Fixed-Term and Permanent Labor Contracts as Bargaining Outcomes: Theory and Evidence
(PRELIMINARY AND INCOMPLETE)∗

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Abstract

We construct and estimate a theory in which employers choose to offer workers contracts with different firing conditions. In a labor market characterized by search and matching frictions, firms find optimal to offer some workers a contract in which firing is costly - we label this a permanent contract, while offering other workers the alternative - temporary - contract in which dismissal is free. The trade-off a firm faces is to offer a temporary contract to a worker with a good match-specific value at the risk of losing the worker if she finds a better match; or offering a permanent contract when firing is costly to a worker with a low match-specific value. We analytically characterize a cut-off rule which determines the type of contract offered. We use matched employer-employee data from Canada to structurally estimate the model and decompose the wage distribution into search and matching frictions, workers’ and firms’ characteristics, and types of contracts signed.

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1 Introduction

The existence of two-tiered labor markets in which workers are segmented by the degree of job protection they enjoy, is typical in many OECD countries. Some workers, which one could label temporary workers, enjoy little or no protection. Meanwhile, other workers enjoy positions where at dismissal the employer faces a firing tax or a statutory severance payment.

This paper asks the following question. In an environment featuring ex-ante identical workers and firms, what would lead some worker-firm pair meetings to materialize in a “temporary” contract and others in a “permanent” one? Intuitively firms should always opt for offering workers the contract in which dismissal is free, not to have their hands tied in case the worker under-performs. Our answer is based on different contracts resulting from differences in match-quality revealed at the meeting, and we provide and test a theory in which firms discriminate against low-quality matches. Match quality is represented by a component of a worker’s productivity that remains fixed as long as the firm and the worker do not separate. Firms offer not-so-good workers a low-wage contract that can be terminated at no cost after one period. If it is not terminated, the firm agrees to upgrade the contract into a permanent one, which features a higher wage and it is relatively protected by a firing tax. Firms find optimal to offer high-quality matches a permanent contract because temporary workers search on the job. Facing the risk of losing a good worker, the firm ties its hands promising to pay the tax in case of termination and remunerating the worker with a higher wage. Endogenous destruction of matches, both permanent and temporary, results from a change in the time-varying component of a worker’s productivity: if this change is negative enough, it will force the firm to end the relationship.

Our set-up is simple enough to allow us to characterize three cut-off rules. First, a cut-off point in the distribution of match-specific shocks above which, the firm offers a permanent contract and below which, the firm offers a temporary contract. A second
cut-off point in the distribution of the time-varying component of productivity below which, the relationship between a temporary worker and a firm ends, and above which it continues. Finally, a third cut-off point also in the distribution of the time-varying component of productivity below which, the relationship between a permanent worker and a firm ends, and above which it continues.

Naturally, workers stay longer in jobs for which they constitute a good-match. Permanent workers enjoy stability and higher pay. Temporary workers on the other hand experience high job-to-job transition rates in lower-paid jobs, while they search for better opportunities. We emphasize that our theory delivers all of these results endogenously.

We use matched employer-employee data from Canada to structurally estimate the model. We make parametric assumptions on the technologies for producing goods and to match workers and firms, and on the distributions for the two types of shocks. We estimate the wage equations and the production function by GMM. The model delivers a predicted wage distribution that is affected by three components: worker and firm heterogeneity, search and matching frictions, and different types of contracts.

To the best of our knowledge, the literature lacks a pure theory of the existence of two-tiered labor markets in which some workers hold temporary contracts while others hold permanent contracts. Our study is clearly not the first one that analyzes this question within a theoretical framework, so by pure theory we mean not assuming an ex-ante segmentation of a labor market into temporary workers or permanent workers. This segmentation can occur for a variety of reasons: related to technology (e.g. assuming that workers under different contracts are different factors in the production function); due to preferences - assuming that workers value being under a permanent contract differently than being under a temporary contract), or that they are subject to different market frictions. There are several examples which feature such an assumption: Wasmer (1999), Alonso-Borrego, Galdon-Sanchez, and Fernandez Villaverde (2006), or Bentolila and Saint-Paul (1992). Blanchard and Landier (2002) take a slightly different route, associating temporary contracts with entry-level positions: a worker begins a relationship.
with a firm in a job with a low level of productivity. After some time, the worker reveals her true - perpetual - productivity level. If such level is high enough, the firm will retain the worker offering her a contract with job security. Our model is somewhat closer to Cahuc and Postel-Vinay (2002) but again with a key difference: in the theory we present the fraction of temporary workers in the workforce is endogenous, while in their study is a parameter. It is intuitive that changes in the firing costs affect the mix of temporary versus permanent worker, and for this reason we argue it should be a necessary ingredient in any model that analyzes policies in two-tiered labor markets.

There is a related branch of the literature that looks at the effect of increasing firing taxes on job creation, job destruction and productivity. An example is Hopenhayn and Rogerson (1993). They find large welfare losses of labor protection policies as they interfere with labor reallocation from high productivity firms to low productivity firms. Other examples would be Bentolila and Bertola (1990) or Alvarez and Veracierto(2000,2006).

2 Economic Environment

Our main departure from the standard Mortensen and Pissarides (1994) framework is the coexistence of two types of labor contracts. The first type of contract is a temporary contract, which lasts for one period and it is costless to rescind. However, at its expiration this temporary contract can be converted to a permanent one by paying a small cost \( c \). The permanent contract has no pre-determined length and it is costly to rescind: firms must face a cost \( f \) if they dismiss a worker under a permanent contract. Firms pay this cost to the government who redistributes it among individuals through a lump-sum transfer \( \tau \). The production technology is the same for the two types of contracts. If a firm hires a worker it produces \( y_t + z_i \) units of output, where \( y \) is an aggregate time-varying productivity shock which drives endogenous separations and \( z \) is a match-specific shock that will allow for different contracts being offered. The supports of the distributions of both types of shocks are given by \([y_{\text{min}}, y_{\text{max}}]\) and \([z_{\text{min}}, z_{\text{max}}]\).

The technology that matches searching workers and firms displays constant returns to
scale and results in a job-finding probability $\alpha^u(\theta)$ and a vacancy-filling probability $\alpha^f(\theta)$ defined as functions of market tightness $\theta$. The latter is defined as the ratio of the number of vacancies to the sum of workers under temporary contracts plus those unemployed. Firms must pay a cost $k$ per vacancy posted, irrespective of the type of contract the matched worker will work under. If a worker begins with a firm as a temporary she can be promoted to a permanent contract. However, these temporary workers are allowed to search before the firm’s retention decision. Consequently, firms face a trade-off when hiring someone as a temporary worker: although unconditionally temporary contracts are preferred to permanent contracts, a temporary offer risks losing the worker if she finds a better match before the firm’s retention decision. Finally, both firms and workers have risk-neutral preferences.

Let us first fix some notation:

- $Q$: Value of a vacancy.
- $U$: Value of being unemployed.
- $V^P$: Value of being employed under a permanent contract.
- $V^R$: Value of being employed following promotion from a temporary position to a permanent one.
- $V^T$: Value of being employed under a temporary contract.
- $J^P$: Value of a filled job under a permanent contract.
- $J^R$: Value of a filled job that in the previous period was temporary and has been converted to permanent.
- $J^T$: Value of a filled job under a temporary contract.

