríos-Rull and Santaeulalia-Llopis (2007). We build upon Merz (1995), Andolfatto (1996), Cheron and Langot (2004), among others, who analyze models where factor prices are not set competitively, employment lags productivity, and labor share falls with productivity innovations.

In these models, frictions exist in the labor market and wages are non-competitive, in the tradition of the Mortensen-Pissarides search and matching model. More specifically, these models are characterized by labor markets where search frictions prevent the seamless allocation of workers to jobs; also, the non-competitive wage setting (usually, Nash Bargaining) introduces some rigidity in real compensations which in turn creates a wedge between wages and average labor productivity.

---

1See Pissarides (1990)
We see this model as a natural extension of standard business cycle models and a place where we can depart from constant factor shares of income by assuming non competitive wages in a transparent and intuitive way. Moreover, and as we will show below, the response of labor share to innovations in technology is hump shaped, driven in a big part by the hump shaped response of employment and total hours. This is not possible to replicate in models where labor inputs to production adjust instantaneously to the cycle, hence the need for some kind of frictions in the determination of aggregate employment. An additional benefit of taking the search and matching framework as a baseline is that it naturally replicates the fact that the immediate response of the labor share to productivity innovations is negative.\textsuperscript{2} Since labor share is comprised of the wage bill (real wages times labor input) over total output, the negative instantaneous response of the simulated labor share to positive innovations in technology is reproduced in the model because labor inputs are fixed (given the search frictions) and wages react sluggishly to the cycle (given Nash bargaining).

Our results show that this class of models fails in replicating the overshooting property of the labor share: unlike the data, responses in the model economy are short-lived. Even though the model presented here is tailored to mimic the behavior of labor share 5 to 10 periods after a shock in technology, it moves very little afterwards. In other words, the model reverses almost immediately to one that looks more like competitive pricing.

We calibrate our baseline model to average properties of the data, such as employment rates, fraction of GDP devoted to creating vacancies and the like. We also calibrate our model so that it matches the observed response of employment and the immediate response of labor share to productivity innovations. We do so not so much to ask how good a model is this of labor fluctuations but to see whether a version of this model capable of generating employment fluctuations generates the observed dynamics of labor share. As we have said, it does not.

There has been a recent controversy on the ability of the Mortensen-Pissarides search and matching type of models to replicate volatility of vacancies and unemployment as seen in US data. This discussion relates closely to the work presented here, since it addresses the same

\textsuperscript{2}This fact has been well documented in Merz (1995), Andolfatto (1996), Cheron and Langot (2004), Ríos-Rull and Santeulalia-Llopis (2007), among others.
margins we study.

The main point was raised by Shimer (2005), who performs a standard calibration of the Mortensen-Pissarides model and finds that it cannot replicate the volatility of vacancies and unemployment as seen in US data. Shimer (2005) blames Nash bargaining, arguing that equilibrium wages in that model are not “rigid” enough: productivity increases are followed closely by real wages, eroding profits and firms’ incentives to post vacancies during the cycle. This point is underscored by Hall (2005) who studies a model where rigid wages (in fact, wages in his paper are static) generates high volatility of vacancies and unemployment.

An alternative view is presented by Hagedorn and Manovskii (2008) who show that a different calibration of the model can deliver the facts: They point out that Shimer’s calibration (high bargaining power of workers and low utility from leisure) effectively kills incentives for vacancy posting: the surplus of the match\(^3\) is high (given low value of leisure), so increases in labor productivity increase the surplus by a proportional small amount. Moreover, much of these increases are taken by workers with high bargaining power. Hence, by setting a high value of leisure and a low bargaining power, Hagedorn and Manovskii (2008)’s version of the model produces accurate cyclical responses of vacancies and unemployment.

Our findings shed some light on the controversy, since our calibration strategy imposes the response of employment observed in the data while freeing the value of home production when not working. Our results confirm (in the context of the present model) that in order to generate large employment responses, the outside option value for the worker has to be quite large.\(^4\)

We do explore some alternative calibration strategies to see whether the additional discipline implied by the dynamics of labor share sheds some lights on the properties of calibration strategies. We find that changing the baseline calibration to an alternative in the spirit of Hagedorn and

---

\(^3\)In the Mortensen-Pissarides model, the static surplus of a match is labor productivity minus utility from not working for the workers. In steady state, the surplus also depends on the average duration of a match.

\(^4\)An important alternative finding is that of Nakajima (2007) who poses a slight variation of the model wherein workers who separate from a match, go to the pool of prospective hires immediately, and not on the next period as is traditional in the literature. When the model is posed like this, the value of home production implied solely by the utility of leisure is supplemented by a higher continuation value for the workers, which in turn decreases the value of each match. By the logic presented in Hagedorn and Manovskii (2008), this helps in creating the incentives for high vacancy posting.
Manovskii (2008) where we target an extremely low value of the Nash weight of the worker increases the cyclical response of vacancies and employment; nevertheless, increased responses of employment are matched with a lower-than-the-baseline response in real wages, leaving the response of the labor share close to what is seen in the baseline. The use of this alternative calibration strategy could be seen as a way of affecting the hiring margin of the model.

We also explore a variant of the baseline, designed to increase real wages after a positive technology shock and that we refer to as the "Garrison" preferences where we pose curvature in bodies making it increasing costly in utility terms to increase the fraction of the population working. This is an attempt to vary the wage margin in the model. However, again we find that we cannot generate accurate dynamic responses of labor share to productivity innovations, the reason being that while wages move more employment move less.

