Information Acquisition in Shareholder Voting*

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Abstract

We study information acquisition by investors prior to voting and trading. Extant work on information acquisition and governance ties the calculus of information acquisition to the potential value of information when voting. By considering a setting where shareholders can both vote and trade, we find that previous work misses the potentially more important value of information acquisition - informing trading choices. Thus, incentives to acquire information may be higher than previous theoretical work suggests. However, the opportunity to generate trading rents might distort voting incentives and reduce the quality of governance. We find that these negative incentives are stronger when more voters are informed. As a result, the quality of firm governance is eventually decreasing in the number of shareholders that become informed. Accordingly, there may be previously unrecognized costs to policies that generally lower the cost of information for investors. While governance quality can be improved by limiting information acquisition, it is true that allowing trade generally improves governance quality.

Keywords: strategic voting; common values voting; shareholder voting, corporate governance, information aggregation, Condorcet Jury Theorem

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1 Introduction

The justification for reliance on shareholders to vote on corporate policy (or provide a mandate for decisions by management) hinges largely on the idea that shareholders will obtain information that is germane to the decision at hand. In the setting of a common values problem where all shareholders simply evaluate whether a given policy increases firm value, Malenko and Malenko (2019) focus on natural sources of information in the context of publicly traded companies and highlight a natural intuition; a shareholder is willing to gather information up to the point where the marginal cost of additional information balances the expected impact the voter's information will have on the probability that the firm makes the correct choice. That is to say, in settings where shareholders are conceived of as just voters, the incentive to gather information is tied to the likelihood of impacting a vote.

Our point of departure is the recognition that shareholders see themselves as involved in more than just governance, and thus the value of information is not limited to its impact on voting. In reality, shareholders can vote as well as trade. In addition to realizing the value of information on governance by voting informatively, informed shareholders may also capitalize on their information by trading more wisely and extracting informational rents from markets. This suggests that the narrow focus on shareholders' roles as voters in the literature may understate shareholders' incentives to gather information and a theory of information acquisition in governance needs to look beyond just governance.

If the point were just that information levels are higher, then we would not have much new to say. In fact, things are more complicated it turns out. Building off the voting and trading model of Meirowitz and Pi (2022), we allow shareholders to choose whether to acquire information or not before they vote and trade and endogenize the level and distribution of private information. Once information acquisition decisions are tied to equilibrium, margins from both voting and trading equilibrium spillovers are present. We find that when too many shareholders acquire information, voting must be very uninformative and governance is of low quality. Accordingly, the effect of reducing information acquisition costs is ambiguous: if one could fix how shareholders vote, adding additional informed shareholders would improve governance. However, in equilibrium, adding these informed shareholders may cause equilibrium voting strategies to become less informative. Thus, instead of improving the informativeness of the governance decision, adding more informed voters may actually reduce the probability of selecting the correct corporate policy. The non-linear relationship between the number of informed shareholders and the efficiency of governance implies that there is a finite optimal number of informed shareholders and that governance is not always improved by reforms aiming to lower information costs for shareholders.

This paper bridges two recent theoretical contributions. As mentioned, Malenko and Malenko (2019) endogenizes information acquisition decisions. In particular, they look at how shareholders choose between idiosyncratic private signals and a common signal sold by a proxy advisor. The focus is on whether the latter crowds out the former. Meirowitz and Pi (2022) take the distribution of private information as fixed and add a trading stage to the classic voting problem. The key finding here is that shareholder voting is generally less informative because in equilibrium voters must balance incentives to steer firm policy in the correct decision with incentives to mislead the market about their assessment of the value of the shares and then trade on these informational asymmetries. Of course, both effects can be small for a given share, but equilibrium requires balancing these kinds of margins.

A secondary strand of theoretical work is also relevant. A few papers take a mechanism design approach to studying information acquisition and decision-making in committees or collective choice bodies. The focus of these papers is finding optimal ways to acquire and use the information to make a policy decision in common values setting. Here the most relevant papers are Persico (2004) and Gerardi and Yariv (2008). Persico (2004) focuses on characterizing the best equilibrium given a fixed voting rule. Gerardi and Yariv (2008) do not take the institution as fixed and characterize the optimal mechanism. Two important insights come from these papers. First, in contrast to the approach taken by Malenko and Malenko (2019), where the focus is on equilibria in which each voter uses the same mixed strategy in information acquisition decisions, Persico (2004) finds that efficiency requires that voters use asymmetric information acquisition strategies. Some voters opt to acquire information in pure strategies, and some opt to remain uninformative in pure strategies. This is commonly known in equilibria. We build off this insight and focus on equilibria where information acquisition strategies are degenerate and asymmetric because this involves an efficiency gain over the approach in Malenko and Malenko (2019). But, more importantly, we take this approach because many accounts of shareholder voting leave room for different kinds of investors. It is widely believed that some investors are active hands-on participants. They learn what they can and are typically involved voters. On the other hand, there are investors that routinely take a hands-off approach. The fact that one investor generally stays in one of these two categories as opposed to switching her level of engagement from vote to vote justifies this equilibrium selection. [1]

2 Overview of Findings

First, we find that there are two types of informational advantages shareholders may have over the market after voting. The first type of informational advantage occurs when a shareholder acquires information, and the market cannot perfectly infer her private information from the voting results. Moreover, we find that an informed shareholder is able to capitalize on her private signal by trading after every realization of the public voting outcomes (even if she is not the pivotal voter). This implies that the value of acquiring private information does not hinge on the probability of being pivotal. This feature is contrary to the conclusion commonly drawn by the previous literature. The second source of informational advantage is knowing if a vote is correlated with a private signal. Since the market cannot tell if a vote is from an informed shareholder and thus is informative or from an uninformed shareholder and thus is simply noise, the share prices after voting can be thought of as based on an average level of voting informativeness. This point allows us to see the second type of informational advantage which accrues to shareholders who do not acquire information (referred to as "uninformed shareholders" below). An uninformed shareholder knows that her vote is less informative than the market. Accordingly, she recognizes that market prices that treat her vote as having average informativeness are distorted. An uninformed voter thinks the firm is overpriced (underpriced) and thus wants to sell (buy) if she finds that the alternative she votes for is chosen (not chosen).

[1] A second insight, which comes out of the analysis of Gerardi and Yariv (2008) is that once the incentives to acquire information and reveal this information are both considered, an optimal institution will involve inefficient governance given the information that is acquired. This is of course true in our equilibrium as well. It is worth noting that we stop well short of the mechanism design approach. In fact, we don't even select the optimal equilibrium to the game as we do not seek to find equilibria where voters that opt to remain uninformed vote in pure strategies in ways that minimize their contribution to variance in the policy chosen. We choose to proceed in this way because following the insight that information acquisition choices are generally deterministic we follow the empirical literature in asserting that the voting behavior of retail investors is not particularly predictable

Second, we find that how informed shareholders utilize their private information depends on the number of shareholders acquiring information. Each informed shareholder is willing to vote for the alternative favored by her information only if the number of shareholders acquiring information is less than a threshold. If the number of informed shareholders is larger than this threshold, every informed shareholder votes with a mixed strategy and votes against her information with positive probability. This reduces the quality of information aggregation given the information that is acquired. Moreover, as the number of informed shareholders increases, the equilibrium votes of informed shareholders become less correlated with their information.

This second finding introduces a friction between the number of informed shareholders and the probability of making the correct decision via voting. On the one hand, as the number of informed shareholders increases, more information is being added to the voting game. This will increase the efficiency of voting. On the other hand, as the number of informed shareholders increases each informed shareholder's vote becomes less informative. This will decrease the likelihood of making the correct decision through voting. As a result, the overall effect of having more informed shareholders on the quality of the decision made by voting depends on which effect dominates. We find that in equilibrium the second effect eventually dominates the first one. At some point having more informed shareholders actually reduces the probability that the policy chosen by voting is correct.

Finally, when we compare a model with voting and trading to a baseline with only voting, we see that because of the opportunities to extract informational rents from trading the equilibrium levels of information acquisition are higher when trading is possible. Despite the distortions to voting that trading opportunities create, sometimes the increase in information acquisition from the opportunity to trade results in a higher equilibrium probability of making the correct choice. Thus despite distortions in voting that trading introduces, governance can be improved by the presence of trading opportunities after voting.

3 Model

Consider a firm that has n (an odd number) of shareholders and each of them has 1 share. We develop a three-period model in which shareholders can acquire information, vote, and trade. Voting consists of making a decision $x \in \{0, 1\}$ under simple majority rule. The shareholders face uncertainty about which decision is better for the firm. Formally speaking, denote the underlying state by $\omega \in \{0,1\}$ with the interpretation that if $x = \omega$ each share has value 1 and if $x \neq \omega$ each share has value 0. The common prior is that $Pr(\omega = 1) = \frac{1}{2}$.

