The Information in Portfolio Holdings and Investors' Capital Allocations

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Abstract

The paper studies mutual funds' portfolio management and investors' capital allocations in a unified framework under mandatory portfolio disclosure. By modeling fund managers and investors simultaneously, I show that more skill managers produce better performance by trading more actively, which causes investors to care about both fund performance and activeness when evaluating fund managers. This investor's behavior explains the convex flow-performance relation observed in the market. In addition, my model demonstrates that portfolio holdings information is more useful to investors than fund returns because portfolio holdings reveal manager activeness that is not fully captured by fund returns. My model offers three novel empirical predictions for which I find consistent evidence in the data. First, investor flows respond to both fund performance and activeness. Second, investor flows are more sensitive to the performance of illiquid holdings in the portfolio. Finally, in a diff-in-diff analysis, I show that investor flows become more sensitive to fund activeness when portfolios are disclosed more frequently.

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1 Introduction

In 2020, the total assets managed by U.S. mutual funds exceeded U.S. GDP and reached $23.9 trillion. Given that the majority of the wealth is invested in actively managed funds (e.g., 60% in 2020), it is not surprising that a large body of literature has focused on methodologies to test for the skills of fund managers, starting with Jensen (1968) and continuing with Carhart (1997) and others. Within this literature, one of the major approaches to evaluate fund performance utilizes portfolio holdings data, which allows researchers (and investors) to examine fund performance at the security level and potentially improve the precision of inferences about managerial skill (e.g., Grinblatt and Titman (1989), Daniel, Grinblatt, Titman, and Wermers (1997), Kacperczyk, Sialm, and Zheng (2008), and Cremers and Petajisto (2009)).[1] Despite the popularity of holdings-based performance measurement, relatively little is known about how investors process holdings information and allocate their money accordingly. My paper is the first, to my knowledge, to investigate the inference of investors who have access to portfolios holdings and fund returns, and the optimal response of this inference by active fund managers.

In this paper, I take the perspective of investors to study how they evaluate fund managers using portfolio holdings, and the perspective of fund managers to study their optimal response when they know that their investors are "watching" their portfolios. By modeling managers' portfolio construction and investors' inference about manager skill simultaneously, I study the role of portfolio holdings explicitly and compare the information content between portfolio holdings and fund returns. I make three main contributions. First, I demonstrate that portfolio holdings provide more useful information to investors than fund returns. Second, I show that a convex flow-performance relation is obtained endogenously as a result of investors' inference process. Third, I derive three novel empirical predictions for investors' capital allocations, for which I find consistent evidence in the data.

Most of the existing studies on investor flows focus solely on investors' inference problem and simplify the role of fund managers. A common approach taken by the literature is to abstract from portfolio construction and directly assume that fund returns follow a reduced-form process which consists of a fund alpha, a benchmark return, and a residual return (see Berk and Green (2004), Lynch and Musto (2003), Huang, Wei, and Yan (2007), Huang, Wei, and Yan (2012), and Franzoni and Schmalz (2017), among others). Managerial skill is captured by the alpha which is not observable to investors. As a result, investors have to infer skill from past fund returns. In this paper, I argue that this reduced-form return

[1] See Wermers (2011), Ferson (2010), and Ferson (2013) for more comprehensive surveys on returns-based and holdings-based performance evaluation.

process oversimplifies fund managers' behavior and completely overlooks the central role of holdings information. Fund managers are not just endowed with an alpha. To beat the benchmark, managers must form a portfolio that is somewhat different from the benchmark. More skilled managers potentially want to deviate more from the benchmark to pursue better performance. This behavior creates a positive connection between fund performance and activeness, which is largely missing in the extant literature on investor flows. More importantly, my paper shows that this positive connection has a deep impact on investors' inference about manager skill – it affects how investors evaluate fund managers and allocate their money.

To address the missing link, I take a different approach in this paper by studying managers' portfolio construction and investors' inference about manager skill simultaneously in an overlapping generation model. In my model, more skilled managers are able to acquire private signals on future stock returns with higher precision. A key insight of the model is that manager skill affects both fund performance and portfolio risk. High-skill managers produce higher fund alpha by deviating more from their benchmark, which creates a positive association between fund alpha and activeness. Rational investors who are aware of this association extract information from both fund performance and activeness when evaluating fund managers. A direct implication of this behavior is that investor flows are convex in performance. The reason for the convexity is that signals from realized fund returns and activeness do not always agree with each other. When investors observe good performance, the good performance itself signals high skill, and the good performance implies high activeness which also signals high skill. The two signals agree with each other and send a strong combined signal. On the other hand, when investors observe bad performance, the bad performance itself signals low skill. But the underperformance still implies high activeness, since close benchmark trackers (i.e., low-activeness managers) do not significantly underperform their benchmark. In this case, the two signals contradict each other, which makes investors less willing to punish underperforming funds. Combining the two cases together, the model offers a simple and novel explanation to the convex flow-performance relation.

The second contribution of my model is to show, with reasonable assumptions, that portfolio holdings contain more useful information than fund returns. An advantage of my modeling approach is that it allows me to study investors' inference problem under different information sets. I show that fund returns and portfolio holdings, in general, do not carry the same information. The loss of information in fund returns come from two sources. First, investors are not able to accurately infer the differences in the precision of signals from each individual positions in the portfolio when they observe only one signal from fund returns. Second, and more importantly, portfolio holdings reveal all the bets made by managers, which are informative of manager activeness but not fully captured by fund returns. The idea can be illustrated in an example. Suppose that two managers, $m1$ and $m2$, both hold a portfolio of two stocks, A and B. Manager $m1$ chooses to invest 5% in stock A and -5% in stock B (in excess of the benchmark weights), whereas manager $m2$ simply holds the benchmark weights on both stocks. Further, suppose that the next-period realized return is 10% for both stocks, so both funds earn zero abnormal returns. Since the two funds have the same performance, investors who observe only fund returns will infer that the two managers have the same level of skill. Investors who observe portfolio holdings, however, will infer that manager $m1$ is superior to $m2$ because manager $m1$ exhibits higher activeness – manager $m1$ successfully captured the rise of stock A but bet in the wrong direction on stock B, whereas manager $m2$ missed both profit opportunities. In short, portfolio holdings allow investors to gain a more complete picture on managers' activeness and, thus, help investors better evaluate fund managers and make smarter investment decisions. This paper offers a theoretical justification on mandating periodical portfolio disclosure to the public.

To further quantify the benefit of portfolio disclosure, I conduct simulation experiments to compare the usefulness of portfolio holdings and fund returns. I simulate an economy with 1,000 low-skill managers and 1,000 high-skill managers to study the evolution of investors' beliefs over time. The simulation results show that investors who observe lagged portfolio holdings are able to identify the types of managers more than twice as fast as investors who observe only fund returns. In the baseline simulation where each manager observes 10 private signals, investors can successfully identify 35.35% of the managers after 10 periods when they observe portfolio holdings, but the identification rate is only 5.95% when investors do not observe portfolio holdings. Moreover, investors who observe portfolio holdings have much lower chance of misclassifying managers. Throughout the entire 100 periods of simulation, the rate of false discovery never exceeds 2% when investors observe portfolio holdings. But the rate of false discovery is as high as 15% when investors infer managerial skill from fund returns. In short, the simulation experiments show that holdings information dominates returns information in terms of both power and accuracy.

My model delivers three testable predictions on the behavior of investor flows. First, flows respond to both fund performance and activeness. Second, flows are more sensitive to the performance of illiquid holdings. Third, flows become more sensitive to activeness when portfolios are disclosed more frequently. I begin my empirical study with a portfolio-based analysis. At the end of each quarter, I sort funds into five quintile portfolios based on their Active Share (Cremers and Petajisto (2009)). I find that funds in the top activeness quintile, on average, attract 1.5% more flows over the next quarter than funds in the bottom activeness quintile. The result is not driven by fund performance, since the correlation between Active

Share and funds' 12-month 4-factor alpha is only 7%. In addition, the regression results show that fund activeness significantly predicts future fund flows after controlling for fund performance. A one standard deviation increase in a fund's Active Share attracts 2% more flows, on average, over the following year. Therefore, the empirical evidence shows that the activeness of managers has independent predictive power on future flows, which is consistent with my model's prediction.

Importantly, when investors aggregate information from individual holdings in the portfolio, they assign different "importance" to the signals of individual positions. The "importance" that investors assign is proportional to the illiquidity of underlying holdings because illiquid positions are more informative of skill than liquid positions. Intuitively, managers are more conservative and cautious of investing in illiquid assets due to higher trading costs, which makes their choices of illiquid positions more revealing of their true nature. To test this prediction, I explore within-portfolio variations of the flow-performance sensitivity by decomposing each fund portfolio into five liquidity-based subportfolios. Specifically, I rank stock holdings in each portfolio based on their illiquidity and sort them into five subportfolios. In a panel regression with both fund and time fixed effects, I find that investor flows are nearly twice as sensitive to the return of the most illiquid subportfolio as to the return of the most liquid subportfolio. In the regression of institutional flows, the monotonic relation test proposed by Patton and Timmermann (2010) is also passed, indicating that the flow-performance sensitivity increases monotonically with the illiquidity of portfolio holdings, which supports my model's second prediction.