We take the inability of the model, under very distinct calibration strategies, to generate the observed dynamics of labor share as evidence that in non competitive wage setting models the hold up problems that create movements in the wedge between real wages and labor productivity fades too quickly: in the model, firms have rational expectations on what will happen with the cost of employment after a productivity shock and they can act upon that by modifying vacancy posting; hence, the model has embedded strong forces that nullify the effects of non competitive wages.

In addition, the failure of these models shows that the dynamics of the cyclical behavior of factor shares in income poses a strict discipline that may be used to discriminate between alternative classes of models. In this context, we think that a next step is to explore models with technologies that are not Cobb-Douglas, and that are susceptible to induce interesting dynamics: CES technologies, technologies with two types of labor according to experience and technologies with putty clay characteristics are natural candidates that we have started to explore.

This paper is not the first one in modeling an endogenous non-constant labor share⁵, (nor

the first one pointing out the necessity of understanding its cyclical behavior. However, it is (to the best of our knowledge) the first one trying to explain the overshooting property, or in other words, the medium-run frequency movements of endogenous labor market variables, identified as impulse response functions from data. In doing this, we take into account both relative correlations and levels of endogenous variables influenced by technological shocks; most of the existing literature focuses only on correlations (and only in the short-run) and dismisses information contained on impulse response functions.

The structure of the paper is as follows: the next section discusses the data, estimation procedures and facts about the labor share at quarterly frequency. Section 3 describes our baseline model. Section 4 discusses our calibration strategy while section 5 shows properties of the model economy under the proposed calibration. In section 6 we consider different calibration strategies in order to check for robustness; we then propose a simple deviation from the baseline economy, which clarifies the link between non-competitive wage setting through Nash-bargaining and the dynamics of employment, hours and the labor share. We conclude in section last section.

2 Cyclical Behavior of the Labor Share: The Facts

Here we briefly summarize the findings in Ríos-Rull and Santaeulalia-Llopis (2007). The facts we are interested in can be summarized in the following figures and tables that we have calculated from US data, from the first quarter of 1964:I to the last quarter of 2004. The facts can be summarized by

1. Labor share is quite volatile: its standard deviation is 42% that of output.

---

6For example, Gali and Gertler (1999) and Sbordone (2002) argue that the labor share is better suited than the output gap to estimate inflation dynamics in environments where staggered contracts and rigidities in wages are present, hence, the need to understand labor share dynamics in order to understand/predict trends in inflation.

7We use the labor share as constructed by Ríos-Rull and Santaeulalia-Llopis (2007). We use data from NIPA and BLS. Standard data and standard construction of variables...

8Taking logs allows the interpretation of its volatility as percentages.
2. It is countercyclical: its correlation with contemporaneous output is -0.13.

3. It is highly persistent: Autocorrelation of 0.78

4. It lags output by a year

5. It overshoots. When we estimate a vector autoregression (VAR) of order 1 (without constant) between the Solow residual and the labor share and then plot the impulse response (IR) function for the labor share, we get figure 2. The term ”overshooting” is due to the fact that, while the instantaneous response of labor share to a technology shock is negative, the overall effect is positive and long lasting.
Basically, this "hump-shaped" response of the labor share is due to the hump-shaped response of employment during an expansion. Hours per worker and wages have weaker responses. This is shown in figure 3, where each response function is calculated from a bivariate VAR of order 1, between the variable of interest and the solow residual.

We take these VARs as statistics from the data. In the results section, we will take simulated data and construct these same figures and statistics and compare them.

3 The model

Here we describe the model that we take as a baseline. As pointed out in the introduction, this model is appealing for the problem at hand because it has three main ingredients: non competitive wages, a role for frictions in employment and forces that create a countercyclical labor share.\footnote{See Pissarides (1990), Merz (1995), Andolfatto (1996) and Cheron and Langot (2004), among others.}

The environment can be described as follows. Time is discrete and goes on forever. There is...
Figure 3: Response of Labor Share components to a shock in the Solow residual

A continuum of identical and infinitely lived large families, or households, each of measure one. The household consumes \(c\), accumulates assets \(a\) which they rent to the firms and depreciates at rate \(\delta\) (in equilibrium, assets and capital are the same) and provides labor: there is a fraction \(n\) of individuals in the family, or household members, that is matched with firms or employed, the rest, \(1 - n\), is not employed but willing to be assigned to any open job. The amount of hours worked by those employed is determined in a decentralized way in the job.

Firms produce the unique good in the economy, using a constant returns to scale technology, subject to aggregate and persistent productivity shocks \(z\):

\[
y = e^z F(k, nh)
\]  

(1)

Inputs to this technology are capital \(k\) and labor in the form of workers, \(n\), times hours per worker, \(h\). There are search frictions in the labor market: workers and firms need to be matched. Over time there is attrition of matches and firms post vacancies, \(v\), that result in additional future matches according to a standard matching function that depends on the aggregate level of
vacancies and size of the non-working population, \( M = M(V, 1 - N) \). Following the literature, we assume that this function is homogenous of degree one in both arguments \((V \text{ and } 1 - N)\) and that there is an exogenous and constant separation rate \( \chi \) between jobs and workers. The search component of the model makes employment a predetermined state variable at the beginning of each period; also, its value evolves according to the matching function.

When solving their problem, both the household and the firm take as given the interest rate \((r)\), which is determined in a perfectly competitive capital market. On the other hand, the real wage \((w)\) and hours per worker \((h)\) are determined by a pairwise Nash bargaining game between firms and individuals.

The assumption of large families helps us distinguishing employment and hours per worker in equilibrium, as well as the employment status of individuals inside the family. The latter provides a simple framework to understand labor search and matching in this model, since we don’t have to distinguish between employed or unemployed families, but rather employed or unemployed individuals.