We will assume throughout that $y_{\text{min}} < y_{\text{max}} - c - f$ and we define the set $A \equiv \{ z \in [z_{\text{min}}, z_{\text{max}}] | E_y J^P (y, z) \geq E_y J^T (y, z) \}$ to be the set of realizations of $z$ for which
the firm prefers to offer a permanent contract to the worker. For convenience, let \( I_A \) denote an indicator function defined as,

\[
I_A = \begin{cases} 
1 & z \in A, \\
0 & z \notin A.
\end{cases}
\]

A firm and a worker match before \( y \) is realized but after \( z \) is revealed. We now turn to define some recursive relationships that must hold between asset values of vacant jobs, filled jobs, and employment and unemployment states.

\[
Q = -k + \beta \alpha^f(\theta) \int_{z_{\min}}^{z_{\max}} \max (E_y J^P(y, z), E_y J^T(y, z)) \, dG(z) \\
+ \beta (1 - \alpha^f(\theta)) Q,
\]

(1)

This equation just states that the value of a vacant position is the expected payoff net of posting costs \( k \). The expected payoff is discounted with a factor \( \beta \). With probability \( \alpha^w(\theta) \), the vacant position gets matched to a worker and the vacancy will become either a permanent job, a temporary job, or it will remain unfilled, all depending on the realization of the expected realization of the idiosyncratic shock. With probability \( 1 - \alpha^f(\theta) \) the vacant position finds no match. The following equation states the value of the unemployment state as the flow from unemployment benefits \( b \) and the discounted value of being matched to an un-filled job - which happens with probability \( \alpha^w(\theta) \) - or remaining unemployed.

\[
U = b + \tau + \beta \alpha^w(\theta) \int_{z_{\min}}^{z_{\max}} \left[ I_A E_y V^P(y, z) + (1 - I_A) E_y V^T(y, z) \right] \, dG(z) \\
+ \beta (1 - \alpha^w(\theta)) U,
\]

(2)

We now turn to describe the value of being employed, which will depend on the type of contract that governs the relationship between the worker and the firm. We begin by describing the value of being employed under a permanent contract, given by the following equation:

\[
V^P(y, z) = w^P(y, z) + \tau + \beta \int_{y_{\min}}^{y_{\max}} \max (V^P(x, z), U) \, dF(x),
\]

(3)
The flow value of being employed is a wage \( w^P(y, z) \); the discounted flow is the maximum of quitting and becoming unemployed or remaining in the relationship. As the idiosyncratic productivity \( z \) does not vary over time, under a permanent contract a change in the wage is only driven by a change in the aggregate shock. Analogously, the value of being employed following a promotion is given by,

\[
V^R(y, z) = w^R(y, z) + \tau + \beta \int_{y_{min}}^{y_{max}} \max (V^P(x, z), U) \, dF(x), \tag{4}
\]

Finally, the worker employed under a temporary contract earns \( w^T(y, z) \). At the end of the period, she has a chance to search for better matches, the set of which is given by all those matches with a higher idiosyncratic productivity value than \( z \) - the current realization. Absent this better match the two remaining options for the temporary worker are to become unemployed or to be promoted to a permanent worker with her current employer. Formally,

\[
V^T(y, z) = w^T(y, z) + \tau + \beta (1 - \alpha^w(\theta)) \int_{y_{min}}^{y_{max}} \max (V^R(x, z), U) \, dF(x)
\]

\[
+ \beta \alpha^w(\theta) \int_{z_{min}}^{z_{max}} \left[ I_{A} E_y V^P(y, x) + (1 - I_{A}) E_y V^T(y, x) \right] \, dG(x). \tag{5}
\]

Regarding capital values of filled positions, the flow profit for a firm is given by the total productivity of the worker, \( y + z \), net of the wage paid. This wage is contingent on the type of contract the worker is under. In the case of an updated contract, the firm must pay a cost \( c \) to change the contract from a temporary to a contract. The asset values of filled jobs under permanent, upgraded, and temporary contracts are given by,

\[
J^P(y, z) = y + z - w^P(y, z) + \beta \int_{y_{min}}^{y_{max}} \max (J^P(x, z), Q - f) \, dF(x), \tag{6}
\]

\[
J^R(y, z) = y + z - w^R(y, z) - c + \beta \int_{y_{min}}^{y_{max}} \max (J^P(x, z), Q - f) \, dF(x), \tag{7}
\]

\[
J^T(y, z) = y + z - w^T(y, z) + \beta (1 - \alpha^w(\theta)) \int_{y_{min}}^{y_{max}} \max (J^R(x, z), Q) \, dF(x)
\]

\[
+ \beta \alpha^w(\theta) Q, \tag{8}
\]
Using the definition of $I_A$ indicator function, the value of vacancy can be written as:

\[
Q = -k + \beta \alpha^f(\theta) \int_{z_{\min}}^{z_{\max}} \left[ I_A E_y J^P(y, z) + (1 - I_A) E_y J^T(y, z) \right] dG(z) \\
+ \beta \left( I_A \right) Q.
\] (9)

The corresponding total surpluses for each type of contract each contract are given by:

\[
S_P(y, z) = J^P(y, z) - (Q - f) + V^P(y, z) - U,
\]

\[
S_R(y, z) = J^R(y, z) - Q + V^R(y, z) - U,
\]

\[
S_T(y, z) = J^T(y, z) - Q + V^T(y, z) - U.
\]

Since wages are determined by the Nash bargaining, the generalized Nash bargaining solution yields the following surplus sharing rule:

\[
S_P(y, z) = \frac{J^P(y, z) - Q + f}{1 - \phi} = \frac{V^P(y, z) - U}{\phi},
\]

\[
S_R(y, z) = \frac{J^R(y, z) - Q}{1 - \phi} = \frac{V^R(y, z) - U}{\phi},
\]

\[
S_T(y, z) = \frac{J^T(y, z) - Q}{1 - \phi} = \frac{V^T(y, z) - U}{\phi}.
\] (10)

Free entry implies $Q = 0$. Combine equation 9 with free entry condition and use the surplus sharing rule in 10, we can derive the following equation:

\[
\int_{z_{\min}}^{z_{\max}} \left[ I_A E_y S_P(y, z) + (1 - I_A) E_y S_T(y, z) \right] dG(z) = \frac{k + \beta \alpha^f(\theta) \mu_G(A) f}{(1 - \phi) \beta \alpha^f(\theta)}.
\] (11)

where $\mu_G(A)$ is the probability measure of $A$. Using this relationship together with equation (10) to substitute into equation (2), one can derive the expression for $U$ as

\[
U = \frac{1}{1 - \beta} \left\{ b + \tau + \frac{\phi \alpha^w(\theta) \left( k + \beta \alpha^f(\theta) \mu_G(A) f \right)}{(1 - \phi) \alpha^f(\theta)} \right\}.
\] (12)
Substitute equation (12) into equations (6)-(5) and use (10), we have
\[
S^P(y, z) = y + z + \beta \int_{y_{min}}^{y_{max}} \max \left( S^P(x, z), 0 \right) dF(x) + (1 - \beta) f
\]
\[
- b - \frac{\phi\alpha^w(\theta)(k + \beta\alpha f(\theta)\mu G(A)f)}{(1 - \phi)\alpha f(\theta)},
\]
(13)
\[
S^R(y, z) = y + z + \beta \int_{y_{min}}^{y_{max}} \max \left( S^P(x, z), 0 \right) dF(x) - c - \beta f
\]
\[
- b - \frac{\phi\alpha^w(\theta)(k + \beta\alpha f(\theta)\mu G(A)f)}{(1 - \phi)\alpha f(\theta)},
\]
(14)
\[
S^T(y, z) = y + z + \beta (1 - \alpha^w(\theta)) \int_{y_{min}}^{y_{max}} \max \left( S^R(x, z), 0 \right) dF(x) - b.
\]
(15)

Observe that the surpluses are strictly increasing in \( y \), and in fact are linear in \( y \) because \( \partial S^P(y, z)/\partial y = \partial S^R(y, z)/\partial y = \partial S^T(y, z)/\partial y = 1 \). Jobs are endogenously destructed if the match surpluses are negative. This happens when the aggregate productivity falls below some cut-off values \( y^P(z) \) if jobs are permanent, and \( y^R(z) \) if jobs are temporary. The threshold values are determined by \( S^P(y^P, z) = 0 \) and \( S^R(y^R, z) = 0 \). The first proposition shows the existence of such thresholds and establish the job destruction rules.