Finally, I employ a difference-in-differences analysis to examine the change of flow-activeness sensitivity after a fund chooses to voluntarily report its portfolios more frequently to Morningstar . The hypothesis is that investors are more likely to respond to fund activeness when that information is more available to them. Empirically, I use the frequency of portfolio disclosure to measure information availability on fund activeness. The variable of interest is the change of flow-activeness sensitivity after a fund disclose its portfolios more frequently. This setting can be thought of as a standard panel diff-in-diff where the post indicator is interacted with fund activeness. Although the decision to voluntarily report portfolio holdings to Morningstar is endogenous from the perspective of fund families, I argue that the change of disclosure frequency is an exogenous shock from the perspective of investors. To further alleviate the endogeneity concern, the regression controls for the level effect in a basic diff-in-diff setting. The results show a significant positive change of flow-activeness sensitivity for institutional flows after a fund chooses to disclose its portfolio more frequently. A one standard deviation increase in Active Share attracts an additional 1% to 1.5% institutional flows over the following year after the treated funds decide to report their portfolios more frequently to Morningstar. In summary, I show empirical evidence that supports all three predictions of my model.

The rest of my paper is organized as follows. Section 2 discusses related work. Section 3 presents the model. Section 4 describes the simulation experiments. Section 5 explains the empirical strategies and presents the empirical results. Section 6 generalizes the baseline model to a strategic game. Section 7 concludes.

2 Literature Review

My paper extends and contributes to the literature in several important ways. First, my paper relates to the literature on manager activeness. Various activeness measures have been developed in the research on performance analysis, including portfolio concentration by Kacperczyk, Sialm, and Zheng (2005), Active Share by Cremers and Petajisto (2009), $1-R^2$ measure by Amihud and Goyenko (2013), portfolio turnover ratio by Pástor, Stambaugh, and Taylor (2017), and active weight by Doshi, Elkamhi, and Simutin (2015). All these studies have documented a positive association between manager activeness and fund performance. This paper, however, focuses on both the supply side (as studied in these prior papers) and the demand side in concert, and studies the connection between manager activeness and investors' capital allocations, which to my knowledge has not been studied in the literature. I show that investors care about both fund performance and activeness when they evaluate fund managers, and that fund managers take this into consideration when they choose their portfolios (and, specifically, their level of active risk).

Second, my paper offers a novel explanation to the convex flow-performance relation observed among equity mutual funds. Early studies have documented an asymmetric flow-performance relationship that investors invest disproportional more in funds that have performed well (see Ippolito (1992), Gruber (1996), Chevalier and Ellison (1997), and Sirri and Tufano (1998)). Huang et al. (2007) explain this puzzling phenomenon by introducing participation costs that disincentivize investors to withdraw money from underperforming funds. Lynch and Musto (2003) and Dangl, Wu, and Zechner (2008) attribute the asymmetric responses to strategy/manager replacement. In addition, Pegoraro (2021) explains this pattern in a dynamic contracting model. Taking a behavioral perspective, Goetzmann and Peles (1997) attribute the asymmetry in investors' behavior to cognitive dissonance, and Ferreira, Keswani, Miguel, and Ramos (2012) relate the convexity to investor sophistication. In this paper, I demonstrate that the convex flow-performance relation arises simply as a result of that investors infer manager skill from both fund performance and activeness. I also test the unique predictions of my model in the data, which separate my theory from existing studies.

Third, this paper extends the literature on the costs and benefits of portfolio disclosure. Mutual funds holdings are a subject of regulated and costly disclosure. It's important to understand the costs and benefits of portfolio disclosure for its policy implications (Wermers (2001)). The extant literature has largely found evidence that portfolio disclosure reveals trade secrets and impairs fund performance (Agarwal, Mullally, Tang, and Yang (2015), Shi (2017)). Cao, Du, Yang, and Zhang (2021) find evidence of copycat trading and show that copycats impose significant costs on disclosing companies. Another strand of literature in this area studies window dressing. Several studies have documented that fund managers window dress their portfolios before disclosing them to the public. Among equity mutual funds, Meier and Schaumburg (2006) detect increased turnover on the last days of the quarter and show that this trading activity is related to purchasing past winners and selling past losers prior to holdings disclosure. Other window-dressing behaviors have also been discovered among pension fund managers (Lakonishok, Shleifer, Thaler, and Robert (1991)) and retail money fund managers (Musto (1999)). Moreover, Solomon, Soltes, and Sosyura (2014) show that investor flows follow the performance of media-covered holdings. They attribute this reaction to the information salience rather than valuable inferences about managerial ability. All the aforementioned studies document the downsides of portfolio disclosure. Portfolio disclosure not only gives away proprietary ideas and harms performance, but also induces managers to window dress and exacerbates investor biases. My paper takes the other side of the debate, and shows evidence, both in theory and empirics, that portfolio disclosure reveals useful information to investors and thus helps investors make smarter investment decisions.

Finally, this paper contributes to the literature on smart money vs. dumb money effect. Early work by Gruber (1996) and Zheng (1999) finds a smart money effect that flows predict performance at a short horizon. In addition, Keswani and Stolin (2008) document a robust smart money effect in the United Kingdom. More recently, Kang, Sinclair, and Xeno (2020) find that investor flows become smarter after a hedge fund discloses its holdings in 13F filings. On the other hand, Frazzini and Lamont (2008) find a dumb money effect that, on average, retail investors direct their money to funds which invest in stocks that have low future returns. In theory, however, Berk and Green (2004) demonstrate that, with diseconomies of scale, a weak relation between flows and future performance can be consistent with either smart money or dumb money effect. In this paper, I derive new predictions for the smart money effect under the equilibrium condition of Berk and Green (2004) and show empirical evidence supporting the existence of rational investors.

3 Model

In this section, I study managers' portfolio construction and investors' capital allocation simultaneously in an overlapping generation model. The goal of the model is to understand how investors process information and evaluate fund managers. A key ingredient of the model is that high-skill managers produce higher fund alpha by trading more actively. I show that this positive connection between fund performance and activeness has a deep impact on investors' inference about manager skill. Rational investors extract information from both fund performance and activeness to evaluate managers, which results in a convex flow-performance relation. Moreover, my model demonstrates that portfolio holdings provide more valuable information to investors than fund returns.

3.1 Setup

I consider a discrete-time economy with time indexed by t. The model features one riskless asset, n risky assets, and one composite asset. The net return of the riskless asset is normalized to zero without loss of generality. Risky assets $i \in \{1, ..., n\}$ have random returns r_i with respective loadings $\beta_1, ..., \beta_n$ on the return of a systematic risk factor, and face asset-specific shocks $z_1, ..., z_n$. I assume there is a tradable composite asset m that tracks the return of the systematic risk factor $r_{m,t}$ perfectly over time. Formally,

$$
\begin{aligned}
r_{i,t} &= \beta_i r_{m,t} + z_{i,t}, \qquad i \in \{1, ..., n\} \\
r_{m,t} &= \mu_m + z_{m,t},
\end{aligned}
$$

The idiosyncratic risk factors $z_t = (z_{1,t}, z_{2,t}, ..., z_{n,t})'$, is normally distributed as $z_t \sim \mathcal{N}(0, \Sigma)$, where Σ is a diagonal matrix with ith diagonal element τ_i^{-1}. The composite asset m is exposed to only the systematic risk $z_{m,t}$, where $z_{m,t} \sim \mathcal{N}(0, \sigma_m^2)$ and is independent from z.

At each time period, a generation of young fund managers and investors are born and live for two periods. Each generation consists of M active fund managers (indexed by j), one passive fund manager (denoted by p) and a continuum of investors (denoted by I). All agents are risk averse and price-takers. Agents who are born at time period t are referred as Generation t. Figure 1 depicts the timeline for Generation $t-1$ and Generation t. At time period t, young active fund managers of Generation t first inherit skill from Generation t-1 and then acquire private signals on the next-period idiosyncratic shocks z_{t+1}.[2] Fund managers do not observe signals on the systematic shock. Therefore, the skill in the model

[2] Old manager j passes on his skill to the young manager j, so the skill carries over through generations.

measures managers' stock-picking ability. The signals acquired by manager j at time t is denoted by η_t^j.

$$\eta_t^j = \begin{pmatrix} \eta_{1,t}^j \\ \eta_{1,t}^j \\ \vdots \\ \eta_{1,t}^j \end{pmatrix} = \begin{pmatrix} z_{1,t+1} + \epsilon_{1,t}^j \\ z_{2,t+1} + \epsilon_{2,t}^j \\ \vdots \\ z_{n,t+1} + \epsilon_{n,t}^j \end{pmatrix} = z_{t+1} + \epsilon_t^j$$

where ϵ_t^j is a vector of signal noise, and is distributed as $\epsilon_t^j \sim \mathcal{N}(0, I(\tau^j)^{-1})$.[3] The precision of the signals is specific to each manager (i.e., manager j observes signals with precision τ^j), which is the measure of skill in my model. More skilled managers are able to observe signals with higher precision. I assume that the economy is populated with two types of active fund managers: high type (H) and low type (L). High-skill managers are able to observe private signals with high precision τ^H, whereas low-skill managers observe signals with low precision τ^L. Therefore, by observing more precise signals, high-type managers are able to resolve more uncertainty ex ante, which gives them an advantage when forming portfolios. Young managers use investors' money to construct portfolios and receive compensation next period, when they turn old. Investors, on the other hand, allocate their money across funds when they are young and consume all the proceeds of their investment (net of fees) in the next period. Before young investors make their investment decisions, they inherit beliefs from the last generation and update these beliefs based on the new information released in the current period. Define $\phi_{j,t} \equiv Prob_t^I(\tau^j = \tau^H)$ as investors' posterior belief at time t that manager j is a high type. The variable $\phi_{j,t}$ tracks investors' belief over time on manager j. A higher $\phi_{j,t}$ indicates that investors believe that manager j is more likely to be a high-type manager.