In the first period, each shareholder decides to acquire a private signal s_i about the underlying state ω or not, $s_i \in \{0,1\}$. Private signals are imperfectly informative $Pr(s_i = \omega|\omega) = q > \frac{1}{2}$ and conditionally independent. We denote the cost of acquiring a private signal with c. If a shareholder does not acquire a signal, we denote her information set with $s_i = \emptyset$. We denote the acquisition choice of i, by a_i with $a_i = 1$ corresponding to acquiring a signal and $a_i = 0$ corresponding to not obtaining a signal. By $\bar{a}$ we denote the number of agents selecting $a_i = 1$. As our focus is on equilibria in which this choice is in pure strategies, the term is not random.

In the second period, shareholders cast ballots $v_i \in \{0,1\}$. By $v = (v_1, v_2, ..., v_n)$, we denote a profile of votes. Whichever policy receives more votes is selected. By $t = \sum_{i=1}^{n} v_i$ we denote the publicly available vote tally.

$$x = \begin{cases} 1 \text{ if } t \geq \frac{n-1}{2} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

It is convenient to also describe the tally from shareholders other than i, denoted $t_{-i} = \sum_{j \in n - \{i\}} v_j$.

In the third period, after observing the policy x, and the vote count $t = \sum_{i=1}^{n} v_i$, and the common price $P_x(t)$ each trader submits an order $b_i \in \{-1, 0, 1\}$ with the interpretation that $b_i = -1$ denotes selling their share, $b_i = 0$ denotes holding and $b_i = 1$ denotes buying an additional share. Trades are executed at the common price $P_x(t)$ which is assumed to satisfy a no-arbitrage condition,

$$P_x(t) = E[1_{x=\omega}|t]$$

where the expectation is taken over a version of conditional probability that is based on a correct conjecture of the joint probability $Pr(t|s)$. As long as strategies are measurable, we may conveniently write,

$$P_x(t) = Pr(\omega = x|x, t)$$

where the conditional probability satisfies Bayes rule given correct conjectures of the voting strategies. Note that because shareholders can compute the price based on the public information it does not matter whether we assume that the price is posted before or after orders are submitted.[2] Meirowitz and Pi (2022) extend the model to allow orders to impact prices and show tha qualitatively the logic from the posted price model carries over. For reasons of tractability we retain this feature from their baseline model.

Finally, the state is observed, and the value of the share is realized. One interpretation is that the firm provides a one-time dividend of either 1 or 0 for each share, and the game ends. Thus, at the end of the game, the value of each share is given by

$$v(x, \omega) = \begin{cases} 1 \text{ if } \omega = x \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

and an agent that bought a share obtains payoff $2v(x, \omega) - P_x(t)$, an agent that sold a share receives payoff $P_x(t)$ and an agent that made no trades obtains payoff $v(x, \omega)$.

4 Benchmark: Acquiring Information and Voting

We begin with a natural benchmark in which there is no trading opportunity after voting. We characterize the condition on cost, for there to exist an equilibrium in which $\bar{a} = k \leq n$ shareholders buy information and the remaining $n - k$ shareholders do not buy information.

Proposition 1 (Voting Strategy)**.** *Given that k shareholders acquire information, the following voting strategy constitutes a mutual best response: each informed shareholder sincerely votes their signal, $v_i(s_i) = s_i$ for $s_i \in \{0, 1\}$, and every uninformed shareholder votes for each policy with equal probability, $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$.*

Proof. We verify $v_i(s_i) = s_i$ for $s_i \in \{0, 1\}$ and $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$ are mutual best responses.

[2] To focus incentives on how information acquisition and governance can depend on the anticipation of optimal trading we do not explicitly include pre-voting trade. What matters is that at the time of voting, previous market transactions do not fully reveal the private information of the shareholders. The presence of noise traders is sufficient to ensure this feature.

Consider a shareholder with the signal of $s_i = 1$. Conjecturing that she is pivotal, her expected payoff from voting for policy 1 is

$$\begin{aligned}
&EU(v_i = 1|pivotal, s_i = 1) \\
&= Pr(\omega = 1|pivotal, s_i = 1) \\
&= \frac{Pr(t' = \frac{n-1}{2}|\omega = 1)q}{Pr(t' = \frac{n-1}{2}|\omega = 1)q + Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q)} \\
&= \frac{1}{1 + \frac{Pr(t'=\frac{n-1}{2}|\omega=0)}{Pr(t'=\frac{n-1}{2}|\omega=1)}\frac{1-q}{q}}
\end{aligned} \tag{3}$$

, where t' denotes the voting tally of all the other shareholders.

If she votes for policy 0, her expected payoff is

$$\begin{aligned}
&EU(v_i = 0|pivotal, s_i = 1) \\
&Pr(\omega = 0|pivotal, s_i = 1) \\
&= \frac{Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q)}{Pr(t' = \frac{n-1}{2}|\omega = 0)(1-q) + Pr(t' = \frac{n-1}{2}|\omega = 1)q} \\
&= \frac{1}{1 + \frac{Pr(t'=\frac{n-1}{2}|\omega=1)}{Pr(t'=\frac{n-1}{2}|\omega=0)}\frac{q}{1-q}}
\end{aligned} \tag{4}$$

Note that $Pr(t' = \frac{n-1}{2}|\omega = 0) = Pr(t' = \frac{n-1}{2}|\omega = 1)$ because of the symmetry of voting strategies. Then, since $\frac{1}{2} < q < 1$, we have $\frac{q}{1-q} > \frac{1-q}{q}$, and thus we have $EU(v_i = 1|pivotal, s_i = 1) > EU(v_i = 0|pivotal, s_i = 1)$.

Similarly, when $s_i = 0$, we have $EU(v_i = 0|pivotal, s_i = 0) > EU(v_i = 1|pivotal, s_i = 0)$. So, for each informed shareholder, voting sincerely is the best response.

Now consider a shareholder without a signal. The uninformed shareholder's expected payoff from voting for policy 1 is

$$\begin{aligned}
&EU(v_i = 1|pivotal, s_i = \emptyset) \\
&= \frac{Pr(t' = \frac{n-1}{2}|\omega = 1)}{Pr(t' = \frac{n-1}{2}|\omega = 1) + Pr(t' = \frac{n-1}{2}|\omega = 0)}
\end{aligned} \tag{5}$$

Her expected payoff from voting for 0 is

$$EU(v_i = 0|pivotal, s_i = \emptyset) \\ = \frac{Pr(t' = \frac{n-1}{2}|\omega = 0)}{Pr(t' = \frac{n-1}{2}|\omega = 0) + Pr(t' = \frac{n-1}{2}|\omega = 1)} \tag{6}$$

Since we have $Pr(t' = \frac{n-1}{2}|\omega = 0) = Pr(t' = \frac{n-1}{2}|\omega = 1)$, the uninformed shareholder is indifferent with voting for each policy. Thus, $Pr(v_i(s_i = \emptyset) = 1) = Pr(v_i(s_i = \emptyset) = 0) = \frac{1}{2}$ is a best response ■

Of course, their are other mutual best responses. For example holding fixed the acquisition strategies a profile where all voters cast the same ballot is a mutual best response since no voter is voter is ever pivotal. We now examine how the number of informed shareholders affects the probability that the above voting strategy makes a correct decision.

Proposition 2 (Information Aggregation)**.** *The probability that the above voting strategy selects the correct policy is strictly increasing in the number of informed shareholders, k.*

$$\frac{dPr^*(x = \omega)}{dk} > 0$$

Proof.

$$Pr(\omega = x) \\ = Pr(x = 1|\omega = 1)Pr(\omega = 1) + Pr(x = 0|\omega = 0)Pr(\omega = 0) \\ = \frac{1}{2}(Pr(t \geq \frac{n+1}{2}|\omega = 1) + Pr(t \leq \frac{n-1}{2}|\omega = 0)) \tag{7}$$

Note that $Pr(t \geq \frac{n+1}{2}|\omega = 1) = Pr(t \leq \frac{n-1}{2}|\omega = 0))$ due to the symmetry of voting strategies. We have

$$
\begin{aligned}
&Pr(\omega = x) \\
&= \sum_{t=\frac{n+1}{2}}^{n} Pr(t|\omega = 1) \\
&= \sum_{t=\frac{n+1}{2}}^{n} \sum_{i=0}^{t} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{t-i} (\frac{1}{2})^{n-k} \qquad (8) \\
&= 1 - \Phi\left(\frac{\frac{n+1}{2} - (kq + (n-k)\frac{1}{2})}{\sqrt{(kq(1-q) + (n-k)\frac{1}{4})}} \right)
\end{aligned}
$$

Because $\frac{d\frac{\frac{n+1}{2}-(kq+(n-k)\frac{1}{2})}{\sqrt{(kq(1-q)+(n-k)\frac{1}{4})}}}{dk} < 0$, we have $Pr(\omega = x)$ increases in k. ■

Proposition 2 shows that without considering the market, having more informed shareholders improves the probability of selecting the correct policy via shareholder voting. This is because an informed shareholder's vote is more likely to be correct than an uninformed shareholder's vote and the probability an informed voter votes correctly does not depend on the number of informed voters, In the following section, we will see that if the probability that an informed shareholder's vote is correct decreases (as it does in equilibrium when there is also trading) then when more shareholders acquire information, the probability of selecting the correct policy does not necessarily increase.