**Assumption 1.** Let \( \Phi_z(y) = y + z + \beta \int_y^{y_{max}} (1 - F(x)) dx \). For any \( z \in [z_{min}, z_{max}] \), the following inequalities hold
\[
\Phi_z(y_{max}) > b + \frac{\phi\alpha^w(\theta)(k + \beta\alpha f(\theta)f)}{(1 - \phi)\alpha f(\theta)} - (1 - \beta) f,
\]
(16)
\[
b + \frac{\phi\alpha^w(\theta)k}{(1 - \phi)\alpha f(\theta)} - (1 - \beta) f > \Phi_z(y_{min}).
\]
(17)

**Assumption 2.** In addition, \( \forall z \in [z_{min}, z_{max}] \)
\[
\Phi_z(y_{max} - c - f) > b + \frac{\phi\alpha^w(\theta)(k + \beta\alpha f(\theta)f)}{(1 - \phi)\alpha f(\theta)} - (1 - \beta) f.
\]
(18)

**Proposition 1.** Under assumption 1, for any \( z \), there exists an unique cut-off value \( y^P(z) \in (y_{min}, y_{max}) \) and such that \( S^P(y^P(z), z) = 0 \). If assumption 2 is also hold then
the unique cut-off value \( y^R(z) \in (y_{\min}, y_{\max}) \) exists where \( S^R(y^R(z), z) = 0 \). The cut-off values solve the following equations:

\[
y^P + z + \beta \int_{y^P}^{y_{\max}} (1 - F(x)) \, dx = b - (1 - \beta) f,
\]

\[
+ \frac{\phi \alpha^w(\theta) (k + \beta \alpha^f(\theta) \mu_G(A) f)}{(1 - \phi) \alpha^f(\theta)}
\]

\[
y^P + c + f = y^R. \tag{19}
\]

Next we look at the hiring decision and show that the type of a contract being offered depends on the threshold value \( \bar{z} \).

**Proposition 2.** There exists a unique cut-off value \( \bar{z} \in [z_{\min}, z_{\max}] \) such that when \( z > \bar{z} \) the firm only offers a permanent contract, while \( z < \bar{z} \), only temporary contract is offered if the following condition holds:

\[
\beta \left[ \int_{y^P(z_{\min})}^{y_{\max}} (1 - F(x)) \, dx - (1 - \alpha^w(\theta)) \int_{y^R(z_{\max})}^{y_{\max}} (1 - F(x)) \, dx \right]
\]

\[
< \left[ \frac{1}{1 - \phi} - (1 - \beta) \right] f - \frac{\phi \alpha^w(\theta) (k + \beta \alpha^f(\theta) f)}{(1 - \phi) \alpha^f(\theta)}
\]

\[
< \left[ \frac{1}{1 - \phi} - (1 - \beta) \right] f - \frac{\phi \alpha^w(\theta) (k + \beta \alpha^f(\theta) f)}{(1 - \phi) \alpha^f(\theta)}
\]

\[
< \beta \left[ \int_{y^P(z_{\max})}^{y_{\max}} (1 - F(x)) \, dx - (1 - \alpha^w(\theta)) \int_{y^R(z_{\max})}^{y_{\max}} (1 - F(x)) \, dx \right] \tag{20}
\]

To obtain expressions for wages paid under different contracts we can substitute the relevant value functions into the the surplus sharing rule (10), which gives:

\[
w^P(y, z) = \phi (y + z) + (1 - \phi) b
\]

\[
+ \phi \left[ (1 - \beta) f + \frac{\alpha^w(\theta)}{\alpha^f(\theta)} (k + \beta \alpha^f(\theta) \mu_G(A) f) \right], \tag{22}
\]

\[
w^R(y, z) = w^P(y, z) - \phi (c + f), \tag{23}
\]

\[
w^T(y, z) = \phi (y + z) + (1 - \phi) b. \tag{24}
\]

Finally, we need to explicitly state how the stock of employment evolves over time. Let \( u_t \) denote the measure of unemployment, and \( n_t^P \) and \( n_t^T \) be the measure of permanent workers and temporary workers. Let’s begin by deriving the law of motion of
the stock of permanent workers, which is given by the sum of three groups of workers. First, unemployed workers and temporary workers can search and match with other firms and become permanent workers. This happens with probability $\alpha^w(\theta_t) \mu_G(A)$. Second, after the realization of the aggregate shock, the permanent worker remains at the current job. The aggregate quantity of this case is $\int_{z_{\min}}^{z_{\max}} \mu_F\left([y_P(z), y_{\max}]\right) dG(z) n_t^P$. Third, some of temporary workers who cannot find other jobs get promoted to permanent workers which adds to the aggregate employment pool for permanent workers by $(1 - \alpha^w(\theta_t)) \int_{z_{\min}}^{\bar{z}} \mu_F\left([y_R(z), y_{\max}]\right) dG(z) n_t^T$. Notice that $\mu_G(A) = 1 - G(\bar{z})$ and $\mu_F([y, y_{\max}]) = 1 - F(y)$. The law of motion for permanent workers is then:

$$n_{t+1}^P = (u_t + n_t^T) \alpha^w(\theta_t) (1 - G(\bar{z})) + \int_{z_{\min}}^{z_{\max}} \left[ 1 - F(y_P(z)) \right] dG(z) n_t^P$$

$$+ (1 - \alpha^w(\theta_t)) \int_{z_{\min}}^{\bar{z}} \left[ 1 - F(y_R(z)) \right] dG(z) n_t^T. \quad (25)$$

For the temporary workers, only those unemployed workers and temporary workers matching with low productivity firms will become temporary for the next period. Therefore the temporary workers evolve according to:

$$n_{t+1}^T = (u_t + n_t^T) \alpha^w(\theta_t) G(\bar{z}). \quad (26)$$

Since the aggregate population is normalized to unity, the unemployment is given by:

$$u_t = 1 - n_{t+1}^T - n_{t+1}^P.$$ 

The market tightness is captured by:

$$\theta = \frac{v}{u + n_T^T}.$$ 

3 Partial Equilibrium Analysis

To understand the intuition behind some of the results we show below, we perform here some comparative statics in “partial” equilibrium. By “partial” equilibrium we mean fixing $\theta$ and analyzing the impact of changing some variables and parameters on the hiring decision and the job destruction decision.
Proposition 3. The hiring rule has the following properties:

1. $d\bar{z}/df > 0$,

2. \[
\begin{cases}
    d\bar{z}/d\alpha^w < 0 & \text{when } \phi < \bar{\phi} \\
    d\bar{z}/d\alpha^w > 0 & \text{when } \phi > \bar{\phi},
\end{cases}
\]

3. $d\bar{z}/dc < 0$.

The intuition behind proposition 3 can be illustrated in the following figures. Figure 1 shows the effects of an increase in the firing cost. An increase in the firing cost has two effects on the (net) value of a filled job. The direct effect causes the value of a permanent job to go down because the firm has to pay more to separate from the worker. Hence the permanent contract curve shifts downward. On the other hand, an increase in $f$ also increases the job destruction rate of temporary workers by raising the threshold value $\bar{y}^R$ so that the value of a temporary job falls as well. In equilibrium, that the effect of an increase in the firing cost on decreasing permanent jobs dominates the effect on decreasing temporary jobs causes firms to offer fewer permanent contracts.