The overlapping-generation structure allows me to solve the model as a sequence of one-period static problems, but still incorporate dynamic ingredients. The dynamics in the model originate from the features that investors update their beliefs on manager skill over time as they learn from new information. The dynamics of investor beliefs and flows are the focal point of the model.

3.2 Managers' Problem

At each time period, young managers construct portfolios to maximize the expected utility over next-period compensation. In the baseline model, I assume managers receive a performance-based compensation contract where their compensation consists of a base salary and a bonus salary proportional to fund performance. Specifically, manager j's compensa-

[3]Matrix I is a $n \times n$ identity matrix, and τ^j is the precision of signals observed by manager j.

tion at time $t+1$ is a fraction of $S_{j,t}(1+R_{j,t+1})$, where $S_{j,t}$ is the beginning-of-period fund size and $R_{j,t+1}$ is the return of fund j at time $t+1$.[4] Denote the fraction (i.e., the expense ratio) that manager j takes as f_j, which is assumed to be an exogenous parameter. So the model does not take a stand on the optimal contract of fund managers. But this compensation contract is largely consistent with the common practice in the mutual fund sector (Ma, Tang, and Gómez (2019)).

Based on the structure of returns, any portfolio of risky assets can be written equivalently as a portfolio of risk factors and the composite asset (Kacperczyk, Nieuwerburgh, and Veldkamp (2016)). Therefore, I use $w = (w_1, ..., w_n, w_m)'$ to denote a portfolio of n risk factors and the composite asset m, with $1 - w'\mathit{1}$ invested in the risk-free asset. Under this set up, the portfolio weight w_i can be interpreted as the active weight in asset i which measures the deviation from the benchmark weight. A portfolio of the n risk factors $w = (w_1, ..., w_n)'$ is the active portfolio which contains the positions that deviate from the benchmark (Koijen (2014)).

Given the portfolio w_t^j chosen by manager j at time t, the return follows:

$$R_{j,t+1} = w_{m,t}^j r_{m,t+1} + (w_t^j)' z_{t+1} - C_{j,t} \tag{1}$$

where $C_{j,t}$ captures a quadratic trading cost and is in the form:

$$C_{j,t} = \frac{1}{2} S_{j,t} \sum_{i=1}^{n} \lambda_i (w_{i,t}^j)^2 \tag{2}$$

where λ_i is the per unit trading cost for stock i. Trading cost of this form can be rationalized by a linear price impact (Kyle (1985)).[5] Since trading cost $C_{j,t}$ increases with fund size, it generates decreasing returns to scale (DRS) at fund level (Berk and Green (2004)).

Assuming that all managers have power utility with relative risk aversion coefficient $\gamma > 1$, the optimal weights chosen by manager j at time t are:

$$w_m^j = \frac{\mu_m}{\gamma \sigma_m^2} \tag{3}$$

$$w_{i,t}^j = \frac{\tau^j \eta_{i,t}^j}{\gamma + (\tau_i + \tau^j)\lambda_i S_{j,t}} \ , \quad i \in \{1, ..., n\} \tag{4}$$

Since the systematic risk factor is unpredictable, all fund managers (including the passive

[4] I use lowercase r to denote security returns , and uppercase R to denote portfolio returns.

[5] Trading Sw_i dollar of stock i moves the (average) price by $\frac{1}{2}\lambda_i S w_i$, and this results in a total trading cost of Sw_i times the price move, which is $\frac{1}{2}\lambda_i S^2 w_i^2$. Finally, dividing the total trading cost by the fund size, we get the percentage trading cost of stock i.

fund manager p) choose the same level of exposure to it (Equation (3)).[6] In addition, passive fund manager p does not invest in any individual stocks since he does not observe any private signals (i.e., $w^p_{i,t} = 0$). I consider the portfolio of the passive fund as the benchmark for all other funds. As for active fund managers, it's optimal for them to deviate from the benchmark and trade individual stocks based on their private information. Equation (4) shows that the optimal active weight invested in stock i is positively associated with the skill of the manager (τ^j), and the observed signal ($\eta^j_{i,t}$), but negatively associated with manager's risk aversion (γ), fund size ($S_{j,t}$), and the illiquidity of the underlying stock (λ_i). Given signal, more skilled managers trade more actively by deviating more from the benchmark to pursue performance. Trading costs constrain managers from trading too aggressively, and the effect increases with fund size and the illiquidity of underlying assets.

In the baseline model, when managers receive a performance-based salary, the managed portfolios achieve "first best" in the sense that the same portfolios are obtained if the managers just give their private information to investors and let the investors construct their own portfolios (given that investors and managers have the same utility). A performance-based contract makes managers focus solely on achieving the best reward-to-risk trade off for their portfolios, and do not care about the amount of flows they could attract from investors. Despite that it's interesting and important to understand how investors evaluate managers when managers are doing their due diligence, one limitation of the baseline model, admittedly, is that managers do not internalize the effect of their behavior on future flows. In Section 6, I study a strategic version of the model where managers receive a size-based compensation contract which incentivizes managers to also compete for investor flows. All the key results established in the baseline model still hold in the strategic game.

To facilitate further analysis, I define the abnormal return of fund j at time $t+1$ as $\delta_{j,t+1} \equiv R_{j,t+1} - R_{p,t+1}$, which is also referred as benchmark-adjusted return in empirical research. Further define fund alpha as the expectation of the abnormal return given skill and fund size, $\alpha_j \equiv E[\delta_{j,t+1}|\tau^j, S_j]$. Finally, the residual return is defined as difference between the abnormal return and alpha, $u_{j,t+1} \equiv \delta_{j,t+1} - \alpha_j$. Proposition 1 shows that fund returns can be written as a "reduced-form" process.

Proposition 1. *Given the skill level and fund size, the return of fund j can be written as*

$$R_{j,t+1} = \alpha_j + R_{p,t+1} + u_{j,t+1} \tag{5}$$

where α_j is a function of manager skill and fund size, $\alpha_j = f_1(\tau^j, S_{j,t})$, and the variance of $u_{j,t+1}$ is also a function of manager skill and fund size, $\sigma^2_{u_j} = f_2(\tau^j, S_{j,t})$. In addition,

[6] All derivations and proofs are in Appendix A.

$\frac{\partial \alpha_j}{\partial \tau^j} > 0$, $\frac{\partial \alpha_j}{\partial S_{j,t}} < 0$; $\frac{\partial \sigma^2_{u_j}}{\partial \tau^j} > 0$, $\frac{\partial \sigma^2_{u_j}}{\partial S_{j,t}} < 0$.

Proposition 1 connects manager skill to fund returns. Similar to the reduced-form process assumed by the extant literature (e.g., Berk and Green (2004), and Huang et al. (2007)), Proposition 1 shows that more skilled managers produce higher fund alpha. The key difference is that Proposition 1 also proves the positive connection between manager skill and fund residual risk. More skilled managers produces superior performance by deviating more from the benchmark. In short, by studying managers' portfolio construction, I establish the positive association between fund performance and activeness. In the next section, I show how this association affects investors' inference about manager skill.

3.3 Investors' Problem

Investors' capital allocations depend on their beliefs – they want to invest more into funds whose managers they believe have higher skills. This section focuses on the process of how investors form their beliefs based on the information they observe, which is the primary interest of my model.