Proposition 3 (Information Acquisition Strategies and Information Cost)**.** *The value of acquiring a private signal is only realized when a shareholder is pivotal. To support an equilibrium in which k shareholders buy information, the cost of acquiring information, c must satisfy*

$$(q - \frac{1}{2}) \sum_{i=0}^{k-1} \binom{k-1}{i} q^i (1-q)^{k-i-1} \binom{n-k+1}{\frac{n+1}{2} - i} (\frac{1}{2})^{n-k+1} \leq c$$

$$(q - \frac{1}{2}) \sum_{i=0}^{k} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{\frac{n-1}{2} - i} (\frac{1}{2})^{n-k} \geq c$$

Proof. To sustain an equilibrium in which k shareholders buy information we must rule out two deviations, one additional shareholder acquiring information and one less shareholder acquiring information. To rule out the first we need c to exceed the change in the probability that an uninformed voter's vote is correct if she acquires a signal, $q - \frac{1}{2}$ weighted by the odds of being pivotal if you are supposed to acquire a signal when k of n voters acquire signals,

$$(q - \frac{1}{2}) \sum_{i=0}^{k-1} \binom{k-1}{i} q^i (1-q)^{k-i-1} \binom{n-k+1}{\frac{n+1}{2} - i} (\frac{1}{2})^{n-k+1} \leq c$$

To rule out the second we need the cost to be less than the change in probabilty of voting correctly weighted by the odds a voter not acquiring a signal is pivotal

$$(q - \frac{1}{2}) \sum_{i=0}^{k} \binom{k}{i} q^i (1-q)^{k-i} \binom{n-k}{\frac{n-1}{2} - i} (\frac{1}{2})^{n-k} \geq c$$

■

The takeaway here is that more information is always better when there is no liquidity. We thus are tempted to call for reducing the price of information.

5 Acquiring Information, Voting, and Trading

Now we consider the full model in which shareholders can trade after they vote. Because votes may reveal information and influence inferences that impact trading we seek Perfect Bayesian Equilibrium. In the information acquisition period, k shareholders acquire private information while the remaining $n - k$ shareholders do not buy information. We focus on equilibria with type-symmetric voting strategies in which in the voting period, each informed shareholder votes for her private signal with probability $m \in [\frac{1}{2}, 1]$. In such an equilibrium the probability that an informed shareholder's vote is correct conditional on the underlying state is $z := Pr(v_i = \omega|\omega) = mq + (1-m)(1-q)$. We also look for an equilibrium in which the uninformed shareholders votes for each policy with equal probability. Recall in the model in which trading is possible we assume that in the trading period, every shareholder can choose to buy one share, hold, or sell one share, $b_i \in \{-1, 0, 1\}$.

We analyze the model starting with subforms in which trading occurs.

5.1 Trading Stage

The stock price in the trading stage depends on the probability that voting selects the correct policy.[3] Given a selected policy, the price depends on the number of votes for the selected policy. For example, when $x = 1$, a larger voting tally t implies that more informed shareholders are receiving the signal of 1, and thus the stock price based on a larger t is higher than the stock price based on a smaller t.

The following lemma gives the pricing function that describes how the price changes with x and t.

Lemma 1 (Stock Price After Voting)**.** *The price after voting depends on the chosen policy x and voting tally t.*

$$P(x,t) = E[v(x,\omega)|x,t] = \begin{cases} Pr(\omega = 1|t), & \text{if } x = 1 \\ 1 - Pr(\omega = 1|t), & \text{if } x = 0 \end{cases} \tag{9}$$

where

$$Pr(\omega = 1|t) = \frac{Pr(t|\omega = 1)}{Pr(t|\omega = 1) + Pr(t|\omega = 0)} \tag{10}$$

$$Pr(t|\omega = 1) = \sum_{i=0}^{t} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k} \tag{11}$$

and

$$Pr(t|\omega = 0) = \sum_{i=0}^{t} \binom{k}{i} (1-z)^i z^{k-i} \binom{n-k}{t-i} \left(\frac{1}{2}\right)^{n-k}. \tag{12}$$

Proof. The expression is obtained by substituting into Bayes rule. ■

5.1.1 Trading Strategy

In the trading stage, each shareholder buys (sells) one share if her expectation of firm value is higher (lower) than the price. After shareholder voting, how a shareholder interprets the voting results (x, t) and thus her expectation of firm value depends

[3] To simplify the model without losing intuitions, we assume that the price does not depend on the number of trading orders. Meirowitz and Pi (2022) show that an informed shareholder's trade-off between voting for the policy she thinks is best and voting against it to maximize trading rents still holds when the pricing function also depends on net trading orders from shareholders and noise traders.

on whether the shareholder buys information or not, how she votes, and her private information (if she has any). Therefore, it is natural that shareholders take various sides of the market after observing the voting results. In particular, different information positions ($a_i = 0$ v.s. $a_i = 1$) cause uninformed shareholders and informed shareholders to take different trading strategies, and informed shareholders with different private information ($s_i = 1$ v.s. $s_i = 1$) also trade differently. But as we now show the driving equilibrium is stark. Uninformed shareholders bet against their vote because the recognize that the market will interpret their vote for 1 (0) as weak evidence in favor of 1 (0), while they know the vote is based on a coin toss only. Informed shareholders bet in line with their signal because they recognize that their signal is not fully capitalized into market prices due to the fact that in equilibrium voting is not fully informative.

Proposition 4 (Trading Strategy)**.** *At the trading stage, each uninformed shareholder buys one share if $x \neq v_i$ and sells one share if $x = v_i$. Every informed shareholder buys one share if $x = s_i$ and sells one share if $x \neq s_i$.*

Proof. Suppose without the loss of generality that $x = 1$. When $x = 1$, the price is given by $Pr(\omega = 1|t)$. Each shareholder's trading strategy depends on the compassion between her expectation of the firm value and the price.

First, consider an uninformed shareholder who votes for 0. It is easy to see that $Pr(\omega = 1|v_i = 0, t, s_i = \emptyset) > Pr(\omega|t)$, because the uninformed shareholder knows that t voters among k informed shareholders and $n - k - 1$ uninformed shareholders vote for 1, yet the market thinks that t voters among k informed shareholders and $n - k$ uninformed shareholders vote for 1. So, the uninformed shareholder privately knows that each vote for policy 1 has a higher chance of being cast by an informed shareholder than what the market believes ($\frac{k}{n-1}$ v.s $\frac{k}{n}$). Therefore, the uninformed shareholder wants to buy one share.

Second, we prove that an uninformed shareholder who votes for 1 wants to sell. To do this we show that $Pr(\omega = 1|t, v_i = 1, s_i = \emptyset) \leq Pr(\omega = 1|t)$ for $t \in [\frac{n+1}{2}, n]$.[4]

[4]Perhaps interestingly, the above arguments proving $Pr(\omega = 1|v_i = 0, t, s_i = \emptyset) > Pr(\omega|t)$ cannot directly prove $Pr(\omega = 1|v_i = 1, t, s_i = \emptyset) > Pr(\omega|t)$. If the uninformed shareholder vote for 1 and observes t, she knows that $t - 1$ voters among k informed shareholders and $n - k - 1$ uninformed shareholders. Thus, although the uninformed shareholder still believes each vote has a higher probability of being cast by an informed shareholder, the uninformed shareholder also realizes there may be fewer informed voters voting for policy 1. It is easier to notice this via an example. Suppose that $t \geq k$. Then, given t and $v_i = 0$, it is possible that t informed shareholders vote for 1. But, if given t and $v_i = 1$, there are at most $t - 1$ shareholders voting for 1.

When $t = \frac{n+1}{2}$, shareholder i knows that she is the pivotal voter, which implies that there are exactly half of the other $n-1$ vote for 1 and half of them vote for 0. Due to the symmetry of voting strategies, the pivotal event is not informative to shareholder i. Since she does not have private information, we must have

$$Pr(\omega = 1|t = \frac{n+1}{2}, v_i = 1, s_i = \emptyset) = \frac{1}{2}.$$

On the other hand, we also know that the price

$$Pr(\omega = 1|t = \frac{n+1}{2}) > \frac{1}{2}$$

because of the existence of k informative votes ($z > \frac{1}{2}$).