The aggregate state in this economy is the aggregate shock, the amount of capital and the fraction of the population matched to firms, \( S = \{z, K, N\} \). The household state is the aggregate state \( S \) and its own state variables which are its assets and the fraction of its members that have a job, i.e. that are attached to a firm, \( s_H = \{a, n\} \). Assets in this economy are both physical capital and firms. The latter have value because they are matched to workers by posing costly vacancies in exchange for future profits. Without loss of generality we close the stock market in this economy and post the dividends (not the rental income of capital) as an endowment of households. This simplifies dramatically the definition of equilibrium and avoids cumbersome notation. Consequently, the household solves the following recursive problem:
\[
V(S, s_H) = \max_{\{c, a\}} u(c) + n\nu(1 - h) + (1 - n)\nu(1) + \beta E [V(S', s_H')|z] \tag{2}
\]

s.t.
\[
c + a' = w(S, s_H)h(S, s_H)n + (1 + r - \delta)a + \pi(S) \tag{3}
\]
\[
n' = (1 - \chi)n + \Psi(S)(1 - n) \tag{4}
\]
\[
S' = G(S) \tag{5}
\]
\[
z' = \rho z + \epsilon', \quad \epsilon \sim iid(0, \sigma^2_{\epsilon}) \tag{6}
\]

where \(u(c)\) is utility of consumption, \(a\) are the assets of the household, \(\nu(\cdot)\) is the utility of leisure, \(\pi(S)\) are dividends from the firm to the households. We denote with primes next period’s values of variables. \(G(S)\) is the law of motion of aggregate variables, and \(\Psi(S)\) is the job-finding rate. This rate is derived from the usual matching function, i.e.,

\[
\Psi = \frac{M(V(S), 1 - N)}{(1 - n)} \tag{7}
\]

The household takes as given by functions \(w(S, s_H)\) and \(h(S, s_H)n\) the allocation of hours and the determination of wages. As we will see below they are set by decentralized bargaining.

Since we use separable utility between consumption and leisure, the intra-household consumption level doesn’t depend on employment status, that is, the household perfectly insures its non-working members. In equilibrium, this means that unemployed individuals are better off than employed ones: they receive the same consumption stream \(c\) and enjoy all the leisure, while the employed agents spend \(h\) working for the firms. This issue was discussed by Cheron and Langot (2004)\(^{10}\) but has no implications for our goal of replicating the cyclical properties of the factor shares.

Turning now to the problem of firms, they rent the capital from the households and post vacancies on the job market, which turn into matches with one period delay, an intrinsically dynamic

\(^{10}\)In their paper, they introduce non-separable utility between consumption ad leisure, a la Rogerson and Wright (1988)
problem so the firm considers the future paths of vacancies and unemployment in order to make its decisions today. The representative firm takes the aggregate and individual state variables, $S$ and $s_F = \{n\}$ respectively, and solves the following dynamic problem:

$$
\Omega(S, s_F) = \max_{\{v, k\}} y - w(S, s_F) h(S, s_F)n - rk - c_v v + E \left[ \tilde{R}(S') \Omega(S', s_F'|S) \right] \tag{8}
$$

s.t.

$$
y = e^z F(k, nh) \tag{9}
$$

$$
n' = (1 - \chi) n + \Phi(S) v \tag{10}
$$

$$
S' = G(S) \tag{11}
$$

$$
z' = \rho z + \epsilon', \quad \epsilon \sim iid(0, \sigma^2) \tag{12}
$$

where $c_v$ is the cost of posting a vacancy, $\Phi$ is the job-filling rate ($\Phi \equiv M/V$), $F$ is the production function, and $G$ is the law of motion of aggregate state variables that in equilibrium is determined by the actions of individual households. The firm also takes as given functions $w(S, s_F)$ and $h(S, s_F)n$ that determine the allocation of hours and the level of wages.

Note that, the discount factor for the firm is none other than the rate of return of the economy which in equilibrium is given by the standard FOC using aggregate variables

$$
\tilde{R}' \equiv \beta \frac{u_c[C(S')]}{u_c[C(S)]}. \tag{13}
$$

where $u_c(.)$ is the marginal utility of consumption for the households.

From the problem of the firm, we can derive the first order condition for vacancies

$$
c_v = \beta \Phi(S) E \left\{ \frac{u_c[C(S')]}{u_c[C(S)]} \Omega_n(S', n')|S \right\} \tag{14}
$$

where $\Omega_n$ (and now we start omitting arguments to avoid exhausting notation) is the value of an
additional worker to the firm, i.e.

$$\Omega_n = \frac{\partial y}{\partial n} - w\, h + (1 - \chi)\, \beta\, E \left[ \frac{u'_c}{u_c} \Omega'_n \right]$$  \hspace{1cm} (15)$$

Equations (14) and (15) say that the firm posts vacancies until the (constant) marginal cost of the vacancy equates the probability of getting a new worker times the marginal benefit of that additional worker.

As stated before, in this model wages are not equal to the marginal productivity of labor, but are determined by the outcome of a bilateral Nash bargaining game between the firm and the individual.\superscript{11} As opposed to the standard Pissarides (1990) model where the outside option for the workers is a fixed parameter, in this model the value of not engaging in production for a worker is determined endogenously; the outside option for the worker is related to the extra leisure that unemployed individuals enjoy (remember that their consumption is insured by the family unit) and the option value of being matched to another job next period.