The effect of an increase in the job finding probability on the hiring decision is ambiguous and depends on the worker’s bargaining power. First, if it is easier for unemployed workers to find a job, the value of unemployment will increase because the unemployment spell is shortened. The match surplus will go down since the worker’s outside option rises. Therefore, the value of filled jobs must fall and (both permanent and temporary) contract curves will shift outward. We call this “unemployment” effect. However, there are two extra effects on temporary jobs. Since the temporary worker can search on-the-job, the higher job finding probability favors the temporary worker to remain employed. Therefore, the match surplus will go up due to the rise in the value of employment. We call this “job continuation” effect. For the temporary job, unemployment effect and job continuation effect exactly cancel out. On the other hand, the higher job finding probability causes the current match of temporary contract to be broken up more often. This so-called “job turnover” effect will reduce the value of a temporary job which moves the
temporary contract curve outward. If a worker has more bargaining power, then the unemployment effect dominates the job turnover effect which is the same case as in Figure 1. However if the worker’s bargaining power is small, the job turnover effect dominates the unemployment effect which leads the firm to hire less temporary workers as a result, see Figure 2 for illustration.

Finally, the effect of an increase in promotion costs is depicted in Figure 3. Promotion costs has only an effect on the temporary contracts. An increase in $c$ reduces the incentive for promoting a temporary worker to a permanent one. As a result, the value of a temporary job decreases and the temporary contract curve shifts downward.

![Figure 1: Effect of Firing Costs on Temporary Contracts](image)

**Proposition 4.** Suppose that the firm has most of the bargaining power, the job destruction rule has the following properties:

1. $dy^P/df < 0$ and $dy^R/df > 0$, 

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2. \( \frac{dy^P}{d\alpha_w} > 0 \) and \( \frac{dy^R}{d\alpha_w} > 0 \).

3. \( y^P \) is weakly increasing in \( c \) and \( \frac{dy^R}{dc} > 0 \).

The first part of Proposition 4 shows the firing cost has opposite effects on the separation of permanent jobs and temporary jobs. An increase in the firing cost induces the firm less willing to pay the cost to fire the permanent worker, but more willing to separate from the temporary worker now in order to avoid the rising firing cost in the future. The second part of the proposition mainly comes from the changes of hiring threshold. The last part is quite straightforward: an increase in promotion costs discourages the firm to retain the temporary worker.

Next, we can take the hiring and job separation decision as given and ask how the changes in the firing cost and promotion cost affect the job creation (vacancy posting) decision. The following proposition summarizes the results.
Proposition 5. Given the hiring rule and permanent job destruction rule, i.e. $\bar{z}$ and $y^P(z)$ are fixed, $d\theta/df < 0$ if $\beta$ is not too small and $d\theta/dc < 0$.

The explanation of this proposition is that an increase in firing costs and promotion costs discourages firm to post more vacancies by reducing the expected profits of jobs.

4 Data

We used the Workforce Employee Survey, a Canadian matched employer-employee dataset collected by Statistics Canada. It is an annual, longitudinal survey at the establishment level, targeting establishments in Canada that have paid employees in March, with the exceptions of those operating in the crop and animal production; fishing, hunting and trapping; households’, religious organizations, and the government sectors. In 1999, it consisted of a sample of 6,322 establishments drawn from the Business Register maintained
by Statistics Canada and the sample has been followed ever since. Every odd year the sample has been augmented with newborn establishments that have become part of the Business Register. The data are rich enough to allow us to distinguish employees by the type of contract they hold. However, only a sample of employees is surveyed from each establishment. The average number of employees in the sample is roughly 20,000 each year. Workers are followed for two years and provide responses on hours worked, earnings, job history, education, and demographic information. Firms provide information about hiring conditions of different workers, payroll and other compensation, vacancies, and separation of workers.

Given the theory laid out above, it is important that the definition of temporary worker in the data matches as close as possible the concept of a temporary worker in the model. In principle, it is unclear that all establishments share the idea of what a temporary worker is when they respond to the survey: it could be a seasonal worker, a fixed-term consultant hired for a project or a worker working under a contract with a set termination date. As a result, Statistics Canada implemented some methodological changes to be consistent in its definition of a temporary worker. This affected the incidence of temporary employment in the survey forcing us to use data only from 2001 onwards. The definition of temporary workers we use, it is of those receiving a T-4 slip from an employer but who have a set termination date. For instance, workers from temporary employment agents or other independent contractors are not included in our definition. With the use of this definition the fraction of temporary workers among all workers is 14%.

Table 1 displays some descriptive statistics on workers’ compensation by type of contract held. All quantities are in Canadian dollars and we use three different measures of compensation: total earnings reported by the employee, hourly wages with reported extra-earnings, and hourly wages without the reported extra earnings. According to the three measures, permanent workers earn more but they do work more as well. As a result,

1All establishments with less than four employees are surveyed. In larger establishments, a sample of workers is surveyed, with a maximum of 24 employees per given establishment.
Table 1: Worker’s Compensation by Type of Contract

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Permanent</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Earnings</td>
<td>$21,847</td>
<td>$33,525</td>
</tr>
<tr>
<td>Real Hourly Wage (No Extra)</td>
<td>$21.43</td>
<td>$11.75</td>
</tr>
<tr>
<td>Real Hourly Wage</td>
<td>$22.57</td>
<td>$14.40</td>
</tr>
<tr>
<td><strong>Temporary</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Earnings</td>
<td>$9,737</td>
<td>$26,469</td>
</tr>
<tr>
<td>Real Hourly Wage (No Extra)</td>
<td>$18.87</td>
<td>$15.22</td>
</tr>
<tr>
<td>Real Hourly Wage</td>
<td>$19.54</td>
<td>$18.85</td>
</tr>
</tbody>
</table>

while total earnings of permanent are roughly double of those earned by temporary workers, when converted to hourly measures, that ratio drops to 1.14-1.15. The cross-sectional distribution of wages per hour has a larger variance in the case of temporary workers than of permanent workers. The standard deviation of permanent workers’ hourly wages is about half of mean hourly wages. This ratio rises to 81% for temporary workers.

In Canada, job turnover is higher for temporary workers than for permanent workers, as extensively documented by Cao and Leung (2010). We reproduce some of their turnover statistics on Table 2. As it is typical, we measure turnover by comparing job creation and job destruction rates. If we denote by $EMP_t$ the total level of employment at time $t$, the creation and destruction rates between periods $t$ and $t + 1$ are calculated as:

$$\text{Creation} = \frac{Emp_{t+1} - Emp_t}{0.5(Emp_{t+1} + Emp_t)}$$

(27)

if $Emp_{t+1} - Emp_t > 0$ and 0 otherwise. And,

$$\text{Destruction} = \frac{|Emp_{t+1} - Emp_t|}{0.5(Emp_{t+1} + Emp_t)}$$

(28)

if $Emp_{t+1} - Emp_t < 0$ and 0 otherwise.

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Given the emphasis of our work on a job market segmented by temporary and permanent workers, we provide measures of job destruction and creation by type of contract held. These measures are analogous to the previous two expressions, however, we measure creation and destruction of temporary (or permanent) workers relative to the average total employment level. In other words, we measure the change in the stock of workers by contract type relative to the stock of total employment. These rates are given on the first two lines of Table 2. The job destruction rates are 6.2% for permanent workers and 6.4% for temporary workers. The creation rates are 8.4% and 5.4%. As the fraction of temporary workers is only 14% of the workforce, these rates point to a much higher degree of turnover for temporary workers.

The reader might have noticed that the sum of the destruction rates for temporary and permanent workers is not equal to the destruction rate for all workers. The same can be said for the creation rates. The reason is that establishments can change the number of temporary and permanent workers without altering the stock of all workers. If we restrict the sample to those establishments that increase or decrease the stock of both permanent and temporary workers, the rates for all workers are the sum of the rates of the two types of workers. These measures are reported in Table 2 under the “Alternative Definition” cell. Turnover decreases under this alternative definition, with creation and destruction rates for all workers that are 2% lower than using the conventional definition. The total job creation rate is 8.2% and the job destruction rate is 4.1%.