At each period, a continuum of investors who are risk averse and endowed with an initial wealth of W_0 is born and lives for two periods. Young investors choose a portfolio of funds, including both active and passive funds, to invest and consume all proceeds of their investment next period, when they turn old.[7] Young investors' capital allocations at each period determine the size of funds. Following Berk and Green (2004), equilibrium fund sizes are obtained at each period such that all investors earn zero expected excess return in equilibrium. Let $E^I_t[\cdot]$ and $V^I_t[\cdot]$ denote the (representative) investor's expectation and variance conditional on all the information available to her at time t. Proposition 2 defines the equilibrium formally:

Proposition 2. *Fund j's equilibrium size $S_{j,t}$ at time t is given by the zero net alpha condition: $E^I_t[\alpha_j] = \frac{f_j}{1-f_j}$. Specifically, equilibrium fund size $S_{j,t}$ is the solution to the equation $G(\phi_{j,t}, S_{j,t}) = 0$, where $\phi_{j,t} = Prob^I_t(\tau^j = \tau^H)$, and*

$$G(\phi_{j,t}, S_{j,t}) = \phi_{j,t} Alpha(\tau^H, S_j) + (1 - \phi_{j,t}) Alpha(\tau^L, S_j) - \frac{f_j}{1 - f_j}$$

$Alpha(\tau^j, S_j)$ is the alpha earned by manager j with fund size S_j. According to the implicit

[7] In theory, investors can also invest in individual assets. But since investors do not observe any private signals, their portfolio of individual assets is the same as the passive fund's portfolio. Thus, there is no need to model investors' portfolio choice of individual assets separately. Moreover, as shown in Appendix A, the exact utility function and the initial wealth W_0 of investors do not matter in obtaining equilibrium.

function theorem, it follows that

$$\frac{\partial S_j}{\partial \phi_t} = -\frac{\partial G/\partial \phi_t}{\partial G/\partial S_j} > 0$$

Hence, fund size increases with the investor's posterior belief.

Proposition 2 characterizes the equilibrium fund size and its connection with investors' posterior beliefs. Intuitively, investors choose to invest money in a fund up to the point where a marginal dollar in the fund equals the outside option of risk-free rate, which is normalized to zero. Competition among investors leaves no "free" alpha on the table. More money is invested into funds that are perceived to deliver superior performance, which is determined jointly by fund size and manager skill (Proposition 1). Since fund sizes are observable to investors in real time, the primary interest of investors is to infer the unobserved skills of fund managers. Therefore, investors' capital allocations are determined by their inference about manager types given the information available to them.

Next, I study how investors' beliefs, ϕ_t, are formed. I assume that investors are rational and update their beliefs in a Bayesian way. The thinking process of investors is described as follows. Given a piece of information (e.g., an realized fund return), investors evaluate how likely the information is generated by a high-type manager as opposed to how likely it's generated by a low-type manager. If this piece of information is more likely to generated by a high-type manager, investors will update their belief towards that this manager is high-type, and vice versa. This thinking process is captured mathematically in a likelihood ratio, $\mathcal{L}(x_t) \equiv \frac{f_t(x_t|\tau^j=\tau^L)}{f_t(x_t|\tau^j=\tau^H)}$, where $f_t(x_t|\tau^j)$ is the conditional probability density function of outcome x_t given manager type τ^j. The ratio of the two densities tells us whether it is more or less likely that outcome x_t is associated with a low-skill or a high-skill manager. $\mathcal{L}(x_t) > 1$ indicates that the observed outcome is more likely to be associated with a low-skill manager, whereas $\mathcal{L}(x_t) < 1$ indicates that the observed outcome is more likely to be associated with a high-skill manager. Once investors form the likelihood ratio given the new information, they update their beliefs following Bayes' rule. Given the observed new information x_t and the prior belief $\phi_{j,t-1}$, the posterior belief of investors for manager j is given by

$$\phi_{j,t} = \frac{\phi_{j,t-1}}{\phi_{j,t-1} + (1-\phi_{j,t-1})\mathcal{L}(x_t)} \tag{6}$$

From Equation (6), we can see that the key determinant of the investor's posterior belief is the new information x_t (i.e., what information is actually available to investors). Next, I consider two cases. In section 3.3.1, I study investors' inference problem when portfolio holdings are available to investors with one-period delay. In section 3.3.2, I study investors' inference problem when holdings information are not available so that investors have to rely on fund returns to infer manager skill.

3.3.1 Learning from Portfolio Holdings.

At time $t+1$, the information set of investors includes all realized security and fund returns up to time $t+1$, and portfolio holdings of all funds up to time t. In this case, information on fund returns become redundant, since they can be perfectly inferred from portfolio holdings and security returns. I start by studying how investors evaluate a single portfolio weight. From the perspective of investors, each observed portfolio weight $w_{i,t}^j$ follows a normal distribution conditional on manager type (see Equation (14)). Therefore, the likelihood ratio of observing $w_{i,t}^j$ can be written as a ratio of two normal distributions, which can be written in the form of

$$\mathcal{L}(w_{i,t}^j) = a_{i,t}^j exp\{-(\tau^H - \tau^L)[S_{j,t}\lambda_i \underbrace{\delta_{i,t+1}^j}_{\text{performance}} + b_{i,t}^j \underbrace{(w_{i,t}^j)^2}_{\text{activeness}} - \frac{1}{2}z_{i,t+1}^2]\} \tag{7}$$

where $a_{i,t}^j$ and $b_{i,t}^j$ are variables based on only exogenous parameters. From Equation (7), we can see that investors use the observed portfolio weight in two different ways. First, investors use portfolio weights to measure performance (i.e., the abnormal return $\delta_{i,t+1}^j$). A good performance is a sign of high skill, which makes investors more inclined to believe that the manager is a high type. Second, investors use squared weights to measure the activeness of the manager, regardless of the actual performance. High activeness is a sign of conviction, which reflects a manager's willingness to make large bets on potential profit opportunities. Since making bets, especially large bets, are costly, managers whose signals are more precise are more willing to take the risk. Thus, higher activeness indicates higher skill, which explains why investors also respond to activeness.[8] The behavior of investors is consistent with Proposition 1. Investors extract information from both fund performance and activeness because they are aware of that high-skill managers produce better performance by trading more actively.

After investors extract information from each of the positions in the portfolio, they aggregate all pieces of information to form a total likelihood ratio of observing the entire portfolio.

[8] Admittedly, taking large bets itself does not require skill. In Section 6, I study an extension of the model where managers also compete for flows. In this strategic game, low-type managers have an incentive to trade aggressively to mimic the behavior of high-type managers in order to attract more flows. Indeed, I show that low-type managers trade more aggressively in equilibrium. In the meantime, however, high-type managers also trade more aggressively in equilibrium to separate themselves from the low-type managers. Therefore, the important result still holds – higher activeness indicates higher skill.

Lemma 1 *The total likelihood ratio of observing multiple independent signals equals to the product of the likelihood ratios of each signal.*

According to Lemma 1, we can derive the total likelihood ratio of observing the entire portfolio of fund j at time $t+1$ as

$$\mathcal{L}(w_{1,t}^j, w_{2,t}^j, ..., w_{n,t}^j) = \prod_{i=1}^{n} \mathcal{L}(w_{i,t}^j) \tag{8}$$

If the total likelihood ratio is less than one, investors will upgrade their belief that manager j is high-type, and vice versa. Importantly, when investors aggregate information from each positions, they assign different "importance" to these signals based on their precision. As shown in Equation (7), the "importance" that investors assign is proportional to the illiquidity of underlying stocks because signals from illiquid holdings are more informative of skill. Intuitively, managers are more conservative and cautious of investing in illiquid stocks due to higher trading costs, so their choices of illiquid positions are more revealing of their true nature. This idea is illustrated in Figure 2. As shown in Panel A, given a 10% profit opportunity of a stock, both the expectation and the volatility of the portfolio weight chosen by a manager deceases as the illiquidity of the underlying stock increases. Therefore, the signals from illiquid positions are more precise from the perspective of investors. For example, given that investors observe a 1% position on this stock (i.e., a good bet for a 10% profit opportunity), the likelihood ratio decreases as the illiquidity of the stock increases (Panel B of Figure 2), indicating that investors are more convinced that this manager is high-type when the underlying stock is more illiquid. On the other hand, given that investors observe a -1% position on this stock (i.e., a bad bet for a 10% profit opportunity), the likelihood ratio increases with the illiquidity of the stock, indicating that investors are more convinced that this manager is low-type when the underlying stock is more illiquid. Therefore, investors optimally pay more attention to the signals of illiquid positions because they are more informative.

The final step for investors to update their beliefs is to plug the total likelihood ratio from Equation (8) into Bayes formula in Equation (6). Investors choose their capital allocations based on their posterior beliefs and the equilibrium fund sizes are determined. Intuitively, the more investors believe that a manager is a high type, the more they will invest in his fund (Proposition 2).

3.3.2 Learning from Fund Returns.

In the section, I study the case where portfolio holdings are not disclosed to investors, so investors rely on only fund returns to infer manager skill. At time $t + 1$, the information set of investors include all realized security returns and fund returns up to time $t + 1$. Similar to the thinking process of evaluating portfolio weight (Section 3.3.1), investors compare the likelihood that an observed fund return is generated by a high-type manager with the likelihood by a low-type manager. If investors think that the observed fund return is more likely to be generated by a high-type manager, they will update their belief towards that the manger is high-type, and vice versa. Conditional on the realized security returns and manager type, fund returns follow a distribution which is the sum of a normal distribution and a non-central chi-squared distribution. The likelihood ratio from this non-standard distribution cannot be solved in closed-form. Hence, I solve and simulate investors' inference problem numerically to obtain their posterior beliefs.