Note that $\frac{d|Pr(\omega=1|t) - Pr(\omega=1|t, v_i=1, s_i=\emptyset)|}{dt} < 0$ for $t \in [\frac{n+1}{2}, n]$. This is because both the market's posterior beliefs and shareholder i' posterior beliefs are affected by the public information t. As the public signal becomes more convincing, the differences in agents' beliefs shrink. The voting tally t is most noise when it is around $\frac{n+1}{2}$ and becomes more convincing as it approaches to the two tails, 0 and n. Thus, when t is between $\frac{n+1}{2}$ and n, the difference between the market's Bayesian posterior belief and shareholder i' Bayesian posterior beliefs, $|Pr(\omega = 1|t) - Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)|$, monotonically decreases with t.

Then, we focus on $t = n$. When $t = n$, both the market and shareholder i know that every shareholder, including shareholder i, must vote for 1. Therefore, when $t = n$, the uniformed shareholder and the market essentially have the same information set, and thus $Pr(\omega = 1|t) = Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)$ when $t = n$.

Therefore, we know that $Pr(\omega = 1|t) > Pr(\omega = 1|t, v_i = 1, s_i = \emptyset)$ when $t \in [\frac{n+1}{2}, n]$. As a result, the uninformed shareholder wants to sell when she votes for 1 and observes that $x = 1$.

Now consider an informed shareholder who buys information and has $s_i = 0$. If she votes for 1, we know that $Pr(\omega = 1|t = \frac{n+1}{2}, v_i = 1, s_i = 0) = 1 - q < Pr(\omega = 1|t = \frac{n+1}{2}) = \frac{1}{2}$. We also know that $Pr(\omega = 1|t = n, v_i = 1, s_i = 0) < Pr(\omega = 1|t = n)$, because the informed shareholder has private information that her vote for 1 actually is strategically against her signal 0. Recall that the difference between the market's belief and the shareholder's belief shrinks when t increases from $t = \frac{n+1}{2}$ to n. Therefore, we know that $Pr(\omega = 1|t, v_i = 1, s_i = 0) < Pr(\omega = 1|t)$ for $t \in [\frac{n+1}{2}, n]$, and thus the shareholder wants to sell. Now consider the case that the informed shareholder has $s_i = 0$ but votes for 0. When observing t, both the market maker and the informed shareholder know that $n - t$ shareholders votes for 0. For every vote for policy 0, the market maker cannot tell the type of shareholder, $s_i \in \{\emptyset, 0, 1\}$, casting it. However, the shareholder knows that her vote for policy 0 is based on her private signal of s_i. Thus, for every $t \in [\frac{n+1}{2}, n-1]$, the shareholder's expectation of firm value is lower than the price. So, the shareholder with the signal of $s_i = 0$ still wants to sell if she votes for 0.

■

In equilibrium, each uninformed shareholder compares her vote and the chosen policy, and then buys (sells) if her vote is different from (the same as) the chosen policy. Every informed shareholder compares her private signal and the chosen policy, and then buys (sells) if her private signal is the same as (different from) the chosen policy. we now move back to the voting period.

5.2 Voting Period

Each informed shareholders' voting strategies depend on the difference between $EU(v_i = s_i|s_i)$ and $EU(v_i \neq s_i|s_i)$ when these expected utilities correctly anticipate equilibrium trading strategies and market price as a function of t. We have

$$\begin{aligned} & EU(v_i = s_i|s_i) - EU(v_i \neq s_i|s_i) \\ & = \underbrace{Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)}_{Pivotal\ Effect} \\ & - \underbrace{\sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t' + 1) - Pr(\omega = 1|t'))}_{Signaling\ Effect} \end{aligned} \tag{13}$$

Since the first term measures the gains from voting for one's signal over voting against one's signal when an informed shareholder i is pivotal, we call it the "Pivotal Effect". The second term sums the gains from voting against one's signal over voting for one's signal and trading to capatialize on this informational advantage in all cases. Thus, we call the second term "Signaling Effect".

An informed shareholder's voting strategy depends on which effect dominates. If *Signaling Effect(z)-Pivotal Effect(z)* ≤ 0 when $z = q$, then voting sincerely is a best response for an informed shareholder. However, if *Signaling Effect(z)-Pivotal Effect(z)* ≥ 0 when $z = q$ then voting against one's signal is a best response for an informed shareholder. If at z^* both inequalities hold with equality then mixing is a best response and we can support the mixed strategy m with $z^* = mq + (1-q)m$. In particular, z^* is determined by the following indifference condition.

$$\begin{aligned}&\sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(Pr(\omega = 1|t'+1) - Pr(\omega = 1|t')) \\ &= Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1)\end{aligned} \tag{14}$$

Obviously, both the signaling effect and pivotal effect are a function of the number of informed shareholders, k. Thus, in equilibrium, the number of informed shareholders, k, affect the strength of the pivotal and signaling effects, and thus influence informed shareholders' voting strategies.

We may gain traction on how $\frac{k}{n}$ impacts voting and information aggregation by considering the case of large n. Accordingly, we close this section by assuming n is large and employing asymptotic methods. In the next subsection we help fix ideas by presenting a small n example.

Proposition 5 (Voting Strategies)**.** *Let $n \to \infty$. For each n there is a threshold $\kappa(n)$ which converges to $\frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$ for which in an equilibrium voting strategies of the informed shareholders are as follows*

If the ratio between the number of informed shareholders and the number of all shareholders is weakly smaller than the threshold, $\frac{k}{n} \leq \kappa(n)$, then every informed shareholder sincerely votes for her signal. If $\frac{k}{n} > \kappa(n)$, every informed shareholder takes a mixed voting strategy. The probability that each informed shareholder's vote is correct is converges to

$$z^*(n,k) = Pr(v_i = \omega|\omega) = \frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}erf\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

In both cases every uninformed shareholder votes for each policy with probability one half.

Proof. We firstly simplify the signaling effect and pivotal effect in algebra.

$$
\begin{aligned}
&\text{Signaling Effect} \\
&= \sum_{t'=0}^{n-1} Pr(t'|s_i = 1)(P_{x=1}(t'+1) - P_{x=1}(t')) \\
&= \sum_{t'=0}^{n-1} (Pr(t'|\omega = 1)q + Pr(t'|\omega = 0)(1-q))(P_{x=1}(t'+1) - P_{x=1}(t')) \\
&= q \sum_{t'=0}^{n-1} (Pr(t'|\omega = 1)(P_{x=1}(t'+1) - P_{x=1}(t')) \\
&+ (1-q) \sum_{t'=0}^{n-1} Pr(t'|\omega = 0)(P_{x=1}(t'+1) - P_{x=1}(t')) \\
&= \sum_{t'=0}^{n-1} (Pr(t'|\omega = 1)(P_{x=1}(t'+1) - P_{x=1}(t'))
\end{aligned}
\tag{15}
$$

The last step is because $\sum_{t'=0}^{n-1} Pr(t'|\omega = 1)(P_{x=1}(t'+1) - P_{x=1}(t')) = \sum_{t'=0}^{n-1} Pr(t'|\omega = 0)(P_{x=1}(t'+1) - P_{x=1}(t'))$ due to the symmetry of binomial distribution.

$$
\begin{aligned}
&\text{Pivotal Effect} \\
&= Pr(t' = \frac{n-1}{2}|s_i = 1)(2Pr(\omega = 1|t' = \frac{n-1}{2}, s_i = 1) - 1) \\
&= (Pr(t' = \frac{n-1}{2}|\omega = 1)q + Pr(t' = t' = \frac{n-1}{2}|\omega = 0)(1-q))(2q-1) \\
&= Pr(t' = \frac{n-1}{2}|\omega = 1)(2q-1)
\end{aligned}
\tag{16}
$$

Thus, the indifference condition implying z^* is

$$
\sum_{t'=0}^{n-1} Pr(t'|\omega = 1)(P_{x=1}(t'+1) - P_{x=1}(t')) = Pr(t' = \frac{n-1}{2}|\omega = 1)(2q-1) \tag{17}
$$

Dividing both sides by $Pr(t' = \frac{n-1}{2}|\omega = 1)$, we have

$$
\sum_{t'=0}^{n-1} \frac{Pr(t'|\omega = 1)}{Pr(t' = \frac{n-1}{2}|\omega = 1)}(P_{x=1}(t'+1) - P_{x=1}(t')) = 2q-1 \tag{18}
$$