Specifically, the setup of the bargaining game has the following components. First, the value of an additional worker for the household ($\equiv \frac{\partial V}{\partial n}$) is given by

$$V_n = u_c\, w\, h - \nu(1) + \nu(1 - h) + (1 - \chi - \Psi)\, \beta\, E [V'_n]$$  \hspace{1cm} (16)$$

and the value of an additional worker for the firm ($\equiv \frac{\partial \Omega}{\partial n}$) is given by

$$\Omega_n = y_{nh}\, h - w\, h + (1 - \chi)\beta E \left[ \frac{u'_c}{u_c} \Omega'_n \right]$$  \hspace{1cm} (17)$$

where $y_{nh} = \frac{\partial y}{\partial (nh)}$ denotes the output produced by someone who works $h$ hours. This is an important assumption, that goes to the nature of the bargaining protocol.

With these elements, we can define the axiomatic Nash bargaining problem for which the\superscript{12}This is important, specially if the production function is Cobb-Douglas, since competitive pricing of the factors under that particular production function gives rise to constant factor shares.
outcome determines both the wage and the hours per worker:

\[(w, h) = \arg \max_{w, h} \left( \frac{V_n}{u_c} \right)^\mu \left( \Omega_n \right)^{1-\mu} \tag{18} \]

where \(\mu\) is the bargaining power of the worker. Note that the value of the marginal worker for the household \(V_n\) is multiplied by \(u_c\) (the marginal utility from consumption) in order to transform everything into units of the consumption good.

Taking the derivatives with respect to real wages and hours, we get two conditions: the first one is the "sharing rule" of production surplus and the second is a static condition for determining the length of the workweek,

\[
\begin{align*}
\mu u_c V_n &= (1 - \mu) W_n \tag{19} \\
u_c y_nh &= \nu h (1 - h) \tag{20}
\end{align*}
\]

To solve for the equilibrium wage, we have to use the first order condition for vacancies (14) as well as the sharing rule (19) to get

\[
w h = \mu \left[ \frac{\partial y}{\partial n} + c_v \frac{v}{1-v} \right] + (1 - \mu) \left[ \frac{\nu(1) - \nu(1-h)}{u_c} \right] \tag{21}
\]

This is an analog to the wage equation derived by Pissarides (1990) in the simpler setup where productivity and the outside option of the worker are constants. In words, the wage bill is a weighted average of (i) the marginal productivity of the worker plus the average savings in vacancy postings per unemployed individuals and (ii) the outside option of the worker, which in this case is simply the forfeited leisure incurred by the individual who works \(h\) hours at the firm.

The bargaining protocol that we have posed implicitly assumes that in case that the negotiations break down between any specific worker and firm, the negotiations of the firm (and the worker) with other workers and (firms) are unaffected. This is an extreme assumption and an alternative has been posed nicely by Stole and Zwiebel (1996) where firms and workers internalize that any breakdown of negotiations affects negatively the position of the firm with other workers.
Unfortunately, the simultaneous determination of hours with this alternative bargaining protocol seems intractable. To finalize we should impose the equilibrium conditions that both households and firms are representative. We omit them since they are very well known.

4 Calibration of the Model

The calibration process consists of selecting functional forms, specifying parameter values and the targets that the model economy has to satisfy that restrict the values of those parameters. Of course, the number of targets has to be at least that of parameters.

4.1 Functional Forms and Parameters

A model period is taken to be one quarter. The production function is Cobb-Douglas with exponential depreciation. The utility function is separable in consumption (where we use log utility) and leisure, with

\[
\nu(\ell) = \begin{cases} 
\gamma^{-\eta}_{u} \frac{\ell^{1-\eta}}{1-\eta} & \text{if } \ell \in [0, 1) \\
\gamma_{u} & \text{if } \ell = 1
\end{cases}
\] (22)

We follow Andolfatto (1996) and introduce a differentiated parameterization for the leisure in the household, where \(\gamma_{u}\) is a constant. The different values for leisure of the employed versus the non-employed can be interpreted as differential efficiency in home production given the labor force status\(^{12}\), commuting time or search costs. In practical terms, this extra parameter allows for more flexibility in the calibration of the model since it can be set independently from the leisure of the employed. \([[[\text{The additional flexibility becomes quite handy when noticing that it allows for accommodating the large lag implied by the model for workers that have been just separated from a match, in joining the set of prospective hires (12 weeks) and the large group of non-workers in this economy. As stated before, Nakajima (2007) uses a disciplined value of home production in an environment}}\)

\(^{12}\text{See Andolfatto (1996), page 115.}\)
where if a worker breaks up with a firm, it becomes immediately eligible to search for another job.]

The matching function has constant returns to scale, and it is also assumed to be of the Cobb-Douglas form\textsuperscript{13}.

\[
M(V, 1 - N) = \omega V^\psi (1 - N)^{1-\psi}
\]

(23)

Given these expressions, the equilibrium of the model is characterized by the following system of nonlinear equations:

\[
\begin{align*}
Y &= e^z K^\theta (Nh)^{1-\theta} \\
N' &= (1 - \chi)N + \omega V^\psi (1 - N)^{1-\psi} \\
K' &= (1 - \delta)K + I \\
Y &= I + C + c_v V \\
\Phi &= \omega V^{\psi-1} (1 - N)^{1-\psi} \\
1 &= \beta E \left[ \frac{C}{C'} \left( 1 - \delta + \theta \frac{Y'}{K'} \right) \right] \\
\frac{c_v}{\Phi} &= \beta E \left[ \frac{C}{C'} \left( (1 - \theta) \frac{Y'}{N'} - w' h' + (1 - \chi) \frac{c_v}{\Phi} \right) \right] \\
(1 - \theta) \frac{Y}{Nh} &= C \gamma (1 - h)^{-\eta} \\
wh &= \mu \left[ (1 - \theta) \frac{Y}{N} + c_v \frac{V}{1 - N} \right] + (1 - \mu) C \left[ \gamma_u - \gamma \frac{(1 - h)^{1-\eta}}{1 - \eta} \right] \\
z' &= \rho z + \epsilon'
\end{align*}
\]

Given any parametrization, we solve this model with standard local approximations. Specifically, we use the very convenient Dynare package (Thanks to the authors).