5 Parameterization

In motivating our work, we have stressed as one of our main contributions structurally estimating and validating our model by the use of statistical methods. In the next section we describe with some detail our estimation strategy and discuss its results. However, before launching into that task it is illustrative to perform a standard calibration in which the relevant parameters of the model are chosen to match some features of the Canadian economy. We perform that calibration in this section.
Table 2: Job Creation and Job Destruction (%)

<table>
<thead>
<tr>
<th></th>
<th>Conventional Definition</th>
<th>Alternative Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Workers</td>
<td>Permanent</td>
</tr>
<tr>
<td>Job Creation</td>
<td>10.2</td>
<td>8.1</td>
</tr>
<tr>
<td>Job Destruction</td>
<td>9.2</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Let's begin with the concept of time in the model. As most of the data we have is at the annual frequency we set one model period to be one year. For this reason we set the discount factor $\beta$ to 0.97. Lacking data on promotion costs we set $c$ to zero, which seems an innocuous assumption as these costs can’t be very large. These two parameters, $c$ and $\beta$ are the only ones we fix ex-ante. The remaining ones - $\{y_{\min}, y_{\max}, z_{\min}, z_{\max}, \xi, \phi, k, b, f\}$ are chosen to minimize the squared-sum of deviations of model moments from empirical targets.

We choose the targets in an ad-hoc way, finding moments we think the model should deliver. We begin by targeting the 14% fraction of temporary workers in the Canadian workforce. Estimates of the job-finding probability in Canada find values close to 0.3 on a monthly frequency. As our model period is one year, we set this rate to 0.9 per-year. We target a few moments related to turnover: the aggregate job creation rate of 8.3%, a destruction rate for permanent workers of 4.0%, a total destruction rate of 7.1% and a creation rate of temporary workers of 3.1%. To calibrate $f$ we need a measure of the average firing cost per salary earned by a permanent worker. In Canada the severance pay is one week worth of wages per year worked. The average tenure is slightly more than 4 years, so total payments at dismissal would be about 10% of a year’s salary. Hence, $f/w_P$ should be about 0.1. the unemployment benefit has a pure pecuniary interpretation in

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2See, for instance, Zhang (2008).
our model, so we set it to be 55% of temporary workers’ wages, which is that statutory replacement ratio in Canada.

So far we have specified eight moments but need to estimate nine parameters. In the theory described in the previous section, the total scale of the economy measured as output $y$ and $z$ is irrelevant. If the distributions of productivity shift to the right by a certain amount, and $b$, $f$, $k$, and $c$ all change proportionally, the results are unchanged. Consequently we need to bound the distribution of the sum of the two shocks. It is nevertheless important that we do not constrain where the distributions of $y$ and $z$ lie in any other way. We choose to set the mean of the sum of the two distributions to 1 as a normalization, i.e. $(y_{\text{min}} + y_{\text{max}})/2 + (z_{\text{min}} + z_{\text{max}})/2 = 1$.

Our parameterization is summarized in Table 3. To further assess the model’s performance Table 4 displays the quantities for other - non-targeted - moments of interest and their empirical counterparts. By construction, in our theory the wage of permanent workers $W_P$ is higher than those of temporary workers $W_T$. Quantitatively however, the ratio between the two we obtain in the baseline calibration is close to the data: it is 14% higher in the data and 19% higher in the model. The unemployment rate is also
extremely close. To get the empirical unemployment rate we computed the average annual rate from 1999-2000 to be consistent with the period used to calculate job creation and job destruction rates. This average is 7.08% and the value in the model is 7.15%. When we turn to job destruction rates for temporary contracts, the model is somewhat detached from the data. Even though we targeted to minimize the distance between the creation and destruction of permanent workers and all workers, it was difficult to achieve those targets exactly. The result is a job destruction rate for temporary workers that is about 10% of that observed in the data. The creation rate of temporary workers is, on the contrary, larger in the model (4.92%) than in the data (3.10%).

5.1 Effects of Increasing Labor Protection

Most previous work has found ambiguous effects of changing the degree of labor protection on labor market outcomes. For instance, it can increase or decrease the unemployment rate depending on the size of the changes in the firing and hiring rates. This ambiguity translates into our framework, but we can nevertheless ask under the baseline calibration described above what are the effects of increasing the firing tax \( f \). We show the consequences of increasing \( f \) by 25% and 50% from its calibrated value of 0.126. We report the results of Table 5.

An increase in the firing tax decreases unemployment. Even increasing \( f \) by 25% decreases unemployment by more than 7%. As expected, the resulting increase in employment falls more on temporary workers than permanent workers. The latter barely
Table 5: Effects of Increasing the Firing Tax

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta f = 25%$</th>
<th>$\Delta f = 50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>-7.49%</td>
<td>-15.58%</td>
</tr>
<tr>
<td>$N^P$</td>
<td>0.21%</td>
<td>0.76%</td>
</tr>
<tr>
<td>$N^T$</td>
<td>2.87%</td>
<td>4.01%</td>
</tr>
<tr>
<td>$JC_{All}$</td>
<td>-8.23%</td>
<td>-16.90%</td>
</tr>
<tr>
<td>$JD_{All}$</td>
<td>-8.37%</td>
<td>-17.12%</td>
</tr>
<tr>
<td>$JC_{Perm}$</td>
<td>-17.09%</td>
<td>-32.36%</td>
</tr>
<tr>
<td>$JD_{Perm}$</td>
<td>-9.39%</td>
<td>-19.00%</td>
</tr>
<tr>
<td>$JC_{Temp}$</td>
<td>-4.56%</td>
<td>-10.51%</td>
</tr>
<tr>
<td>$JD_{Temp}$</td>
<td>15.18%</td>
<td>26.64%</td>
</tr>
<tr>
<td>$W^P/W^T$</td>
<td>0.04%</td>
<td>0.07%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-2.13%</td>
<td>-3.69%</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>0.32%</td>
<td>0.61%</td>
</tr>
</tbody>
</table>

increase (0.21%) while the stock of temporary workers increase by almost 3%. Turnover in the aggregate decreases, but the destruction rate decreases more than the creation rate contributing to the lower level of unemployment. According to one’s intuition the increase in $f$ increases the job destruction rate for permanent workers as the rate of promotion is lower (it is costlier to enter into permanent contracts), and it decreases the rate of creation of temporary jobs but much less than the decrease in the creation of permanent jobs.

6 Estimation

The empirical estimation is intended to test whether the theoretical model and assumptions are consistent with the labor market in actuality. A precisely estimated model also allows one to understand the wage dynamics of workers with different types of employment contract, and the job destruction and creation.

Previous estimation of search and match models, Cahuc, Postel-Vinay and Robin (2006), Postel-Vinay and Robin (2002), Flinn and Mabli (2009). These papers follow the
In our estimation, besides investigating the wage determination and wage distribution, we also focus on individual employment contracts. In Canadian matched employee-employer data, both firms and workers display much heterogeneity on wages and employment contract types. For example, around 60 percent of firms do not have temporary employment contract, while temporary jobs account for at least 30 percent of gross job reallocation.

Our estimation consists of two components. First, we estimate both the marginal productivity of firms and the worker productivity. Second, we (jointly) estimate the distribution of employment contracts, and wage equations of both permanent workers and temporary workers.

### 6.1 Labor productivity estimation

For the firm productivity, we use Workplace and Employee Survey (WES) (1999 - 2007). The data contains employment contract type, number of employees, and sales, and industry index. We estimate $s_j$ and $\gamma$ from the production function

$$y_{jt} = s_j e^{\varepsilon_{jt}} (n_{jt} + \lambda l_{jt})^\gamma,$$

where $\varepsilon_{jt}$ is the unobserved heterogeneity. Firm’s labor productivity $p_{jt}$ can be estimated as the average labor product, $y_{jt}/(n_{jt}+l_{jt})$. The production function cannot be directly estimated because of endogeneity problem. Both $n_{jt}$ and $l_{jt}$ are determined by productivity shocks. Further, in WES 35 percent firms hire both permanent and temporary workers, while 65 percent of firms only hire permanent workers. To overcome this problem, we estimate the production function together with the firm’s hiring decisions.
We can think $s_j$ as the match quality $z$ in the model, and $\varepsilon_{jt}$ as the time-varying component of the match. We rely on data to find the correlation between employment contract types and these productivity measures, particularly whether there exists some cut-off values of $s_j$ below which the firm offers temporary contracts.