Note that conditional on $\omega = 1$, the voting tally t is the convolution of two binomial distributions with different success rates (z and $\frac{1}{2}$). When n is large, this convolution approximates to a normal distribution with the mean of $kz + (n-k)\frac{1}{2}$ and the variance of $kz(1-z)+(n-k)\frac{1}{4}$. Similarly, conditional on $\omega = 1$ ($\omega = 0$), t' is normally distributed with with the mean of $(k-1)z+(n-k)\frac{1}{2}$ ($(k-1)(1-z)+(n-k)\frac{1}{2}$) and the variance of $(k-1)z(1-z)+(n-k)\frac{1}{4}$. Thus, the left hand side of the difference condition becomes

$$\int_{t'=0}^{n-1} \frac{\phi(t';\mu_1,\sigma_1)}{\phi(\frac{n-1}{2};\mu_1,\sigma_1)} \cdot \left(\frac{\phi(t'+1;\mu_2,\sigma_2)}{\phi(t'+1;\mu_2,\sigma_2)+\phi(t'+1;\mu_3,\sigma_3)} - \frac{\phi(t';\mu_2,\sigma_2)}{\phi(t';\mu_2,\sigma_2)+\phi(t';\mu_3,\sigma_3)} \right) dt' \tag{19}$$

, where
$\mu_1 = (k-1)z + (n-k)\frac{1}{2}$, $\sigma_1 = \sqrt{(k-1)z(1-z)+(n-k)\frac{1}{4}}$
$\mu_2 = kz + (n-k)\frac{1}{2}$, $\sigma_2 = \sqrt{kz(1-z)+(n-k)\frac{1}{4}}$
$\mu_3 = k(1-z) + (n-k)\frac{1}{2}$, $\sigma_3 = \sqrt{kz(1-z)+(n-k)\frac{1}{4}}$

We view the equation in the integral symbols as a function of z and denote it as $f(z)$. We then take Taylor expansions of of $f(z)$ around $z = \frac{1}{2}$.[5] Then, we have

$$\begin{aligned} & f(z) \\ &= f(\frac{1}{2}) + \frac{f'(\frac{1}{2})}{1!})(z-\frac{1}{2}) + \frac{f'(\frac{1}{2})}{2!})(z-\frac{1}{2})^2 + \ldots\ldots \\ &= 0 + \frac{2k\left(z-\frac{1}{2}\right)e^{-\frac{(n-2t-1)^2}{2(n-1)}}}{n} - \frac{4\left(z-\frac{1}{2}\right)^2\left((k-1)ke^{-\frac{(n-2t-1)^2}{2(n-1)}}(n-2t-1)\right)}{(n-1)n} + O\left(\left(z-\frac{1}{2}\right)^3\right) \end{aligned} \tag{20}$$

Accordingly, we have

$$\frac{\sqrt{\frac{\pi}{2}}k\sqrt{n-1}(2z-1)\text{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}{n} = 2q - 1 \tag{21}$$

In an equilibrium in which informed shareholders sincerely vote for their information ($z = q$), it must be the signaling effect is weakly smaller than the pivotal effect when $z = q$. This implies that

[5] There are two reasons for expanding at $z = \frac{1}{2}$. First, as the number of informed shareholders, k, increases to n (the case that every shareholder has private information), our model becomes to the model of Meirowitz and Pi (2022). According to Propostion XXX of Meirowitz and Pi (2022), we know that we must have $\lim_{n(k)\to\infty} z \to \frac{1}{2}$. Second, note that every term contains $(z-\frac{1}{2})^i$. Recall that $z \in (\frac{1}{2}, q)$. Then, we must have $0 < z - \frac{1}{2} < q - \frac{1}{2} < 1$. Since $0 < z - \frac{1}{2} < 1$, we know that, for any z in $(\frac{1}{2}, q)$, $(z-\frac{1}{2})^i$ must exponentially vanish towards 0 as i increases to ∞.

$$\frac{k}{n} \leq \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

That is to say, to ensure that all informed shareholders sincerely vote for the information they own in voting ($z = q$), we cannot have too many informed shareholders. In particular, the ratio between the amounts of informed shareholders (k) and the amounts of all shareholders (n) cannot be too high.

On the other hand, if $\frac{k}{n} > \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$, then we have equilibria in which informed shareholders strategically vote with $z < q$. In particular, z is given by

$$\frac{1}{2} + \frac{n(2q-1)}{\sqrt{2\pi}k\sqrt{n-1}\mathrm{erf}\left(\frac{\sqrt{n-1}}{\sqrt{2}}\right)}$$

■

5.3 Information Aggregation

We first give a numerical solution. Suppose $n = 9$, $q = \frac{4}{5}$, and $b = \frac{1}{2}$. We solve z and the range of c for each $k \in \{0, 1, ..., 8, 9\}$.

k	c_L	c_h	z	Pr(x=w)
0	0.382031	N/M	N/M	0.5
1	0.303365	0.315365	0.80	0.582031
2	0.209935	0.235375	0.80	0.664063
3	0.139089	0.168149	0.780375	0.726551
4	0.144461	0.165745	0.709116	0.723005
5	0.148089	0.164801	0.666364	0.720304
6	0.15058	0.164317	0.638041	0.718366
7	0.152368	0.164025	0.617935	0.716943
8	0.153707	0.163829	0.602933	0.715863
9	N/M	0.163689	0.591313	0.715017
N/M means Not Meaningful				

Figure 1: Equilibrium Voting in which k shareholders acquire information

We can use the above calculs to exhibite how voting and information aggregation depend on k in a small n example. The efficiency of information aggregation is measured by the probability of selecting the correct policy, $Pr(x = \omega)$. As shown in Figure 1, this probability is concave on the number of informed shareholders. Information aggregation depends on two things: the number of informed shareholders and the informativeness of informed shareholders' votes. When $k = 1, 2$, informed shareholders sincere votes for their private information ($z = q$), and thus the informativeness of informed shareholders' votes reaches its maximum. When $k > 2$, there is no sincere-voting equilibrium, and thus $z < q$.

The growth of the number of informed shareholders brings two effects. On the one hand, more signals are being added to the voting game, which helps information aggregation. On the other hand, the informativeness of informed shareholders' votes is decreasing ($\frac{\partial z}{\partial k} < 0$), which hurts information aggregation. Thus, the overall effect of having more informed shareholders on information aggregation depends on which effect dominates. Figure 1 implies that as the number of shareholders grows, the second effect gradually dominates the first effect. Thus, the probability of selecting the correct policy via voting can decrease with the number of informed shareholders. In this example, $Pr(x = \omega)$ increases with k when $k \geq 2$, reaches its maximum when $k = 3$, and decreases with k when $k > 3$. We now show that this pattern generalizes.

Proposition 6 (Information Aggregation). *If n is large enough, then when $\frac{k}{\sqrt{n}} \leq \frac{1}{\sqrt{2}\int_0^{\frac{n-1}{\sqrt{2n}}} e^{-x^2}dx}$, so that informed shareholders sincerely vote ($z = q$) $Pr(x = \omega)$ increases with k. However, when $\frac{k}{\sqrt{n}} > \frac{1}{\sqrt{2}\int_0^{\frac{n-1}{\sqrt{2n}}} e^{-x^2}dx}$, so that informed shareholders strategically vote ($z < q$), $Pr(x = \omega)$ decreases with k. Thus, $Pr(\omega = x)$ is a concave function of k, and k^* maximizing $Pr^*(x = \omega)$ is given by $\frac{\sqrt{n}}{\sqrt{2}\int_0^{\frac{n-1}{\sqrt{2n}}} e^{-x^2}dx}$.*

Proof. From above we have in the limit

$$Pr(x = \omega) = 1 - \Phi\left(\frac{\frac{n+1}{2} - (kz + (n-k)\frac{1}{2})}{\sqrt{kz(1-z) + (n-k)\frac{1}{2}}}\right)$$

First, as k increases, there are more informative votes ($z > \frac{1}{2}$), which helps information aggregation. Second, as k increases, each informative vote becomes less informative ($\frac{dz}{dk} < 0$), which hurts information aggregation. As a result, the aggregate effect of having more informed shareholders on information aggregation efficiency depends on which effects dominate.

After we substitute $z = \frac{1}{2} + \frac{\sqrt{n}(2q-1)}{\sqrt{2\pi}k\cdot\text{erf}\left(\frac{n-1}{\sqrt{2n}}\right)}$ into $Pr(x = \omega)$, we get

$$Pr(x = \omega) = 1 - \Phi\left(\frac{\sqrt{\pi}\text{erf}\left(\frac{n-1}{\sqrt{2n}}\right) + \sqrt{2n}(1-2q)}{\text{erf}\left(\frac{n-1}{\sqrt{2n}}\right)\sqrt{n\left(\pi - \frac{2(1-2q)^2}{k\cdot\text{erf}\left(\frac{n-1}{\sqrt{2n}}\right)^2}\right)}}\right)$$

Note that $\sqrt{\pi}\text{erf}\left(\frac{n-1}{\sqrt{2n}}\right) + \sqrt{2n}(1-2q) < 0$. Then, as k increases, $Pr(\omega = x)$ decreases in the number of informed shareholders.