There are 13 parameters in this model that we have to specify. There are preference, matching technology and bargaining parameters.

\textsuperscript{13}See Shimer (2005) and Blanchard and Diamond (1989)
4.1.1 Preference parameters

1. $\beta$ Discount factor.

2. $\gamma$ Multiplicative coefficient of leisure.

3. $\gamma_u$ Utility level of unemployment.

4. $\eta$ Exponential Coefficient of leisure.

4.1.2 Production parameters

5. $\theta$ Coefficient of capital in the production function. Under competitive factor prices, $1 - \theta$ is labor share. This is not the case in this model.

6. $\delta$ Depreciation rate.

7. $\sigma^2$ Standard deviation of the innovation to the productivity shocks.

8. $\rho$ Autocorrelation of the productivity shocks.

4.1.3 Matching Technology

9. $c_v$ Cost of posing a vacancy.

10. $\omega$ General level of job creation.

11. $\psi$ Coefficient of vacancies in the matching function.

12. $\chi$ Job destruction rate.

4.1.4 Bargaining protocol

13. $\mu$ The weight of the firm in the bargaining process.
4.2 Calibration Targets

We turn now to specify the targets that we chose to determine the values of these parameters. Unlike traditional work in business cycle research we pose, not only targets that are first moments, but also targets that are second moments. We defend this choice below.

4.2.1 Household Targets

1. Annual rate of return of the economy, 4%.

2. Steady state hours per worker, 433 per quarter (1733 per year, about 1/3 of total time).

3. Frisch elasticity of hours for those that work, .5.

4. Steady state employment rate, 75%. Depending on the definition of employment and unemployment, different authors target different values for this variable. Andolfatto (1996), Merz (1995) and Cheron and Langot (2004), among others, take 0.57 as their target; Shimer (2005), Hagedorn and Manovskii (2008) use a much higher number (around 0.94). Our number is on the very high end of the historical employment rate. We follow in this regard Kydland and Prescott (1991) and Osuna and Ríos-Rull (2003). Although the simulated volatility of vacancies and unemployment are sensitive to this target, the responses of labor share to technology are not, so we settle on an average employment rate.\textsuperscript{14}

4.2.2 Production Targets

5. Consumption to output ratio, 75%.

6. Measured labor share of output, 67%.

7. Standard deviation of measured Solow residual, .64%.

8. Autocorrelation of measured Solow residual, .93.\textsuperscript{15}

\textsuperscript{14}Alternatively we could have targeted the duration of unemployment or the transition of unemployment to employment (as Andolfatto (1996) and Cheron and Langot (2004) do).

\textsuperscript{15}This targets arise from our estimation of a VAR(1) between labor share and productivity, as described in Ríos-Rull and Santaeulalia-Llopis (2007).
4.2.3 Employment Turnover Targets

For the targets below, we follow Andolfatto (1996), Abowd and Kramarz (1998) and Cheron and Langot (2004).

9. Separation rate, 15% per quarter.

10. Vacancy expenditures to output ratio, .5%.

11. Job filling rate in the steady state of 0.9.

4.2.4 Business Cycle Targets

The targets listed so far are steady state targets (except for the implied process for the Solow residual). For the last two targets, we impose two business cycle targets:

13. The immediate response of labor share to a productivity shock, $-0.1263\%$.

14. The size of the response in employment to a productivity shock, $0.4326\%$.

We compute these last two targets (both with real and model simulated data) by way of estimating vector autorregresions between the technological shock and log-detrended variables (construction procedures described in Ríos-Rull and Santaeulalia-Llopis (2007)). We identify the immediate response of labor share to a productivity shock as the first element of the impulse response function of labor share in the VAR; the size of the response in employment is taken as the maximum value of the corresponding impulse response function. We choose these targets since they provide information on the workers’ bargaining weight and the elasticity of the matching function to the number of vacancies, respectively.

Our calibration strategy entails looking for parameter values in order to match all of the targets above. There is a number of parameters that we can set in advance: $\{\beta, \delta, \chi, \eta\}$ while for the rest, we solve a system of 11 non-linear equations (model targets) in 11 unknowns (model
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.990</td>
</tr>
<tr>
<td>$\delta$</td>
<td>dep. rate</td>
<td>0.012</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive parameter of tech. shock</td>
<td>0.930</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>Std. dev. of tech. shock</td>
<td>0.0064</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Share of Capital in Cobb-Douglas</td>
<td>0.327</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Separation rate</td>
<td>0.150</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of matching fnc. wrt vacancies</td>
<td>0.669</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Scale parameter in matching function</td>
<td>0.718</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Curvature parameter in utility of leisure</td>
<td>4.000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Scale parameter in utility of leisure</td>
<td>0.663</td>
</tr>
<tr>
<td>$\gamma_u$</td>
<td>Leisure utility of unemployed</td>
<td>0.326</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Cost of posting a vacancy</td>
<td>0.037</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Bargaining power of workers</td>
<td>0.410</td>
</tr>
</tbody>
</table>

Table 2: Parameter values

parameters). We hit all steady state targets almost perfectly, while we approximate the business cycle ones very closely.\textsuperscript{16}

4.3 Properties of the Calibrated Economy

It is useful to comment on the parameters obtained in the calibration process which are $\rho \ \sigma_\epsilon \ \theta \ \psi \ \omega \ \gamma \ \gamma_u \ c_v \ \mu$. The properties of the shock while not identical are quite similar to the ones that result from a direct estimation of the Solow residual. The value of leisure is very high, it is so high in fact that there is no way to describe its value in terms of commuting costs ((for those who work)). We can however describe its value in terms of the consumption value of the

\textsuperscript{16}The immediate response of labor share is $-0.106\% \ (-0.1263\% \text{ in the data})$, and the size of the response in employment is $0.4258\% \ (0.4326\% \text{ in the data})$. 