From the perspective of investors, portfolio holdings and fund returns do not carry the same information. The difference in the information content comes from two sources. First, investors are not able to accurately infer the differences in the precision of signals from each individual positions in the portfolio when they get only one signal from fund returns, which makes inference from fund returns less efficient. Second, and more importantly, portfolio holdings reveal all the bets made by the manager, which are informative of manager activeness, but are not fully captured by fund returns. The idea can be illustrated in an example. Suppose that two managers, $m1$ and $m2$, each hold a portfolio of two stocks, A and B. Manager $m1$ chooses to invest 5% in stock A and -5% in stock B in excess of the corresponding benchmark weights, whereas manager $m2$ simply holds the benchmark weights on both stocks (i.e., zero active weights). Further, suppose that the next-period realized return is 10% for both stocks, so both funds earn zero abnormal returns. Since the two portfolios have the same performance, investors who observe only fund returns will infer that the two managers have the same skill. Investors who observe portfolio holdings, on the other hand, will infer that manager $m1$ is superior to $m2$, since manager $m1$ exhibits higher activeness than manager $m2$. Revealed by portfolio holdings, manager $m1$ successfully captured the rise of stock A but bet in the wrong direction on stock B, whereas manager $m2$ missed both profit opportunities. Portfolio holdings reveal more useful information than fund returns because holdings depict a more complete picture on the activeness of managers. In short, by observing each individual positions in the portfolio, investors can get a more accurate inference on manager skill. In Section 4, I perform simulation experiments to compare the usefulness of information between portfolio holdings and fund returns, and confirm the intuition that holdings disclosure helps investors make better investment decisions.

3.4 Flow-Performance Relation (FPR)

I define flows as the change of fund size scaled by the beginning-of-period fund size.

$$flow_{j,t+1} = (S_{j,t+1} - S_{j,t})/S_{j,t} \tag{9}$$

Since fund size is determined by investors' beliefs which are determined jointly by fund performance and activeness, flows can be thought as a function of performance and activeness. In this section, I show that the behavior of investors generates a convex flow-performance relation (see Ippolito (1992), Gruber (1996), Chevalier and Ellison (1997), and Sirri and Tufano (1998)).

First, as comparative statistics, Panel A of Figure 3 plots the partial relation between flows and performance. Given activeness, flows increase monotonically with performance – a better performance always attracts more flows from investors. The FPR becomes flat on both tails because manager skill is bounded. When the observed performance are so extreme that it's strong enough to convince investors that the manager is a high or low type, the marginal effect of performance diminishes to zero so the FPR becomes flat. The tail behaviors, however, are not so relevant in practice as the gray area in Panel A represents the 1^{th} to 99^{th} percentile range of performance. Within this more realistic range of performance, the FPR is almost linear, as shown in Panel B which zooms in the gray area in Panel A.

Next, I show how flows are determined jointly by performance and activeness in Panel C of Figure 3. The 3D graph plots the simplest case that there is only one holding in the portfolio. The graph admits the interrelation between performance and activeness as they are both functions of portfolio weights. From the graph, we can observe that investor flows respond to both fund performance and activeness. The shape of the flow-performance relation depends on the comovement between performance and activeness. To see this, Panel D plots the vertical section of the 3D graph in Panel C along the performance axis, which denotes the model-implied FPR. The graph shows that investor flows are more sensitive to good performance than bad performance. The varying sensitivity is due to that performance and activeness do not always agree with each other. For instance, when investors observe good performance, the good performance itself signals high skill. In the meantime, the good performance implies high activeness, which also signals high skill. The two signals agree with each other and send a strong combined signal. On the other hand, when investors observe bad performance, the bad performance itself signals low skill. But the underperformance still implies high activeness, since close benchmark trackers (i.e., low-activeness managers) do not significantly underperform their benchmark. In this case, the two signals contradict each other, which makes investors optimally less willing to punish underperforming funds.

Combing the two cases together, the model shows that a convex flow-performance relation arises endogenously as a result of investors' inference process – investors infer manager skill from both fund performance and activeness. Thus, the model offers a simple and novel explanation to the convex flow-performance relation observed in the data.

In Panel D of Figure 3, we can also observe a region of performance that corresponds to two different levels of flows. The non-unique mapping between flows and performance is due to that each given performance in this region corresponds to two different levels of activeness. This graph is a graphical illustration on why portfolio holdings contain more useful information than fund returns. Although manager in point a (manager a) and manager in point c (manager c) produce the same performance, manager a is more active so that he attracts more flows. But investors can only make this distinction when they observe portfolio holdings. When investors do not observe portfolio holdings, however, they cannot distinguish manager a from manager c so that they have to take a probability-weighted average of the two possible scenarios which ends up at point b in between of point a and c. Panel D demonstrates the informational advantage of portfolio holdings over fund returns, echoing the argument in Section 3.3.2.

It's worth noting that graphs in Figure 3 are all model-implied relations which do not take into account distribution probabilities. Figure 4 presents the simulated FPR in a more realistic case that each manager acquires private signals on 10 stocks. Panel A shows the simulated FPR when investors have access to portfolio holdings, and Panel B shows the FPR when investors do not have access to portfolio holdings. We can observe a clear convexity from both graphs, which is consistent with my model as well as the empirical findings in Figure 6.

3.5 Model Predictions

In the section, I derive three testable predictions from my model to guide my empirical analysis in Section 5. These novel predictions can further separate my theory from the existing stories.

Prediction 1 Flows respond to both performance and activeness.

Prediction 1 is a direct implication of investors' inference about manager skill. As shown in the likelihood ratio in Equation (7), investors use both fund performance and activeness when they evaluate fund managers because they are aware of that manager skill is positively associated with both performance and activeness. Despite that several empirical studies on performance analysis have documented that activeness predicts fund performance (Cremers and Petajisto (2009), Kacperczyk et al. (2005)), my model predicts that activeness should also predict investor flows, which has not been studied in the literature.

Prediction 2 Flows are more sensitive to the performance of illiquid holdings.

When investors aggregate information from different holdings in the portfolio, they assign higher weights to illiquid holdings because the signals from illiquid holdings are more informative of skill. More formally, we can see this relation by taking the total differential of flows:

$$\mathrm{d}flow_{j,t+1} = g_t^j \sum_{i=1}^{n} \lambda_i(\underbrace{\mathrm{d}w_{i,t}^j r_{i,t+1}}_{\text{hypothetical return}} + h_{i,t}^j \underbrace{\mathrm{d}(w_{i,t}^j)^2}_{\text{activeness}}))$$

where g_t^j and $h_{i,t}^j$ are variables based only on exogenous parameters. Since we do not observe the transaction costs of trading each stock in the data, I decompose holdings return into hypothetical return $w_{i,t}^j r_{i,t+1}$ and trading costs, and rearrange terms to get the equation above. The testable prediction is that flows are more sensitive to the hypothetical return of the illiquid holdings as indicated by the λ term (illiquidity) in the equation. Hypothetical return ($w_{i,t}^j r_{i,t+1}$) is the buy-and-hold return of investing a stock without considering the interim trading and trading costs, which can be calculated empirically in the data (Kacperczyk et al. (2008)). Since the sign of variable $h_{i,t}^j$ is ambiguous (depending on the value of exogenous parameters), Prediction 2 only applies to the return of illiquid holdings but not to activeness.

Existing studies on the convex flow-performance relation only make fund-level predictions since they do not consider portfolio holdings. To generate convexity, the extant literature introduces external mechanism to discourage investors from punishing underperforming funds, including search costs (Huang et al. (2007)), strategy switching (Lynch and Musto (2003)), or manager replacement (Dangl et al. (2008)). Prediction 2 of my model is unique in the sense that it makes prediction for the within-portfolio variations of the flow-performance sensitivity. Therefore, empirical findings that support Prediction 2 can separate my theory from existing stories.

Prediction 3 Flows are more sensitive to activeness when portfolios are disclosed more frequently.

As discussed in Section 3.3.2, the primary advantage of observing portfolio holdings over fund returns is that investors are able to assess manager activeness more precisely. Prediction 3 explores the connection between the flow-activeness sensitivity and the frequency of portfolio disclosure. The hypothesis is that investors are more likely to respond to activeness when portfolio holdings are disclosed more frequently so that the information on activeness becomes more available to investors. Prediction 3 uses variations of the flow-activeness sensitivity to test how portfolio disclosure can help investors better evaluate fund managers and guide their capital allocations.

4 Simulation Experiment

In this section, I simulate the economy modeled in Section 3 with calibrated parameters. The goal is to compare the information content between portfolio holdings and fund returns. I simulate investors' inference problem under two different information sets – with portfolio disclosure and without portfolio disclosure. I show that investors can distinguish high-skill managers from low-skill managers at a faster rate and with higher accuracy when they observe portfolio holdings compared to the case when they observe only fund returns.