Overall, when $\frac{k}{\sqrt{n}} \leq \frac{1}{\sqrt{2}\int_0^{\frac{n-1}{\sqrt{2n}}} e^{-x^2}dx}$, informed shareholders sincerely vote ($z = q$), and $Pr(x = \omega)$ increases with k. However, when $\frac{k}{\sqrt{n}} > \frac{1}{\sqrt{2}\int_0^{\frac{n-1}{\sqrt{2n}}} e^{-x^2}dx}$, informed shareholders strategically vote ($z < q$), and $Pr(x = \omega)$ decreases with k. Thus, $Pr(\omega = x)$ is a concave function of k. ■

5.4 Endogenizing the Number of Informed Shareholders

So far we have treated k as exogenous. We now consider the initial stage of the game where shareholders make information acquisition decisions. This involves characterizing a condition on cost,c to support investment in acquiring information by k and only k shareholders. To sustain an equilibrium in which k shareholders buy information and the rest $n - k$ shareholders do not buy information, the cost of acquiring a signal must satisfy the condition that none of the k informed shareholders wants to deviate by being uninformed and none of the $n - k$ uninformed shareholders want to deviate by being informed. In working through the calculations, one key feature that surfaces is that a shareholder can realize some value from here private signal for any realization of t. That is to say, information provides some benefits even if the shareholder is not pivotal.

Proposition 7 (Information Value and Information Costs)**.** *In an equilibrium in which k shareholders buy information and $n - k$ shareholders do not buy information,* *the cost of information must satisfy*

$$\underbrace{EU(a_i = 0) - EU(a_i = 0 \xrightarrow{d} 1)}_{\underline{c}(k)} \leq c \leq \underbrace{EU(a_i = 1) - EU(a_i = 1 \xrightarrow{d} 0)}_{\bar{c}(k)} \qquad (22)$$

The proof is in Appendix.

Proposition 7 gives the minimum cost and maximum cost that can sustain in equilibrium in which exactly k informed shareholders buy information. In particular, the cost cannot be cheaper than $\underline{c}(k)$, otherwise an uninformed shareholder would deviate from the equilibrium and buys information. The cost cannot be more expensive than $\bar{c}(k)$, otherwise, an informed shareholder would deviate from the equilibrium and not invest in acquiring information

We continue the example above and calculate each interval of costs $[\underline{c}(k), \bar{c}(k)]$ that can sustain the equilibrium in which $k = \{0, 1, 2....7, 8, 9\}$ shareholders buy information.

k	$EU(a_i = 1)$	$EU(a_i = 1 \xrightarrow{d} 0)$	$\bar{c}(k)$	$EU(a_i = 0)$	$EU(a_i = 0 \xrightarrow{d} 1)$	$\underline{c}(k)$
0	-	-	-	0.5000	0.8199	0.3199
1	0.8487	0.5333	0.3154	0.6154	0.9187	0.3034
2	0.8795	0.6442	0.2354	0.7256	0.9356	0.2099
3	0.8998	0.7317	0.1681	0.8026	0.9417	0.1391
4	0.9117	0.7460	0.1657	0.7970	0.9415	0.1445
5	0.9182	0.7534	0.1648	0.7932	0.9412	0.1481
6	0.9223	0.7579	0.1643	0.7904	0.9410	0.1506
7	0.9250	0.7610	0.1640	0.7885	0.9408	0.1524
8	0.9270	0.7632	0.1638	0.7870	0.9407	0.1537
9	0.9285	0.7648	0.1637	-	-	-

6 Information Acquisition and Aggregation With and Without Trading

The above analysis has shown that the opportunity to trade after voting incentives shareholders to strategically vote against their information and thus may distort the policy choice. Yet, this section shows that the probability of selecting the correct policy via voting may be higher when trading is possible than when there is no trading, $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$.

When there is no trading, a shareholder can benefit from investing in information only if her vote is pivotal. In contrast, when there is trading, a shareholder can realize informational advantages in all cases; In addition to increasing $Pr(x = w)$ when she happens to be pivotal, she can always capitalize on her private information through strategic trading on the market when she is not pivotal. Thus, trading, as the other channel to extract information value, encourages more shareholders to invest in information than when there is no trading, which may improve the probability of selecting the correct policy via voting.

Formally speaking, the opportunity to trade generates two effects. First, given a fixed information cost, we should see more shareholders invest in information when trading is allowed. Second, recall that the opportunity of trading incentives shareholders to strategically vote against their information, and thus the informativeness of informed shareholders' votes may be smaller when trading is possible than when trading is impossible. As a result of these two effects, $Pr(x = \omega; Trading)$ can be larger (smaller) than $Pr(x = \omega; No\ Trading)$ if the first effect (second effect) dominates.

Before rigorously analyzing this, we illustrate the phenomena by continuing the example. In particular, suppose that shareholders cannot trade, we find the minimum cost and maximum cost that supports an equilibrium in which $k \in \{0, 1, 2...7, 8, 9\}$ shareholders acquire information and calculate the corresponding $Pr(x = \omega; No\, Trading)$. The results are shown in the left three columns of the table below.

k	Trading			No Trading		
	Max Cost	Mini Cost	$Pr(x = \omega)$	Max Cost	Mini Cost	$Pr(x = \omega)$
0	-	0.31989	0.50000	-	0.08203	0.50000
1	0.31537	0.30336	0.58203	0.08203	0.08203	0.58203
2	0.23538	0.20994	0.66406	0.08203	0.07781	0.66406
3	0.16815	0.13909	0.72655	0.07781	0.06938	0.74188
4	0.16575	0.14446	0.72301	0.06938	0.05763	0.81125
5	0.16480	0.14809	0.72030	0.05763	0.04440	0.86888
6	0.16432	0.15058	0.71837	0.04440	0.03187	0.91328
7	0.16402	0.15237	0.71694	0.03187	0.02150	0.94515
8	0.16383	0.15371	0.71586	0.02150	0.01376	0.96666
9	0.16369	0.00000	0.71502	0.01376	0.00000	0.98042

Table 1: Cost Ranges and $Pr(x = \omega)$ When Trading is Possible/Impossible

Consistent with our expectations, these numerical results show that the opportunity of trading largely incentives shareholders to buy information. As long as $0.082031 \leq c \leq 0.16369$, all of the shareholders want to buy information when trading is allowed. However, when trading is impossible, none of the shareholders wants to buy information. Thus, as long as $0.082031 \leq c \leq 0.16369$, $Pr(x = \omega; Trading) = 0.71502$, which is larger than $Pr(x = \omega; No\ Trading) = 0.5$. But if the information is very expensive ($c > 0.31989$), then, no matter whether trading exists or not, no shareholder wants to acquire information. Therefore, when $c > 0.31989$, we have $Pr(x = \omega; Trading) = Pr(x = \omega; No\ Trading) = 0.5$. In addition, if the information is very cheap ($c \leq 0.01376$), then, all shareholders want to buy information regardless of the existence of trading. In this case, we have we have $Pr(x = \omega; Trading) = 0.71502$, which is smaller than $Pr(x = \omega; No\ Trading) = 0.98042$.

Proposition 8 (Information Aggregation With/Without Trading)**.** *When information is sufficiently cheap ($c \to 0$), we have $Pr(x = \omega; No\ Trading) < Pr(x = \omega; Trading)$. When the information cost is exorbitant ($c \to \infty$), then $k(Trading) = k(No\ Trading) = 0$, and we have $Pr(x = \omega; No\ Trading) = Pr(x = \omega; Trading) = \frac{1}{2}$.*

Furthermore, there exists a cost threshold c_T such that when the information cost is $c \geq c_T$, we must have $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$, which *means for a non-empty set of information costs information aggregation is better when trading is possible than when trading is not possible.*

Proof. When the cost of information is tiny ($c \to 0$), then we can have an equilibrium in which all shareholders buy information regardless of whether trading exists or not. When $k(Trading) = k(No\ Trading) = n$, we know $Pr(x = \omega; No\ Trading) < Pr(x = \omega; Trading)$. This is because all informed shareholders sincerely vote when there is no trading, while informed shareholders will strategically vote when there is trading.[6]

The other trivial case is that the information is exorbitant ($c \to \infty$). Then, we can have an equilibrium in which no shareholder wants to acquire information, no matter whether trading is possible or not. When $k(Trading) = k(No\ Trading) = 0$, $Pr(x = \omega; No\ Trading) = Pr(x = \omega; Trading) = \frac{1}{2}$.

Now we turn to prove that there must exist $c_T \in (0, \infty)$ such that $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$. We first focus on the case in which shareholders cannot trade. To sustain the equilibrium in which no shareholder wants to buy information, the cost of information must satisfy

$$EU(not\ buy, k = 0, No\ Trading) \leq EU_D(not\ buy \to buy, k : 0 \to 1, No\ Trading) - c$$

Because when trading is not allowed, shareholders get payoffs if and only if the chosen policy x is the same as the underlying state ω, we have

$$EU(not\ buy, k = 0, No\ Trading) = Pr(x = \omega, k = 0) = \frac{1}{2}$$

[6] Meirowitz and Pi (2021) prove the impossibility of an equilibrium in which all shareholders are informed and vote sincerely.