Sekyu Review this

[[...]]
discontinuity that it implies by solving the following equation for $x$

$$u(c^*) + \gamma \lim_{\ell \to 1} \frac{\ell^{1-\eta}}{1-\eta} = u(c^* + x) + \gamma_u.$$  

(24)

where $c^*$ is the steady state value of consumption. The implied value turns out to be about 90% of $c^*$.

Another important feature of the calibrated parameters is that the Nash weight of workers is .41, on the lower end of those in the literature. This value is right between those used in the Shimer (2005) vs. Hagedorn and Manovskii (2008) debate. The former pairs a high weight for workers (0.72) with a low value of home production or unemployment insurance; the latter pair a high value of home production with a very low value of workers’ weight (0.052). Hence, our calibration lays in between these two opposites: we have a very high value for the utility of the non-employed and a bargaining weight for workers that is a midpoint of the values used by Shimer (2005) and Hagedorn and Manovskii (2008).

Compare the values used in our environment (where upon the breakdown of a match a worker has to stay out of the workforce for 3 months and afterwards joins the 25% of the population that is willing to be matched) with those in Nakajima (2007), who poses that when bargaining breaks, workers can join the rank of employable within the same period. He models utility in the household without any sort of discontinuities with respect to leisure\footnote{Nakajima (2007) has non-separable utility between consumption and leisure, so equation (24) looks differently in his setup:}

$$\lim_{\ell \to 1} u(c^*, \ell) = u(c^* + x, 1)$$

Since there is no form of discontinuity between $u(\cdot, \ell)$ and $u(\cdot, 1)$, $x = 0$. 

20
to sit-out one period after a match breaks, reduces the surplus of each match by increasing the continuation value of the worker.

5 Business Cycle Behavior of the Baseline Model Economy

We solve the model numerically by local approximation of the non-linear system of equations presented above and simulate paths for all endogenous variables. We then calculate statistics in the same manner for both real and model simulated data. Table 3 compares the cyclical properties of the model against quarterly US data, while figure 5 compares impulse response functions. Table 3 shows that the model replicates well the main features of the US business cycle: standard deviation of output is closely replicated, as well as the cross-correlation of output with all endogenous variables in the table. This is not surprising as we targeted the volatility of employment and as the model has built in the propagation mechanism of a delayed response in employment.

As in Merz (1995), Andolfatto (1996) and Cheron and Langot (2004), key cyclical facts about the labor market are captured in the baseline: total hours moves more than in a standard RBC model, but still less than in the data; employment and hours per worker are pro-cyclical, with employment lagging the cycle slightly by one quarter; labor share is countercyclical, again as targeted.
### Table 3: Cyclical Properties of U.S. Economy (1964:I, 2004:IV) and Model