6.2 Worker mobility and employment contract

We estimate worker flows from Labor Force Survey (LFS) and WES data. Specifically, we estimate the probabilities of being permanent worker, temporary worker or unemployed. The job finding rate $\alpha^w(\theta)$ can be estimated with LFS or SLID. Vacancy-filling rate $\alpha^f(\theta)$ is estimated with WES firm data, which surveys the vacancies.

6.3 Wage equations

We estimate wage equations for three types of employment contract, by taking into consideration the endogeneity of other variables, including employment contract type, and production function. The equilibrium wage equations show that the wage is a linear function of matched productivity. Hence wage distribution should be same as the matched productivity distribution. We are not able to identify all parameters in the wage equations, so we focus on bargaining power $\phi$, reservation wage $b$, firing cost $f$, and $\overline{z}$. The system of equations to be estimated consists of wage equations, contract type, and production function. We implement the estimation with GMM on WES matched data.

7 Estimation Results

[TO BE ADDED]
8 Concluding Remarks

This study provides a theory of the co-existence of labor contracts with different firing conditions. Consistent with empirical evidence that points to employers choosing among contracts with different degrees of labor protection, firms here choose to offer *ex-ante* identical workers different contracts, and as a result, different wages. The reason is match-quality that varies among worker-firm pairs and that is revealed at the moment firms and workers meet. Firms offer permanent contracts to “good” matches, as they risk losing the worker should they offer them a temporary contract. This risk results from the different on-the-job search behavior by the two types of workers: temporary workers search while permanent workers do not. Not-so-good matches are given a temporary contract under which they work for a lower wage but they are allowed to search for better opportunities. After one period, temporary workers have to be dismissed or promoted to permanent status.

The existence of search and matching frictions implies that workers might work temporarily in jobs with an inferior match quality, before transferring to better - and more stable - matches. Our assumption of including a time-varying component in the total productivity of a worker allows our environment to generate endogenous destruction rates that differ by type of contract. Our environment is simple enough to deliver several analytical results regarding cut-off rules for the type of relationship firms and workers begin and when and how they separate. Yet, it is rich in its implications and provides a new way to examine the consequences of firing taxes on the wage distribution.

Currently, our work is still in progress, particularly the quantitative part. We have outlined our strategy for estimating and validating the model using statistical methods but results are not yet available.
References


A Appendix: Proof of Propositions

Proof of Proposition 1. Equation (13) can be written as
\[ S^P (y, z) = y + z + \beta \int_{y^P}^{y_{max}} S^P (x, z) \, dF (x) + (1 - \beta) f \]
\[ -b - \frac{\phi \alpha^w (\theta) (k + \beta \alpha^f (\theta) \mu_G (A) f)}{(1 - \phi) \alpha^f (\theta)} . \]  

(29)
From the fact that \( \partial S^P / \partial y = 1 \) and \( \partial^2 S^P / \partial y \partial z = 0 \), it implies that \( S^P (y, z) = y + \varphi (z) \). The integral on the right-hand side of (29) is then
\[ \int_{y^P}^{y_{max}} S^P (x, z) \, dF (x) = \int_{y^P}^{y_{max}} \varphi (x) \, dF (x) , \]
\[ = (x + \varphi (z)) F (x) \bigg|_{y^P}^{y_{max}} - \int_{y^P}^{y_{max}} F (x) \, dx . \]
For any \( z \in Z, S^P (y^P, z) = 0 \) implies \( y^P = -\varphi (z) \). Substitute \( \varphi (z) \) with \( -y^P \), the expression of the integral is
\[ \int_{y^P}^{y_{max}} S^P (x, z) \, dF (x) = \int_{y^P}^{y_{max}} [1 - F (x)] \, dx . \]  

(30)
To pin down \( y^P \), we need to solve the equation \( S^P (y^P, z) = 0 \), thus
\[ y^P + z + \beta \int_{y^P}^{y_{max}} [1 - F (x)] \, dx = b+ \]
\[ \frac{\phi \alpha^w (\theta) (k + \beta \alpha^f (\theta) \mu_G (A) f)}{(1 - \phi) \alpha^f (\theta)} - (1 - \beta) f . \]  

(32)
Denote left-hand side by \( \Phi_z (y) \). Notice that \( \Phi_z (y) \) is increasing in \( y \). If for any \( z \in Z, \Phi_z (y) \) satisfies the inequalities (16) and (17), then together with \( \mu_G (A) \in [0, 1] \), we can conclude there is a unique solution \( y^P (z) \in (y_{min}, y_{max}) \) for equation (32) by the intermediate value theorem. That is, \( y^P (z) \) exists for any \( z \in [z_{min}, z_{max}] \).

Similarly, equation (14) can be rewritten as
\[ S^R (y, z) = y + z + \beta \int_{y^P}^{y_{max}} [1 - F (x)] \, dx - c - \beta f \]
\[ -b - \frac{\phi \alpha^w (\theta) (k + \beta \alpha^f (\theta) \mu_G (A) f)}{(1 - \phi) \alpha^f (\theta)} . \]
Following the same argument for the condition \( S^P (y^P, z) = 0 \), the above equation yields the cut-off value by solving:
\[ y^R + z + \beta \int_{y^P}^{y_{max}} [1 - F (x)] \, dx = b+ \frac{\phi \alpha^w (\theta) (k + \beta \alpha^f (\theta) \mu_G (A) f)}{(1 - \phi) \alpha^f (\theta)} + c + \beta f . \]  

(33)
Comparing equations (32) and (33), we get
\[ y^R = y^P + c + f \]
Then assumption 2 guarantees the existence of \( y^P \in (y_{min}, y_{max} - c - f) \) which implies \( y^R < y_{max} \) exists as well.
Proof of Proposition 2. Step 1. \( E_yJ^P (y, z) \) and \( E_yJ^T (y, z) \) are both strictly increasing in \( z \). From the surplus sharing rule, it is sufficient to show that \( S^P (y, z) \) and \( S^T (y, z) \) are strictly increasing in \( z \). Substitute equation (30) into (29), we obtain

\[
S^P (y, z) = y + z + \beta \int_{y^P(z)}^{y^\text{max}} [1 - F (x)] dF (x) + (1 - \beta) f - b - \frac{\phi \alpha^w (\theta)(k + \beta \alpha^f (\theta) \mu_G (A) f)}{(1 - \phi) \alpha^f (\theta)}.
\]

Take the derivative of \( S^P \) with respect to \( z \), we get

\[
\frac{\partial S^P (y, z)}{\partial z} = 1 - \beta \left(1 - F (y^P (z))\right) y^P (z).
\]  

(34)

From equation (32), the implicit function theorem implies that

\[
y^P (z) = \frac{1}{1 - \beta (1 - F (y^P))} < 0.
\]  

(35)

Plug (35) into (34), we get \( \partial S^P / \partial z > 0 \). Similarly, the total surplus of a temporary contract can be rewritten as

\[
S^T (y, z) = y + z + \beta \left(1 - \alpha^w\right) \int_{y^T(z)+c}^{y^\text{max}} [1 - F (x)] dx - b.
\]

The derivative of \( S^T \) with respect to \( z \) is given by

\[
\frac{\partial S^T (y, z)}{\partial z} = 1 - \beta \left(1 - \alpha^w\right) [1 - F (y^P (z))] y^P (z) > 0.
\]

Step 2. \( E_yJ^P (y, z) \) and \( E_yJ^T (y, z) \) are strictly convex. By the separability of \( y \) and \( z \), it suffices to prove that \( S^P \) and \( S^T \) are convex in \( z \). Twice differentiate \( S^P \) with respect to \( z \), and get

\[
\frac{\partial^2 S^P (y, z)}{\partial z^2} = \beta \left[F' (y^P) \right] \beta - (1 - F (y^P)) y^P (z)\right].
\]

Since \( y''^P = \beta F' y''^P / \left[1 - \beta (1 - F (y^P))\right] ^2 < 0 \) and \( F' > 0 \), it must be the case that \( \partial^2 S^P (y, z) / \partial z^2 > 0 \). Similarly, \( \partial^2 S^T (y, z) / \partial z^2 > 0 \).