Suppose one shareholder i deviates from the equilibrium by choosing to buy information, then her expected payoff is

$$\begin{aligned}
&EU_D(not\ buy \rightarrow buy, k : 0 \rightarrow 1, No\ Trading) - c \\
&= Pr(x = \omega, k = 1, No\ Trading) - c \\
&= Pr(s_i = 1)(Pr(\omega = 1|s_i = 1)Pr(t' \geq \frac{n-1}{2}|\omega = 1) + Pr(\omega = 0|s_i = 1)Pr(t' < \frac{n-1}{2}|\omega = 0)) \\
&+ Pr(s_i = 0)(Pr(\omega = 1|s_i = 0)Pr(t' > \frac{n-1}{2}|\omega = 1) + Pr(\omega = 0|s_i = 0)Pr(t' \leq \frac{n-1}{2}|\omega = 0)) \\
&- c \\
&= qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - c
\end{aligned} \tag{23}$$

Hence, we establish the lemma below.

Lemma 2. *When there is no trading, to sustain an equilibrium in which no shareholder want to buy information, the cost of information must be*

$$c \geq qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - \frac{1}{2}$$

Now we turn to the case in which shareholders can trade after voting. To sustain an equilibrium in which no shareholder invests in information acquisition, the cost of the information must satisfy

$$EU(not\ buy, k = 0, Trading) \leq EU_D(not\ buy \rightarrow buy, k : 0 \rightarrow 1, Trading) - c$$

Suppose that an uninformed shareholder i votes for 1, then her expected payoff at the equilibrium is

$$\begin{aligned}
&EU(not\ buy, k = 0, Trading) \\
&= Pr(t' < \frac{n-1}{2})(2Pr(x = 1|t') - P_0(t'+1)) + Pr(t' \geq \frac{n-1}{2})P_1(t'+1)
\end{aligned} \tag{24}$$

Note that when no one buys information in equilibrium, the voting tally is not informative at all, and thus the price after voting is always $\frac{1}{2}$ regardless of t. So, we have

$$\begin{aligned} & EU(not\ buy, k=0, Trading) \\ &= Pr(t' < \frac{n-1}{2})(2 \cdot \frac{1}{2} - \frac{1}{2}) + Pr(t' \geq \frac{n-1}{2})\frac{1}{2} \\ &= \frac{1}{2} \end{aligned} \tag{25}$$

If the uninformed shareholder i deviates from the equilibrium (buying information), her expected payoff from the deviation is

$$\begin{aligned} & EU_D(not\ buy \to buy, k: 0 \to 1, Trading) - c \\ &= Pr(s_i = 1)Pr(t' < \frac{n-1}{2}|s_i = 1)P_0(t'+1) + Pr(t' \geq \frac{n-1}{2})(2Pr(\omega = 1|s_i = 1, t') - P_1(t'+1)) \\ &+ Pr(s_i = 0)Pr(t' \leq \frac{n-1}{2}|s_i = 0)(2Pr(\omega = 0|s_i = 0, t') - P_0(t')) + Pr(t' \geq \frac{n+1}{2})P_1(t') \\ &- c \\ &= Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - c \end{aligned} \tag{26}$$

Thus, we obtain the lemma below.

Lemma 3. *When there is no trading, to sustain the equilibrium in which no one wants to invest in information, the cost of information must be sufficiently high such that*

$$c \geq Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - \frac{1}{2}$$

Combining Lemma 2 and Lemma 3, we know that if the cost of information satisfies that

$$qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - \frac{1}{2} < c < Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - \frac{1}{2}$$

, then we know that no shareholder wants to buy information in equilibrium when there is no trading (thus $Pr(x = \omega; No\ Trading) = \frac{1}{2}$) and that at least one shareholder wants to buy information in equilibrium when there is trading (thus $Pr(x = \omega; No\ Trading) > \frac{1}{2}$).

Note that the set

$$\{c | qPr(t' \geq \frac{n-1}{2}) + (1-q)Pr(t' < \frac{n-1}{2}) - \frac{1}{2} < c < Pr(t' < \frac{n-1}{2})\frac{1}{2} + Pr(t' \geq \frac{n-1}{2})(2q - \frac{1}{2}) - \frac{1}{2}\} \tag{27}$$

is not empty, because of $\frac{1}{2} > 1 - q$ and $2q - \frac{1}{2} > q$.

Thus, when c_T is in the set above, we must have $Pr(x = \omega; Trading) > Pr(x = \omega; No\ Trading)$, becasue no shareholders buy information when trading does not exist but at least one shareholder buys information when trading is allowed.

■

7 Proxy Advisor

In this section, we examing equilibrium voting and information aggregation when shareholders choose between buying a signal from a proxy advisor or investing in information acquisition on their own or acquiring no information in the information acquisition stage. The signal from the proxy advisor is imperfectly informative, $p = Pr(s^p = \omega|\omega) \in (\frac{1}{2}, 1)$. If she wants to acquire costly information, a sharholder only acquires one type of information, either a private signal or the proxy advisor's signal, but not both. I an not sure this is true like it is in Malenko and Malenko. As in MM a shareholder would defer to the higher quality signal (conditional on being pivotal when voting. When trading she would also defer to the higher quality signal, so the quesiton is can the signal which is more informative conditional on t be different than the one conditional on being pivotal or put differently can the signal which is more informative depend on t

Solving the model numerically, we show that all of main insights from the main model carry over. In particular, we confirm that z^* in equilibrium decreases with the number of shareholders who buy private information, and thus $Pr(x = \omega)$ can decrease with the number of shareholders who buy private information. We also confirm that uninformed shareholders still can extract information rents from voting and trading, as they privately knows which policy they voted for and whether their votes have any informational value.

k	k_p	uninformed	z	z_p	$Pr(x = \omega)$
0	3	6	-	0.55	0.5391
1	3	5	0.80	0.55	0.5438
2	3	4	0.80	0.55	0.5468
3	3	3	0.73	0.55	0.5470
4	3	2	0.67	0.55	0.5467
5	3	1	0.64	0.55	0.5466
6	3	0	0.61	0.55	0.5465

Table 2: Voting Strategies and $Pr(x = \omega)$ with a Proxy Advisor's Signal

We continue the example above, considering $n = 9$ and $q = \frac{4}{5}$. We focus on the case in which 3 shareholders buying the proxy advisor's signal, $k_p = 3$, and the informativeness of the proxy advisor's signal is $p = 0.55$. Table 2 considers various information acquisition strategies and shows that the probability that a vote from a shareholder with a private signal is correct, z, decreases, as the number of shareholders buying private signals increases from $k = 0$ to $k = 6$. When $k \geq 3$, shareholders with private signals vote strategically, which causes the probability of selects the correct policy to decreases with the number of informed shareholders. For the sake of brevity we do not explicitly consider the equilibrium acquisition decisions for particular values of the costs (now for private signals and for the proxy signal). Instead we simply show how voting and aggregation vary with different acquisition strategies.

8 Discussion

The idea that investors would have incentives to acquire information about the firms they invest in is entirely standard, if not obvious, to scholars of finance. This channel has been ignored in theoretical studies of govenrnace. The standard approach when considering voting or governance by investors is to consider the value of information only in impacting governance. We move beyond this narrow perspective by developing a model of information acquisition and governance in which shareholders can also realize rents from trading. The equilibrium analysis illustrates that in general information acquisition will be higher once investing opportunities are considered because shareholders can extract information rents from trading in addition to the rents they extract from potentially helping to select the value enhancing policy. We also document subtle spillover effects. The opportunity to extract informational rents creates governance distortions. Here we determine that overall these distortions need not dominate the benefits of having a more informed group voting on firm policy. In general we find that for some costs to acquiring information govennance is better because of the opportunities to generate trading rents, despite the fact that there are distortions to voting behavior. Moreover, we find that even if it were free information acquisition is not always socially valuable. In equilibrium there can be too much information acquisition in the sense that so many investors are informed that distortions on voting behavior are too strong and the likelihood of selecting the value enhancing policy is lower than it would be in equilibrium if fewer people acquired information. This then justifies the conclusion that governance and welfare of investors is not always enhanced by regulations that make information easy to obtain. The optimal number of informed investors is generally well short of all investors and thus the optimal cost to acquiring information is not 0. Accordingly whether reforms that increase acquisition are good for firm governance depends on features of the governance environment.

References

References

Gerardi, D. and L. Yariv (2008): "Information acquisition in committees," *Games and Economic Behavior*, 62, 436–459.

Malenko, A. and N. Malenko (2019): "Proxy advisory firms: The economics of selling information to voters," *The Journal of Finance*, 74, 2441–2490.