4.1 Experimental Design

The simulated economy is populated with 1,000 high-skill managers and 1,000 low-skill managers. Managers observe private signals on 10 stocks at the beginning of each period.[9] These 10 stocks can be interpreted as 10 profit opportunities that fund managers choose to pursue actively. Besides these 10 stocks, managers could hold more stocks passively following the benchmark. Since investors only care about the active portion of the portfolio when they evaluate managers, it's sufficient to simulate only the portfolios of these 10 stocks.

At each time period, managers construct portfolios given their private signals. The chosen portfolio weights are given by Equation (4), which are interpreted as the weights in excess of the benchmark weights. Investors, on the other hand, allocate their capital at each period based on their beliefs. Intuitively, investors would like to allocate more money to the managers whom they believe are more likely to be high-type, and vice versa. Fund sizes are determined by the condition specified in Proposition 2. In equilibrium, all funds earn zero net alpha due to the competition among investors. Investors update their beliefs over time based on new information observed in each period.

The evolution of investors' beliefs over time is the key subject of the exercise. Specifically, I want to investigate the different paths of investors' posterior beliefs over time when they observe portfolio disclosures and when they observe only fund returns. To facilitate the analysis, I define two information sets:

[9] In the simulation, I assume all managers observe signals on the same 10 stocks. This choice is mainly for efficiency consideration. The results will not change if I instead assume managers observe signals on different stocks, since investors ex-post observe the realized returns of the stocks in the portfolio when they evaluate managers.

Information Set 1 ($\mathcal{F}_1$): $\mathcal{F}_{1,t}$ include all realized asset returns and fund returns up to time t, and all portfolio disclosures up to time $t-1$.

Information Set 2 ($\mathcal{F}_2$): $\mathcal{F}_{2,t}$ include all realized asset returns and fund returns up to time t.

Further denote a manager as a perceived high-type if investors have at least 95% confidence that this manager is high-type (i.e., $\phi \geq 0.95$), and as perceived low-type if investors have at least 95% confidence that this manager is low-type (i.e., $\phi \leq 0.05$). Investors start with a flat prior on all managers (i.e., $\phi_0 = 0.5$). To determine which information set is more useful for investors, I adopt two statistics that are commonly used in scientific fields – the power of a test and the rate of false positive findings. In the context of this exercise, the power of an information set is defined as the rate at which investors successfully identify the type of managers (i.e., $Prob(\phi^j \geq 0.95|\tau^j = \tau^H) \cup Prob(\phi^j \leq 0.05|\tau^j = \tau^L)$). A high power means that investors are able to identify a large portion of managers given the information set, indicating the information set is very useful for investors to infer manager skill. Accuracy is another metric to assess an information set, which is measured as the rate of false positives (i.e., $Prob(\phi^j \geq 0.95|\tau^j = \tau^L) \cup Prob(\phi^j \leq 0.05|\tau^j = \tau^H)$). False positives capture investors' probability of making mistakes by misclassifying fund managers. Misclassifying is costly because it leads to inefficient allocation of capital – over-investing low-type managers and under-investing high-type managers. In summary, a superior information set should have more power and lower rate of false positives.

An implicit assumption made in the simulation experiment is that investors observe fund returns and portfolio holdings at the same frequency, which might seem unreasonable given that we observe fund returns at a much higher frequency than portfolio holdings in practice (e.g., daily vs. quarterly). I justify my approach as follows. In the simulation (and also the model in Section 3), each time period represents a complete cycle of investment for a manager, and whether or not observing more frequent fund returns and portfolio holdings is useful to investors depends on the investment horizon of fund managers. For example, if it takes one month for a manager to develop and carry out an idea, observing very frequent fund returns or portfolio holdings within the month does not bring much new information to investors. In this case, it's much more useful to observe just monthly fund returns and portfolio holdings but over a long horizon. Therefore, I believe that it's reasonable to assume that investors observe fund returns and portfolio holdings as the same frequency in the simulation and compare the information content between the two sources.

4.2 Simulation Results

Period by period, I simulate investors' beliefs on 2000 managers for 100 periods. Table 1 shows the number of perceived high- and low- type managers at given periods and the associated power and rate of false positives. We can observe from Table 1 that Information Set 1 has much higher power and lower rate of false positives than Information Set 2. After 5 periods, investors are able to identify 12.95% of managers as either high-skill or low-skill when they have access to portfolio disclosures. In the meantime, investors can identify only 1.35% of the managers using fund returns. After 20 periods, investors can already identify about 72.7% of the managers if they have access to portfolio disclosures, but the number is just 29.3% if they observe only fund returns. The same pattern carries over throughout all periods. Another observation from Table 1 is that investors are much less likely to misclassify managers under Information Set 1. Throughout the entire 100 periods, the rate of false positives never exceed 2% if investors can observe portfolio holdings. But the rate of false positive stays as high as nearly 15% when investors observe only fund returns.

Figure 5 shows the continuous evolution of investors' beliefs over time. The light green and light red areas represent the number of identified true high-skill and low-skill managers, respectively, under Information Set 2. The combined light and dark green area and the combined light and dark red area represent the number of identified true high-skill and low-skill managers, respectively, under Information Set 1. Therefore, the darker areas correspond to the advantage of portfolio holdings over fund returns. We can see from Figure 5 that holdings information allows investors to identify the type of managers at a much faster rate. The advantage is especially prominent when the number of periods for observation is small. For example, holdings information helps investors to correctly identify more than twice the number of managers than return information after 10 periods. In summary, the results from the simulation experiment show that information in portfolio holdings dominates information in fund returns. Investors who have access to holdings disclosures can identify managers at faster rate and with higher accuracy.

5 Empirical Analysis

In this section, I develop empirical strategies to test three predictions of my model and show empirical evidence supporting these predictions. The goal of the empirical analysis is to study how investors evaluate mutual fund managers and make their investment decisions. The three predictions I test in the data are:

Prediction 1 Flows respond to both performance and activeness.

Prediction 2 Flows are more sensitive to the performance of illiquid holdings.

Prediction 3 Flows are more sensitive to activeness when portfolios are disclosed more frequently.

5.1 Data

The major dataset I use is a fund-level quarterly panel of U.S. actively managed equity mutual funds from 1991 to 2020, which contains information on fund flows, flow returns, and fund characteristics. This dataset is further map to Morningstar mutual fund holdings data. My sample builds upon several datasets. I start with the Center for Research on Security Prices (CRSP) survivorship bias-free mutual fund database. The CRSP database provides comprehensive information about daily and monthly shareclass returns and a host of other shareclass characteristics, including total net assets, age, expense ratio, turnover, and load. I associate and group different shareclasses of a fund into fund-level observations based on primarily CRSP fund identifier (*crsp_cl_grp*). But the CRSP fund identifier is incomplete before 2001 and is known to have mistakes. So I undertook an extensive data project to address these shortcomings. Specifically, I develop a name-matching algorithm that first separate shareclass names from fund names, and then match different share classes of a fund by their common fund names. Finally, I correct the remaining conflicts in shareclass-to-fund mapping manually. I exclude money market funds, index funds, target funds, ETFs, variable annuities from my sample.[10] I also exclude the first 36 months of observations of each fund to alleviate the incubation bias (Evans (2010)). Following the fund flow literature, the flow of a shareclass is defined as the net flow into the shareclass divided by the lagged total net assets (TNA). Formally, the flow is calculated as

$$flow_{j,t+1} = \frac{TNA_{j,t+1}}{TNA_{j,t}} - (1 + R_{j,t})$$

where $R_{j,t}$ is the net return of shareclass j at time t. Shareclass-level variables (e.g., returns, flows, and expense ratio) are aggregated into fund-level variables by taking the TNA weighted averages. When aggregating flows, I exclude flows of the shareclasses that are not open to

[10] I identify bond funds using CRSP style codes following Goldstein, Jiang, and Ng (2017) and Choi and Kronlund (2018), identify index funds using the CRSP flag and the procedures in Berk and van Binsbergen (2015), identify money market funds and target funds by fund name, and finally identify ETFs and variable annuities using CRSP flags.

investors. In addition, I take the flows from the institutional shareclass of a fund as its institutional flows.[11] Non-institutional flows are the fund-level flows excluding institutional flows. I next merge the CRSP dataset with Morningstar following the procedures in Berk and van Binsbergen (2015). For portfolio holdings data, I remove holdings reports that have fewer than 20 stocks, and require funds to invest at least 70% in U.S. equity. This step filters out about 5% of the holdings reports in my sample. I also use CRSP/Compustat stock-level database to get data on stock returns, trading volume, market capitalization, book-to-market ratios, momentum, and industry classification.