These two steps guarantee that if \( E_yJ^P (y, z) = E_yJ^T (y, z) \) holds, the cut-off value \( z \) is unique. The last step is to verify the single crossing property. That is, if

\[
E_yJ^P (y, z_{\text{min}}) < E_yJ^T (y, z_{\text{min}}),
\]

\[
E_yJ^P (y, z_{\text{max}}) > E_yJ^T (y, z_{\text{max}})
\]

hold, then the cut-off value \( z \) exists. Denote

\[
\Delta (z) = \frac{E_yJ^P (y, z) - E_yJ^T (y, z)}{1 - \phi}
\]

\[
= \beta \int_{y^P}^{y^\text{max}} [1 - F (x)] dx - \beta (1 - \alpha^w) \int_{y^T}^{y^\text{max}} [1 - F (x)] dx
\]

\[
- \left[\frac{1}{1 - \phi} - (1 - \beta)\right] f - \frac{\phi \alpha^w}{(1 - \phi) \alpha^f} \left(k + \beta \alpha^f \mu_G (A) f\right).
\]
It is straightforward to verify that if inequalities in (21) hold, $\Delta(z_{\text{min}}) < 0 < \Delta(z_{\text{max}})$. Figure 4 shows the single crossing property.

**Proof of Proposition 3.** The equilibrium condition for $\bar{z}$ is $E_y J^P(y, \bar{z}) = E_y J^T(y, \bar{z})$. By using the sharing rule (10) and equations (13) and (15), it implies that

$$\beta \int_{\bar{y}^P}^{y_{\text{max}}} (1 - F(x)) \, dx - \left[ \frac{1}{1 - \phi} - (1 - \beta) \right] f - \frac{\phi \alpha^w(\theta) (k + \beta \alpha^f(\theta) (1 - G(\bar{z})) f)}{(1 - \phi) \alpha^J(\theta)}$$

$$- \beta (1 - \alpha^w) \int_{\bar{y}^R}^{y_{\text{max}}} (1 - F(x)) \, dx = 0 \quad (36)$$

where $\bar{y}^P \equiv y^P(\bar{z})$ and $\bar{y}^R \equiv y^R(\bar{z}) = \bar{y}^P + c + f$. From equation (19), we have

$$\bar{y}^P + \bar{z} + \beta \int_{\bar{y}^P}^{y_{\text{max}}} (1 - F(x)) \, dx + (1 - \beta) f$$

$$- b - \frac{\phi \alpha^w(\theta) (k + \beta \alpha^f(\theta) (1 - G(\bar{z})) f)}{(1 - \phi) \alpha^J(\theta)} = 0. \quad (37)$$
Denote the left hand sides of equations (36) and (37) by $\Pi (\bar{y}^P, \bar{z})$. Totally differentiate $\Pi$, we get

$$D_{(\bar{y}^P, \bar{z})}\Pi = \begin{bmatrix} 1 - \beta (1 - F (\bar{y}^P)) & 1 + \frac{\phi}{1 - \phi} \alpha^w \beta f G' (\bar{z}) \\ -\beta [1 - F (\bar{y}^P) - (1 - \alpha^w) (1 - F (\bar{y}^R))] & 1 + \frac{\phi}{1 - \phi} \alpha^w \beta f G' (\bar{z}) \end{bmatrix},$$

and

$$D_f \Pi = \begin{bmatrix} 1 - \beta - \frac{\phi}{1 - \phi} \alpha^w \beta (1 - G (\bar{z})) \\ -\left[\frac{1}{1 - \phi} - (1 - \beta)\right] + \beta (1 - \alpha^w) (1 - F (\bar{y}^R)) - \frac{\phi}{1 - \phi} \alpha^w \beta (1 - G (\bar{z})) \end{bmatrix},$$

$$D_{\alpha^w} \Pi = \begin{bmatrix} -\frac{\phi}{(1 - \phi)\alpha f} [k + \beta \alpha f (1 - G (\bar{z}))] \\ \int_{\bar{y}^P}^{\bar{y}^R} (1 - F (x)) dx - \frac{\phi}{(1 - \phi)\alpha f} [k + \beta \alpha f (1 - G (\bar{z}))] \end{bmatrix},$$

$$D_c \Pi = \begin{bmatrix} 0 \\ \beta (1 - \alpha^w) (1 - F (\bar{y}^R)) \end{bmatrix}.$$

The determinant of matrix $D_{(\bar{y}^P, \bar{z})}\Pi$ is

$$\left| D_{(\bar{y}^P, \bar{z})}\Pi \right| = \beta [1 - F (\bar{y}^P) - (1 - \alpha^w) (1 - F (\bar{y}^R))] + [1 - \beta (1 - \alpha^w) (1 - F (\bar{y}^R))] \frac{\phi}{1 - \phi} \alpha^w \beta f G' (\bar{z}) > 0,$$

since $F (\bar{y}^P) < F (\bar{y}^R)$ and $G' (\bar{z}) > 0$. Apply the implicit function theorem, we can calculate the following:

$$\frac{d\bar{z}}{df} = -\frac{a_1 + a_2}{\left| D_{(\bar{y}^P, \bar{z})}\Pi \right|},$$

where

$$a_1 = \beta [1 - F (\bar{y}^P) - (1 - \alpha^w) (1 - F (\bar{y}^R))] \left(1 - \beta - \frac{\phi}{1 - \phi} \alpha^w \beta (1 - G (\bar{z}))\right)$$

and

$$a_2 = \left[1 - \beta (1 - F (\bar{y}^P))\right] \left\{- \left[\frac{1}{1 - \phi} - (1 - \beta)\right] + \beta (1 - \alpha^w) (1 - F (\bar{y}^R)) + \frac{\phi}{1 - \phi} \alpha^w \beta (1 - G (\bar{z}))\right\}.$$

Observe that the numerator $(a_1 + a_2)$ is decreasing in $\phi$. When $\phi = 0$, the numerator becomes

$$a_1 + a_2 = \beta (1 - \beta) [1 - F (\bar{y}^P) - (1 - \alpha^w) (1 - F (\bar{y}^R))] - \beta [1 - \beta (1 - F (\bar{y}^P))] [1 - (1 - \alpha^w) (1 - F (\bar{y}^R))].$$

Because $1 - \beta < 1 - \beta (1 - F (\bar{y}^P))$ and $1 - F (\bar{y}^P) - (1 - \alpha^w) (1 - F (\bar{y}^R)) < 1 - (1 - \alpha^w) (1 - F (\bar{y}^R))$, we must have $a_1 + a_2 < 0$. Hence, for any $\phi$, $d\bar{z}/df > 0$.

$$\frac{d\bar{z}}{d\alpha^w} = -\frac{a_3}{\left| D_{(\bar{y}^P, \bar{z})}\Pi \right|},$$