Meirowitz, A. and S. Pi (2022): "Voting and trading: The shareholder's dilemma," *Journal of Financial Economics*, 146, 1073–1096.

Persico, N. (2004): "Committee design with endogenous information," *The Review of Economic Studies*, 71, 165–191.

9 Appendix

9.1 Proof of Proposition 7: Information Costs

Proof.

$$\begin{aligned}&EU(v_i = 1|s_i = \emptyset) - EU(\text{deviation from not acquiring}) \leq \\ &c \\ &\leq EU(v_i = 1|s_i = 1) - EU(\text{deviation from acquiring})\end{aligned} \tag{28}$$

where

$$\begin{aligned}&EU(\text{deviation from acquiring}) \\ &= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t'; k-1)[2Pr(\omega = 0|t'; k-1) - P_0(t'+1; k)] \\ &+ \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'; k-1)P_1(t'+1; k)\end{aligned} \tag{29}$$

and

$$\begin{aligned}&EU(\text{deviation from not acquiring}) \\ &= \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr(t'|s_i = 1; k+1)P_0(t'+1, k) \\ &+ \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'|s_i = 1; k+1)(2Pr(\omega = 1|t', s_i = 1; k+1) - P_1(t'+1, k))\end{aligned} \tag{30}$$

First, we find the condition under which none of the k informed shareholders wants to deviate by becoming uninformed.

An informed shareholder's equilibrium payoff is

$$\begin{aligned}&EU(\text{acquiring}) - c \\ &= \frac{1}{2}Max\{EU(v_i = 1|s_i = 1), EU(v_i = 0|s_i = 1)\} \\ &+ \frac{1}{2}Max\{EU(v_i = 1|s_i = 0), EU(v_i = 0|s_i = 0)\} \\ &- c\end{aligned} \tag{31}$$

If informed shareholders vote sincerely in equilibrium, then it must be $EU(v_i = 1|s_i = 1) \geq EU(v_i = 0|s_i = 1)$. If informed shareholders play mix strategies, then it must be $EU(v_i = 1|s_i = 1) = EU(v_i = 0|s_i = 1)$. So, we have $Max\{EU(v_i = 1|s_i = 1), EU(v_i = 0|s_i = 1)\} = EU(v_i = 1|s_i = 1)$. Similarly, we have $Max\{EU(v_i = 1|s_i = 0), EU(v_i = 0|s_i = 0)\} = EU(v_i = 0|s_i = 0)$. Thus,

$$\begin{aligned} & EU(\text{acquiring}) - c \\ & = \frac{1}{2}EU(v_i = 1|s_i = 1) + \frac{1}{2}EU(v_i = 0|s_i = 0) - c \end{aligned} \tag{32}$$

Note that $EU(v_i = 1|s_i = 1) = EU(v_i = 0|s_i = 0)$ due to the symmetry of informed shareholders' strategies. That's to say, for a shareholder, no particular signal should have an additional value than the other signal in equilibrium. So, we have

$$EU(\text{acquiring}) - c = EU(v_i = 1|s_i = 1) - c \tag{33}$$

Now we turn to find out the informed shareholders' payoff if she deviates from acquiring information to not acquiring information. Since no particular policy is better than the other policy when shareholder i does not have information about the underlying state, we can conveniently assume that that $v_i = 1$.

$$\begin{aligned} & EU(\text{deviation from acquiring}) \\ & = \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t'; k-1)(2Pr(\omega = 0|t'; k-1) - P_0(t'+1; k)) \\ & + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'; k-1)P_1(t'+1; k) \end{aligned} \tag{34}$$

, where

$$\begin{aligned} & Pr(t'; k-1) \\ & = Pr(t'|\omega = 1; k-1)Pr(\omega = 1) + Pr(t'|\omega = 0; k-1)Pr(\omega = 0) \end{aligned} \tag{35}$$

and

$$\begin{aligned}
&Pr(\omega = 1|t; k-1') \\
&= \frac{Pr(t'|\omega = 1; k-1)Pr(\omega = 1)}{Pr(t'|\omega = 1; k-1)Pr(\omega = 0) + Pr(t'|\omega = 0; k-1)Pr(\omega = 0)} \\
&= \frac{Pr(t'|\omega = 1; k-1)}{Pr(t'|\omega = 1; k-1) + Pr(t'|\omega = 0; k-1)}
\end{aligned} \tag{36}$$

In particular, knowing that she deviates from buying information to not buying information, the shareholder calculates the probabilities in the following way.

$$Pr(t'|\omega = 1; k-1) = \sum_{i=0}^{t'} \binom{k-1}{i} z^i (1-z)^{k-1-i} \binom{n-k}{t'-i} \left(\frac{1}{2}\right)^{n-k} \tag{37}$$

and

$$Pr(t'|\omega = 0; k-1) = \sum_{i=0}^{t'} \binom{k-1}{i} (1-z)^i (1-z)^{k-1-i} \binom{n-k}{t'-i} \left(\frac{1}{2}\right)^{n-k} \tag{38}$$

Therefore, to prevent deviation from acquiring information, we must have

$$c \leq EU(v_i = 1|s_i = 1) - EU(\text{deviation from acquiring}) \tag{39}$$

Second, we find the condition under which none of the $n-k$ uninformed shareholders wants to deviate by acquiring a signal.

in equilibrium, an uninformed shareholder's expected payoff is

$$EU(\text{not acquiring}) = Max\{EU(v_i = 1|s_i = \emptyset), EU(v_i = 0|s_i = \emptyset)\} \tag{40}$$

Recall that $EU(v_i = 1|s_i = \emptyset) = EU(v_i = 0|s_i = \emptyset)$. According to the lemma about trading, we have

$$\begin{aligned}
&EU(\text{not acquiring}) \\
&= EU(v_i = 1|s_i = \emptyset) \\
&= \sum_{t'=0}^{\frac{n-1}{2}-1} Pr(t')(2Pr(\omega = 0|t') - P_0(t'+1, k)) + \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t')P_1(t'+1, k)
\end{aligned} \tag{41}$$

, where

$$\begin{aligned}
&Pr(t') \\
&= Pr(t'|\omega = 1)Pr(\omega = 1) + Pr(t'|\omega = 0)Pr(\omega = 0)
\end{aligned} \tag{42}$$

and

$$
\begin{aligned}
&Pr(\omega = 1|t') \\
&= \frac{Pr(t'|\omega = 1)Pr(\omega = 1)}{Pr(t'|\omega = 1)Pr(\omega = 1) + Pr(t'|\omega = 0)Pr(\omega = 0)}
\end{aligned} \tag{43}
$$

On the other hand, if an uninformed shareholder deviates by acquiring information. Then, her expected payoff from the deviation is

$$
\begin{aligned}
&EU(\text{deviation from not acquiring}) - c \\
&= EU(v_i = 1|s_i = 1; k+1) - c \\
&= \sum_{t'=0}^{t'=\frac{n-1}{2}-1} Pr(t'|s_i = 1; k+1)P_0(t'+1, k) \\
&+ \sum_{t'=\frac{n-1}{2}}^{n-1} Pr(t'|s_i = 1; k+1)(2Pr(\omega = 1|t', s_i = 1; k+1) - P_1(t'+1, k)) \\
&- c
\end{aligned} \tag{44}
$$

, where

$$
\begin{aligned}
&Pr(t'|s_i = 1; k+1) \\
&= Pr(t', s_i = 1|\omega = 1; k+1) + Pr(t', s_i = 1|\omega = 0; k+1)
\end{aligned} \tag{45}
$$

and

$$
\begin{aligned}
&Pr(\omega = 1|t', s_i = 1; k+1) \\
&= \frac{Pr(t', s_i = 1|\omega = 1; k+1)Pr(\omega = 1)}{Pr(t', s_i = 1|\omega = 1; k+1)Pr(\omega = 1) + Pr(t', s_i = 1|\omega = 0; k+1)Pr(\omega = 0)}
\end{aligned} \tag{46}
$$

From the perspective of the uninformed shareholder,

$$
Pr(t', s_i = 1|\omega = 1; k+1) = \sum_{i=0}^{t'} \binom{k}{i} z^i (1-z)^{k-i} \binom{n-1-k}{t'-i} \left(\frac{1}{2}\right)^{n-1-k} q \tag{47}
$$

and

$$
Pr(t', s_i = 1|\omega = 0; k+1) = \sum_{i=0}^{t'} \binom{k}{i} (1-z)^i z^{k-i} \binom{n-1-k}{t'-i} \left(\frac{1}{2}\right)^{n-1-k} (1-q) \tag{48}
$$

To prevent the deviation from not acquiring, we must have c

$$c \geq EU(v_i = 1 | s_i = \emptyset) - EU(\text{deviation from not acquiring}) \tag{49}$$

■