|                  | $\sigma$ | $\sigma(x)/\sigma(y)$ | x(-5) | x(-4) | x(-3) | x(-2) | x(-1) | x | x(1) | x(2) | x(3) | x(4) | x(5) |
|------------------|----------|------------------------|-------|-------|-------|-------|-------|---|-----|-----|-----|-----|-----|-----|
| **US Data**      |          |                        |       |       |       |       |       |   |     |     |     |     |     |     |
| output           | 1.56     | 1.00                   | 0.03  | 0.24  | 0.46  | 0.68  | 0.86  | 1.00| 0.86| 0.68| 0.46| 0.24| 0.03|
| tot. hours       | 1.52     | 0.97                   | -0.14 | 0.04  | 0.25  | 0.48  | 0.71  | 0.89| 0.91| 0.83| 0.69| 0.49| 0.30|
| Employment       | 1.28     | 0.82                   | -0.22 | -0.05 | 0.15  | 0.38  | 0.62  | 0.82| 0.90| 0.88| 0.79| 0.64| 0.45|
| hours            | 0.43     | 0.27                   | 0.14  | 0.29  | 0.42  | 0.57  | 0.68  | 0.70| 0.55| 0.29| 0.08| -0.15| -0.29|
| wages            | 0.79     | 0.51                   | 0.11  | 0.16  | 0.20  | 0.22  | 0.18  | 0.17| 0.05| -0.01| -0.09| -0.09| -0.13|
| labor’s share    | 0.66     | 0.42                   | -0.27 | -0.29 | -0.29 | -0.25 | -0.20 | -0.13| 0.10| 0.27| 0.39| 0.45| 0.45|
|                  |          |                        |       |       |       |       |       |   |     |     |     |     |     |     |
| consumption      | 1.24     | 0.79                   | 0.24  | 0.45  | 0.62  | 0.77  | 0.86  | 0.85| 0.67| 0.48| 0.27| 0.06| -0.10|
| investment       | 7.09     | 4.53                   | 0.12  | 0.28  | 0.45  | 0.62  | 0.78  | 0.90| 0.77| 0.56| 0.32| 0.08| -0.16|
| L. prod.         | 0.74     | 0.47                   | 0.36  | 0.44  | 0.47  | 0.45  | 0.36  | 0.30| -0.04| -0.25| -0.44| -0.50| -0.54|
| **Model**        |          |                        |       |       |       |       |       |   |     |     |     |     |     |     |
| output (y)       | 1.48     | 1.00                   | -0.01 | 0.14  | 0.34  | 0.58  | 0.85  | 1.00| 0.85| 0.58| 0.34| 0.14| -0.01|
| tot. hours       | 1.06     | 0.72                   | 0.00  | 0.14  | 0.31  | 0.52  | 0.75  | 0.95| 0.94| 0.66| 0.38| 0.14| -0.04|
| Employment       | 1.02     | 0.69                   | -0.03 | 0.09  | 0.24  | 0.44  | 0.66  | 0.89| 0.97| 0.71| 0.43| 0.19| 0.00|
| hours            | 0.19     | 0.13                   | 0.18  | 0.29  | 0.43  | 0.57  | 0.68  | 0.56| 0.05| -0.11| -0.17| -0.21| -0.22|
| wages            | 0.46     | 0.31                   | -0.11 | 0.05  | 0.26  | 0.52  | 0.79  | 0.89| 0.59| 0.43| 0.31| 0.21| 0.12|
| labor’s share    | 0.15     | 0.10                   | -0.18 | -0.27 | -0.38 | -0.49 | -0.57 | -0.41| 0.12| 0.23| 0.25| 0.24| 0.22|
|                  |          |                        |       |       |       |       |       |   |     |     |     |     |     |     |
| consumption      | 0.30     | 0.21                   | -0.29 | -0.14 | 0.07  | 0.34  | 0.65  | 0.86| 0.77| 0.67| 0.56| 0.44| 0.33|
| investment       | 7.26     | 4.92                   | 0.04  | 0.19  | 0.38  | 0.60  | 0.85  | 0.99| 0.84| 0.56| 0.30| 0.09| -0.08|
| L. prod. (y/Nh)  | 0.57     | 0.39                   | -0.04 | 0.11  | 0.31  | 0.55  | 0.79  | 0.82| 0.44| 0.29| 0.18| 0.10| 0.03|
The main finding of this paper is shown in Figure 4. Labor share is flat after four periods and most of the initial drop targeted in the calibration has disappeared after one period. Moreover, the response of labor share never changes sign let alone moves in the opposite direction in the amount that it does in the data.

[[The four panels of figure 5 show the complete story. From 5(a), we see that we hit very closely our target of employment response to a technological shock, although the timing is not perfect. The model succeeds in hitting the high levels of employment fluctuations, with a calibration that yields a relatively low value of the worker’s Nash bargaining weight. This is partly due to a low target for employment, as pointed out by Costain and Reiter (2005): we use a target of 0.75, which is above the historical target of 0.57 used by Andolfatto (1996) and Cheron and Langot (2004), but below the one close to 0.95 used more recently by Shimer (2005) and Hagedorn and Manovskii (2008). This is also partly due to the implied high value of home production yielded by parameter $\gamma_{u^*}$]]
Even though employment creation might be sensitive to the choice of this target, the ability of the model to produce "overshooting" of the labor share is not, as it will be clear below.

In 5(c) and 5(d), we see that wages respond similarly to its value in the data but hours do not. In particular, hours in the model respond immediately because they substitute for bodies that cannot be increased. Once bodies are available, hours go back to the steady state level.

Finally, 5(b), shows the response of output. As in the data, the baseline shows a humped-shaped response, but with higher values at the beginning of the time period, due mainly to the difference between responses in the data and the baseline of hours per worker.

The failure of the baseline in replicating the overshooting of the labor share can be attributed to a mix of failed model responses: employment and wages respond less during the medium run (10 to 40 periods after the initial shock) but the main culprit seems to be hours per worker.

6 Robustness and Extensions

We study the extent to which our results are affected by our particular calibration. We recalibrate the model in the spirit of Hagedorn and Manovskii (2008) and compare the implied model responses. Then, we take a small departure from the baseline economy, in order to clarify the possibility of increasing the response of labor share by varying the margins of the wage setting protocol.

6.1 A Different Calibration

The main point argued by Hagedorn and Manovskii (2008) with respect to the calibration of the model, is that in order to increase the ability of the search and matching framework to propagate technological shocks, incentives for the firm to post costly vacancies have to be significant. This implies a calibration where firms have small accountable profits but high bargaining position. Hence, when there is a positive technological innovation, the percentage of increase in profits is large for the firms; moreover, firms want to post vacancies since they have the better bargaining position. By altering this margin in the model, we want to see if we can achieve higher responses
Figure 5: Response to a Technological Shock: Data and Baseline
We achieve this alternative parameterization of the model by setting the bargaining power of workers $\mu$ to 0.05 which is the value used by Hagedorn and Manovskii (2008). We then recalibrate the rest of the parameters by using the steady state targets described in section 4.2; in order to maintain the spirit of the suggested calibration, we replace the business cycle targets with a target for the firms profits of 0.03.\(^\text{18}\). The result of this calibration is presented in table 4. \([\text{Notice that now the value of home production while working increases further although by a lot less than in the original discussion between those authors which may be due in part to the fact that we started with a relatively low bargaining power of workers.}]\]

Comparisons in model responses are shown in figures 6 and 7. As seen in figure 6, the use of this alternative calibration also fails in creating a positive response of the labor share to a technology shock. Notice that since the initial drop in labor share is not a target under the Hagedorn–Manovskii calibration, the model overreacts in this dimension (compared to the baseline), but again, it goes back to its long run average pretty fast.