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where

\[ a_3 = \beta [1 - F(\tilde{y}^P) - (1 - \alpha^w)(1 - F(\bar{y}^R))] \left\{ -\frac{\phi}{1 - \phi} \alpha f \left[ k + \beta \alpha^f (1 - G(\bar{z})) f \right] \right\} \]

\[ + \left[ 1 - \beta (1 - F(\bar{y}^P)) \right] \left\{ \int_{\tilde{y}^R}^{y_{\text{max}}} (1 - F(x)) dx \right\} \]

\[ - \frac{\phi}{(1 - \phi) \alpha f} \left[ k + \beta \alpha^f (1 - G(\bar{z})) f \right]. \]

Again, \( a_3 \) is decreasing in \( \phi \). When \( \phi \to 0 \), \( a_3 \to [1 - \beta (1 - F(\bar{y}^P))] \int_{\tilde{y}^R}^{y_{\text{max}}} (1 - F(x)) dx > 0 \), while \( \phi \to 1 \), \( a_3 \to -\infty \). Therefore there exists \( \bar{\phi} \) such that

\[ \frac{d\bar{z}}{dc} = -\left[ 1 - \beta (1 - F(\bar{y}^P)) \right] \beta (1 - \alpha^w) (1 - F(\bar{y}^R)) \]

\[ < 0. \]

Proof of Proposition 4. Suppose \( \phi \to 0 \), equation (32) implies

\[ \frac{dy^P}{df} = -\frac{1 - \beta - \frac{\phi}{1 - \phi} \alpha^w \beta (1 - G(\bar{z})) + \frac{\phi}{1 - \phi} \alpha^w \beta f G'(\bar{z}) \frac{d\bar{z}}{df}}{1 - \beta (1 - F(\bar{y}^P))} \]

\[ < 0, \]

\[ \frac{dy^P}{d\alpha^w} = -\frac{-\frac{\phi}{(1 - \phi) \alpha f} \left[ k + \beta \alpha^f (1 - G(\bar{z})) f \right] + \frac{\phi}{1 - \phi} \alpha^w \beta f G'(\bar{z}) \frac{d\bar{z}}{d\alpha^w}}{1 - \beta (1 - F(\bar{y}^P))} \]

\[ > 0, \]

\[ \frac{dy^P}{dc} = -\frac{-\frac{\phi}{1 - \phi} \alpha^w \beta f G'(\bar{z}) \frac{d\bar{z}}{dc}}{1 - \beta (1 - F(\bar{y}^P))} \to 0^+. \]

Use these facts and combine equation (33), we derive

\[ \frac{dy^R}{df} = \frac{\beta F(\bar{y}^P) - \frac{\phi}{1 - \phi} \alpha^w \beta \left[ (1 - G(\bar{z})) - f G'(\bar{z}) \frac{d\bar{z}}{df} \right]}{1 - \beta (1 - F(\bar{y}^P))} > 0, \]

\[ \frac{dy^R}{d\alpha^w} = \frac{dy^P}{d\alpha^w} > 0, \]

\[ \frac{dy^R}{df} = 1 + \frac{\phi}{1 - \phi} \alpha^w \beta f G'(\bar{z}) \frac{d\bar{z}}{df} > 0. \]
Proof of Proposition 5. The job creation rule is obtained by equation (11). Substitute equations (13) and (15), we get

\[ E(y + z) - b + (1 - \beta)(1 - G(\bar{z})) f \\
- \frac{\beta + \phi \alpha^w(\theta)(1 - G(\bar{z}))}{(1 - \phi) \beta \alpha^f(\theta)} \left[ k + \beta f (1 - G(\bar{z})) \alpha^f(\theta) \right] \\
+ \beta (1 - \alpha^w(\theta)) \int_{z_{\min}}^{\bar{z}} \int_{y_R(z) + c + f}^{y_{max}} (1 - F(x)) \, dx \, dz \\
+ \beta \int_{\bar{z}}^{z_{\max}} \int_{y_R(z)}^{y_{max}} (1 - F(x)) \, dx \, dz = 0 \quad (38) \]

Denote the left hand side of equation (38) by \( h \) and differentiate it with respect to \( \theta \), \( f \) and \( c \), one gets:

\[ \frac{\partial h}{\partial \theta} = -\beta \alpha^w \int_{z_{\min}}^{\bar{z}} \int_{y_R(z) + c + f}^{y_{max}} (1 - F(x)) \, dx \, dz \\
- \frac{k \left[ \phi (1 - G(\bar{z})) \alpha^f \alpha^w - [\beta + \phi \alpha^w(\theta)(1 - G(\bar{z})) \alpha^f] \right]}{(1 - \phi) \beta (\alpha^f)^2} \\
- \frac{\beta + \phi \alpha^w (1 - G(\bar{z}))}{(1 - \phi) (1 - G(\bar{z}))} \alpha^w < 0, \]

due to \( \alpha^w(\theta) > 0 \) and \( \alpha^f(\theta) < 0 \),

\[ \frac{\partial h}{\partial f} = (1 - \beta)(1 - G(\bar{z})) - \frac{\beta + \phi \alpha^w(1 - G(\bar{z}))}{(1 - \phi)} (1 - G(\bar{z})) \\
- \beta (1 - \alpha^w) \int_{z_{\min}}^{\bar{z}} (1 - F(y^R(z))) \, dz. \]

\( \partial h/\partial f \) is negative provided that \( \beta > 1/2 \). Hence

\[ \frac{d\theta}{df} = -\frac{\partial h/\partial f}{\partial h/\partial \theta} < 0. \]

Finally, since

\[ \frac{\partial h}{\partial c} = -\beta (1 - \alpha^w) \int_{z_{\min}}^{\bar{z}} (1 - F(y^R(z))) \, dz < 0, \]

we can conclude that

\[ \frac{d\theta}{dc} = -\frac{\partial h/\partial c}{\partial h/\partial \theta} < 0. \]
B Solution Algorithm

Having set values for the different parameters of the model and parametric functions for the distributions of the shocks, we use the following iterative procedure to find the equilibrium solution. We begin with guesses for $\theta$, $\bar{z}$, $n^P$, $n^T$, and $u$ and we iterate in the following equations.

- Begin by finding the surplus functions $S^P$, $S^R$ and $S^T$ using value function iteration in equations (13), (14), and (15).
- Update $\theta$ using equation (11). Using the functional form for the matching function specified above, $\theta$ is given by:

$$
\left( \left( \frac{\Phi(y, z; \beta, \phi, c, f, \xi)}{k} \right)^{\xi} - 1 \right)^{\frac{1}{\xi}}
$$

and

$$
\Phi(\beta, \phi, c, f, \xi) = \int_{z_{min}}^{z_{max}} \left[ \mathbb{1}_A E_y S^P (y, z) + (1 - \mathbb{1}_A) E_y S^T (y, z) \right] dG (z) (1 - \phi)\beta - \beta f \mu G (A)
$$

- Update $\bar{z}$ by solving the two-equation system defined by equations (36) and (37), which solve for $\bar{z}$ and $y^P(\bar{z})$.
- With the updated $\bar{z}$ and $\theta$, we can update the employment measures -both temporary and permanent- using the steady-state versions of equations (25) and (26). These are given by:

$$
n^P = \frac{(u + n^T) \alpha^w (\theta) (1 - G (\bar{z}))}{1 - \int_{z_{min}}^{z_{max}} [1 - F(y^P (z))] dG (z)}
$$

and

$$
n^T = \frac{(1 - \alpha^w (\theta)) \int_{z_{min}}^{\bar{z}} [1 - F(y^R (z))] dG (z) n^T}{1 - \int_{z_{min}}^{z_{max}} [1 - F(y^P (z))] dG (z)}
$$

and,

$$
n^T = \frac{u \alpha^w (\theta) G (\bar{z})}{1 - \alpha^w (\theta) G (\bar{z})}
$$

- Update the measure of unemployed people using: $u = 1 - n^P - n^T$.

When the sequence of updated variables changes by a negligible amount, we have found an equilibrium. All integrals are evaluated numerically using quadrature.