5.1.1 Activeness Measure.

In this paper, activeness measures the extent to which fund managers choose to deviate from their benchmarks. Since this definition matches well with the idea of Active Share introduced by Cremers and Petajisto (2009), I use Active Share (AS) as the primary measure of activeness in the empirical analysis. The Active Share is computed as

$$AS = \frac{1}{2}\sum_{i=1}^{N} |w_{fund,i} - w_{index,i}|$$

where $w_{index,i}$ is the benchmark weight invested in stock i. To determine the benchmark for a fund, I employ a data-driven approach following Cremers and Petajisto (2009) and Petajisto (2013). For each given portfolio, I compute AS w.r.t. 15 indexes and assign the index that produces the lowest AS as the benchmark for this fund. These 15 indexes are the most commonly used by equity mutual funds as their benchmarks.[12] Since I do not have data on the index constitutions, I proxy index constituents using the portfolio holdings of the largest ETF that tracks the index. Specifically, I use the portfolio holdings of iShares ETFs to proxy the index constituents of all indexes except for S&P 500, for which I use

[11] I identify institutional shareclass in two steps. First, a shareclass is classified as institutional shareclass if it's shareclass name contains "inst". Second, for the remaining funds that do not have an institutional shareclass identified by step one, I classify the shareclass with the lowest expense ratio as the institutional shareclass given that the fund has multiple shareclasses.

[12] These 15 indexes are from two families: S&P/Barra and Russell. The S&P/Barra indexes I pick are the S&P 500, S&P 500/Barra Growth, S&P500/BarraValue, S&PMidCap 400, and S&PSmallCap 600. The S&P500 is the most common large-cap benchmark index, consisting of approximately the largest five hundred stocks. It is further divided into a growth and value style, with equal market capitalization in each style, and this forms the Barra Growth and Value indexes. The S&P 400 and S&P 600 consist of four hundred mid-cap and six hundred small-cap stocks, respectively. From the Russell family, I use ten indexes: the Russell 1000, Russell 2000, Russell 3000, and Russell Midcap indexes, plus the value and growth components of each except for Russell 3000. The Russell 3000 covers the largest three thousand stocks in the United States and the Russell 1000 covers the largest thousand stocks. The Russell 2000 is the most common small-cap benchmark, consisting of the smallest two thousand stocks in the Russell 3000. The Russell Midcap index contains the smallest eight hundred stocks in the Russell 1000.

SPDR. The AS I compute has 94% correlation with the AS computed in Petajisto (2013) on the overlapping sample.[13]

5.1.2 Summary Statistics.

My main sample has 3725 unique US equity funds with 175,265 fund-quarter observations and 141,494 portfolio disclosures from January 1991 to December 2020. Table 2 presents the summary statistics of my sample. Active equity funds receive 0.53% flows per quarter on average over the full sample period. Institutional flows is, on average, higher than non-institutional flows. The correlation between the institutional flows and non-institutional flows is 0.387, as shown in Panel B of Table 2. This moderate correlation indicates that institutional and non-institutional investors do behave differently. Since my paper focuses on the behavior of rational investors, I expect that the empirical results are stronger for institutional flows which represents the capital allocations of a group of more sophisticated and rational investors. In terms of performance, active equity mutual funds beat the market by 0.8% per year before fees, as indicated by the DGTW characteristic-adjusted returns (since DGTW approach does not account for fees). Net of fees, however, equity funds on average underperform benchmark by -0.53% per year as indicated by the 4-factor alpha (Fama and French (2010)). Finally, in my sample, the average number of portfolio disclosure per fund is 56, and each portfolio contains 118 stocks on average.

5.2 Prediction 1: The Determinants of Flows

I begin my empirical analysis by examining the flow-performance relation in the data. Figure 6 presents the fitted natural splines (with 5 degrees of freedom) in a regression of future flows on past fund performance with a set of control variables and category-by-date fixed effects. The control variables include past flows, expense ratio, log of fund TNA and log of fund age. I use Morningstar category to classify equity funds into nine categories. Fund performance is measured as the past 12-month 4-factor alpha estimated using daily returns (Carhart (1997)). The three panels in Figure 6 plot the flow-performance relation for fund flows, institutional flows, and non-institutional flows, respectively. Regardless of the flow measures, all three plots exhibit a convex shape that investors do not withdraw money from underperforming funds as much as they invest in outperforming funds. The model in Section 3 offers a simple explanation to the convexity: Investors use both fund performance and activeness to infer managerial skill and the two signals do not always agree with each other.

Next, I show empirical evidence that investor flows respond to fund activeness as well

[13] For the data of Active Share in Petajisto (2013), please see "http://www.petajisto.net/data.html".

as fund performance. I first conduct a portfolio-based analysis by sorting funds into five quintile portfolios based on their Active Share at the end of each quarter and computing the TNA-weighted flows for each of the five portfolios over the next quarter. Figure 7 presents the time-series average of flows for each of the five portfolios. From the graph, we can observe a clear upward trend that more active funds receive more flows, and the positive relation holds true for both institutional and non-institutional flows. The economic magnitude is large – the top activeness quintile portfolio, on average, receives 1.6% more flows than the bottom activeness quintile portfolio over the next quarter. This result is not driven by fund performance because the correlation between Active Share and 4-factor alpha is only 7%. Despite that the unconditional correlation between fund performance and activeness is low, it becomes significant and meaningful conditional on performance. Figure 8 plots that correlation between fund performance and activeness within each of the five performance groups. Among funds in the bottom performance quintile, the correlation between fund alpha and Active Share is -25%, whereas the correlation becomes 39% among funds in the top performance quintile. Consistent with my model, performance and activeness tend to agree with each other given good performance, but disagree with each other given bad performance, which explains why investors invest disproportionately more to outperforming funds and are less willing to punish underperforming funds.

Next, I examine the roles of fund performance and activeness jointly in predicting future fund flows and performance. At the end of each quarter, funds are first sorted into five quintile portfolios based on their past 12-month 4-factor alpha, and each quintile portfolio is further subdivided into five Active Share quintiles. This double-sorting procedure results in 25 fractile portfolios, and TNA-weighted flows and 4-factor alpha are computed for each of the 25 portfolios over the next quarter. Table 3 presents the time-series average of flows and alpha for the 25 portfolios. Panel A of Table 3 shows that investor flows chase performance, and the effect is particularly strong among high activeness funds. The difference in quarterly flows between the best performers and the worst performers increases from 6% to 10.7%, moving from the bottom activeness quintile to the top activeness quintile. In addition, portfolios of funds with both good performance and high activeness attract the most flows from investors. Funds in the top performance quintile and the top two activeness quintiles receive 7.4% and 7.3% flows, on average, over the next quarter, respectively. These funds are also the ones that perform well over the next quarter, as shown in Panel B.

It's worth noting that for funds in the bottom performance quintile, the difference in flows between the most active and least active funds is -1%. This result, which might seems contradicted with the intuition that investors respond positively to activeness, is driven by the negative correlation between fund performance and activeness among underperforming funds (Figure 8). Indeed, the most active funds in the bottom performance quintile, on average, earn a 3% lower alpha than the least active funds. Thus, the positive response from activeness is mitigated by the negative response from performance. What's important is that this group of underperforming but highly-active funds do not continue to perform poorly over the next period, as shown in Panel B of Table 3. The reversal indicates that performance is not driving all the results and activeness is also informative of manager skill.

Finally, I test Prediction 1 formally in a panel regression:

$$flow_{j,t+1} = \delta_{c,t} + \beta_1 \text{activeness}_{j,t} + \beta_2 \text{performance}_{j,t} + X_{j,t}\Gamma + u_{j,t+1} \tag{10}$$

where $\delta_{c,t}$ are category-by-date fixed effects, and $X_{j,t}$ is a set of control variables including lag flows, expense ratio, log of fund TNA and log of fund age. I measure *activeness* using Active Share, and $performance$ using the past 12-month 4-factor alpha. For robustness, I also construct an alternative measure of activeness by computing the Active Share using only the top 20 holdings in a portfolio. This simple alternative measure intends to capture the idea that some investors might use the top holdings posted on mutual funds' websites to get a quick approximation for manager activeness. The two measures have 40% correlation.

Table 4 presents regression results that the activeness of managers significantly predicts next-quarter fund flows. Both fund flows and institutional flows respond to fund activeness after controlling for fund performance. A one standard deviation increase in a fund's Active Share attracts 2% more flows on average over the next year. In addition, fund flows also respond positively to the Active Share of the top 20 holdings. In summary, I show empirical evidence that fund activeness has independent predictive power for future investor flows after controlling for fund performance.

5.3 Prediction 2: Flow-Performance Sensitivity and Holdings Illiquidity

In this section, I test Prediction 2 by exploring within-portfolio variations of the flow-performance sensitivity. I start by adjusting fund returns by the DGTW characteristic-based benchmark following Daniel et al. (1997) in a standard flow-performance regression.

$$flow_{j,t+1} = \delta_{c,t} + \beta_1 \tilde{R}_{j,t} + \beta_2 R^b_{j,t} + X_{j,t}\Gamma + u_{j,t+1}$$

where $\tilde{R}_{j,t} = \sum_i w^j_{i,t-1}\tilde{r}_{i,t}$ is the fund-level DGTW-adjusted return and $R^b_{j,t}$ is the fund-level DGTW characteristic-based benchmark return. DGTW-adjusted returns ($\tilde{r}_{i,t}$) measure the relative performance of a stock to a group of peer stocks with similar market value, book-to-market ratio, and momentum. Since managers should not be rewarded for easy bets that represent passive strategies, flows from rational investors should be more sensitive to DGTW-adjusted returns $\tilde{R}_{j,t}$ than benchmark returns $R^b_{j,t}$ (Barber, Huang, and Odean (2016)). Thus, my first hypothesis is that $\beta_1 > \beta_2$.