From figure 7 and its panels, we see that this alternative parameterization affects mostly employment and real wages, leaving hours per worker and output fairly untouched: Indeed, employment has a stronger response as opposed to the baseline, while real wages are more

---

\(^\text{18}\)This target is used by Shimer (2005) and Hagedorn and Manovskii (2008)
sluggish under the new calibration. Combined, these responses leave the labor share as in the baseline. Hence, although the insight provided by Hagedorn and Manovskii (2008) (namely, that a different calibration alone is able to create the incentives for the firm to create more employment during the cycle) also works in this model, it doesn’t affect our goal of matching the dynamics of the labor share.

6.2 The ”Garrison” Effect

In this section we present a small deviation from the baseline model in order to understand better the reason for its inability in replicating the overshooting property of the labor share. In the previous section we analyzed whether a different calibration strategy would deliver. Here we go one step further and force some particular mechanisms on the model. Below we describe this mechanisms, explain how we recalibrate the model and show its performance in terms of impulse response functions.

Recall that the instantaneous utility of the household was given by

\[ U = U(c) + N\nu(1 - h) + (1 - N)\nu(1) \]
Figure 7: Response to a Technological Shock: Data and Baseline with Hagedorn and Manovskii (2008) Calibration
Consider the following alternative:

$$U_g = U(c) + N\nu(1 - h) + (1 - N)^\kappa\nu(1)$$

where we denote $\kappa \in [0, 1)$ as the "Garrison" effect, since it captures the notion that having $N$ going to 1 (most members of the household working), might be increasingly costly in terms of leisure, say, because of increasing returns to scale in household production.

The "Garrison" effect increases the in-household (non-working) option value for the workers, hence affecting the wage bill. Denote $\tilde{b}$ as follows

$$\tilde{b} \left( \equiv -\frac{\partial U}{\partial N} \right) = \nu(1) - \nu(1 - h)$$

With Garrison, we have that

$$\tilde{b}_g \left( \equiv -\frac{\partial U_g}{\partial N} \right) = \kappa(1 - N)^{\kappa-1}\nu(1) - \nu(1 - h)$$

hence, with $\kappa < 1$ the reluctance to work by unemployed increases with $N$. [[Basically, through the "Garrison" effect, we are forcing the firm to pay higher wages during an expansion following a positive technology shock since the outside option value for the worker improves more than in the baseline in an expansion]].

Given the exercise nature of this extension to the baseline, we solve the model for different values of $\kappa$ instead of trying to calibrate it to some data target. We let $\kappa \in \{0.25, 0.50, 0.75\}$ and recalibrate the rest of the parameters in the model as described in section 4. The resulting parameter values are in table 5.

To get a better idea of the margins involved, in the next figures we plot model responses of the option values for the workers under the parameterization when $\kappa = 0.5$. We label these option values as *in-firm* and *in-household* because they represent the values of either working for the firm or staying at home enjoying leisure. They also represent the components of the wage bill, as calculated in equation 21.
In figure 9, panel (a) shows the response to technology of the in-firm value:

$$\frac{\partial y}{\partial n} + c_v \frac{v}{1 - n}$$

while panel (b) shows the response of the in-household value:

$$\tilde{b}_g \equiv \frac{\kappa^{-1} \nu(1) - \nu(1 - h)}{u_c}$$

The last panel in figure 9 shows the impulse response function of the labor share to a technology shock, (while the first two panels show the response of employment and wages. The calibration is not entirely successful. Maybe, a conflict of steady state targets given the presence of \( \kappa \) prevents us from hitting all rbc targets)). As seen in the figure, the introduction of this tweak in the model doesn’t change the fact that the response of the labor share fades too quickly when compared to the data. The more telling story is in panel (b) of figure 9, where the response of the in-household option value is depicted: through the Garrison preferences, we are forcing this value to be higher than the baseline throughout the transition back to the steady state. Nevertheless, we have two mechanisms that work against a positive response of labor share to technology. First, the same linkage between average labor productivity and real wages (through Nash bargaining) depresses employment in the face of higher wages. This is the same insight provided by Shimer (2005). In other words, although we force the model to produce real wages that are higher than the baseline, firms react by posting less vacancies and creating less
Second, 9(b) shows that the effect of this artificial wedge is short-lived, given the convergence of the responses by the baseline and Garrison models just after 15 periods.

[However, again we find that we cannot generate accurate dynamic responses of labor share to productivity innovations, the reason being that while wages move more employment move less.]

7 Conclusion

In this paper we explore the extent to which models tailored to replicate cyclical facts about the labor market, are capable of generating the observed dynamics of labor share as described in Ríos-Rull and Santaeulalia-Llopis (2007). We build upon Merz (1995), Andolfatto (1996), Cheron and Langot (2004), among others, who analyze models where factor prices are not set competitively, employment lags productivity, and labor share falls with productivity innovations.
Figure 9: Response to a Technological Shock: Option Values for Workers
Our results show that the search and matching framework, along with non competitive wage setting fail in replicating empirical responses of the labor share to technological shocks: unlike in the data, responses by the model labor share are short-lived and fail to "overshoot". This negative result is robust to different calibration strategies and is linked to the fact that the wage setting protocol used in these type of models (Nash bargaining) is not able to create a persistent wedge between real wages and labor productivity.

Moreover, the failure of these models shows that the dynamics of the cyclical behavior of factor shares in income poses a strict discipline that may be used to discriminate between alternative classes of models. In this context, we think that a next step is to explore models with technologies other than Cobb-Douglas.
References