Next, I further decompose fund-level DGTW-adjusted returns into returns of several liquidity-based subportfolios. Specifically, for each portfolio, I rank stock holdings based on their liquidity (in a descending order) and sort them into five quintile subportfolios. The returns of the subportfolios are computed as the weighted average of the underlying holdings' returns. The regression specification is:

$$flow_{j,t+1} = \delta_j + \delta_{c,t} + \sum_{p=1}^{5} \beta_p \tilde{R}^p_{j,t} + \beta_2 R^b_{j,t} + X_{j,t}\Gamma + u_{j,t+1} \tag{11}$$

where $\{\tilde{R}^1_{j,t}, ..., \tilde{R}^5_{j,t}\}$ are the DGTW-adjusted returns for the five liquidity-based subportfolios (i.e., $\tilde{R}^1_{j,t}$ is the return of the most liquid subportfolio and $\tilde{R}^5_{j,t}$ is the return of the most illiquid sub-portoflio).[14] The regression includes both fund and category-by-date fixed effects to examine the within-portfolio and within-time variations of the flow-performance sensitivity. Following Prediction 2, the second hypothesis is that $\beta_1, ..., \beta_5$ in Regression (11) increase monotonically.

The primary stock-level illiquidity measure I use is the Amihud Ratio (Amihud (2002)) which captures daily price response associated with one dollar of trading volume. The economic intuition of the Amihud Ratio matches the idea of illiquidity in the model. In addition, several papers which examine liquidity measure at a large scale have documented that the Amihud Ratio is one of the best measures for price impact (Goyenko, Holden, and Trzcinka (2009), Fong, Holden, and Trzcinka (2017)). The Amihud Ratio (AR) is computed as the ratio of daily absolute return to dollar volume, averaged over all positive-volume days.

$$AR_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|R_{idt}|}{Dvol_{idt}}$$

Besides Amihud Ratio, I also use the Price Impact Ratio (PIR) developed by Florackis,

[14] $\tilde{R}^p_{j,t}$ is the weighted return of subportfolio p, where the weights in the subportfolio do not sum to one. By construction, the total of the DGTW-adjusted return of the five subportfolio equals the fund-level DGTW-adjusted returns, $\sum_{p=1}^{5} \tilde{R}^p_{j,t} = \tilde{R}_{j,t}$; the total of DGTW-adjusted return and benchmark return equals fund return, $\tilde{R}_{j,t} + R^b_{j,t} = R_{j,t}$.

Gregoriou, and Kostakis (2011) as robustness check.

$$PIR_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|R_{idt}|}{TR_{idt}}$$

PIR replaces the dollar trading volume in the Amihud Ratio with stock turnover ratio to adjust for the high correlation between dollar volume and the market value of stocks.[15] Finally, I compute portfolio-level illiquidity as the weighted average of stock-level illiquidity.

Panel A of Figure 9 presents the average Amihud Ratio for each of the five subportfolios. We can see from the plot that the illiquidity of the five subportfolios increases monotonically and it spikes at the most illiquid subportfolio, indicating that equity fund managers usually hold a liquid portfolio except for a small portion of illiquid holdings. Panel B presents the average weight for each of the five subportfolios. The most illiquid subportfolio, on average, accounts for 12% weight in the portfolio. According to Prediction 2, investor flows are more sensitive to the performance of the most illiquid portion of the portfolio. To test this prediction, I first construct a subportfolio which contains the top 20% most illiquid holdings in a portfolio and run a regression of future flows on the DGTW-adjusted returns of this illiquid subportfolio and the fund-level DGTW returns.

$$flow_{j,t+1} = \delta_j + \delta_{c,t} + \beta_1 \tilde{R}^{illiquid}_{j,t} + \beta_2 \tilde{R}_{j,t} + \beta_0 R^b_{j,t} + X_{j,t}\Gamma + u_{j,t+1}$$

where $\tilde{R}^{illiquid}_{j,t}$ is the DGTW-adjusted return for the 20% most illiquid holdings of fund j at time t. The coefficient of interest is β_1 which measures the extra sensitivity of flows to the return of illiquid holdings over the sensitivity of flows to the fund-level DGTW return. The regression results are presented in Table 5. The coefficient on the DGTW return of the 20% most illiquid holdings is 0.373% and highly significant, indicating that flows exhibit 0.373% more sensitivity to the return of the 20% most illiquid holdings than to the return of the entire portfolio. The same pattern also holds for institutional flows. As robustness check, I construct an alternative subportfolio that contains only the top 10% most illiquid holdings and rerun the regression. This results are economically the same – investor flows are more sensitive to the return of the top 10% most illiquid holdings. In addition, Table 6 presents the consistent results when the stock illiquidity is measured as Price Impact Ratio.

Finally, following Regression (11), I run a horserace regression of future flows on all of the five liquidity-based subportfolios. The regression results in Table 8 show that flows are more sensitive to the performance of more illiquid subportfolios. The Panel C and D of

[15]Stock turnover ratio is calculated as the number of traded shares divided by the number of shares outstanding.

Figure 9 layout the regression coefficients of the five subportfolios for fund flows and institutional flows, respectively. The difference in the flow-performance sensitivity between the most illiquid subportfolio and the most liquid subportfolio is 0.17 for fund flows and 0.262 for institutional flows, and both differences are statistically significant. To test for the monotonicity, I employ the monotonic relation test (MR test) proposed by Patton and Timmermann (2010) with null hypothesis that the five coefficients are non-increasing. The bootstrapped p-value are presented in Table 8 – the null hypothesis is not rejected for fund flows (column (2)) but strongly rejected for institutional flows (column (2)). Thus, for institutional investors, we conclude that the flow-performance sensitivity increases monotonically with the illiquidity of portfolio holdings.

One alternative explanation of the increasing sensitivity is that the return of the illiquid subportfolio has higher correlation with the fund DGTW return so that it drives out other subportfolios in the horserace regression. Table 7 presents the correlation matrix of the returns of the five subportfolios and the fund-level DGTW returns. The pairwise correlation among the five subportfolios are all below 20%, which alleviates the concern of multicollinearity. More importantly, the returns of the most illiquid subportfolio actually have the lowest correlation with fund-level DGTW returns. Therefore, the result of increasing sensitivity cannot be explained by the high correlation between illiquid subportfolio returns and fund returns. In summary, I show empirical evidence which supports Prediction 2 that flow-performance sensitivity increases with the illiquidity of holdings. The effect is stronger for institutional flows which represent the capital allocations of a group of more sophisticated and rational investors.

5.4 Prediction 3: Flow-Activeness Sensitivity and Frequency of Disclosure

Prediction 3 relates flow-activeness sensitivity with the availability of information. The hypothesis is that investors are more likely to respond to activeness when that information is more available to them. Empirically, I use the frequency of disclosure to proxy information availability on fund activeness. Variable $Freq_{j,t}$ counts the number of portfolio disclosures for fund j over the past six-month period ending at time t, since portfolio disclosures allow investors to assess manager activeness more precisely. The variable $Freq_{j,t}$ ranges between 0 (no disclosure) to 6 (monthly disclosure). Panel A of Figure 10 plots the time-series $Freq_{j,t}$ using Morningstar holdings data and Thomson Reuters holdings data, respectively. We can observe a clear upward trend of disclosure frequency in Morningstar data over the years. The average frequency only increases moderately in Thomson Reuters holdings data because

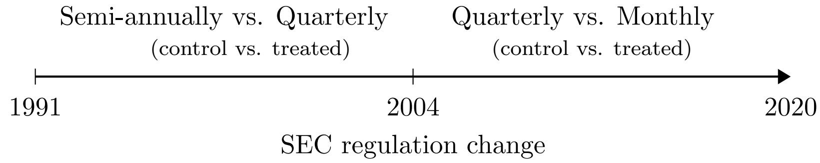
Thomson Reuters only collects quarterly holdings (whereas Morningstar also collects monthly voluntary disclosure). Panel B presents the decomposition of funds that choose different disclosure frequencies in Morningstar. The number of funds that report semiannually almost reduces to zero after 2004 due to the regulation change by SEC that requires mutual funds to disclosure their portfolio quarterly. Both the numbers of funds that voluntarily report quarterly and monthly increase over time. At the end of 2020, there is about the same number of funds that choose to disclose portfolio monthly and funds that choose to disclose quarterly.

The first test I run is a panel regression of flows on past manager activeness interacted with the frequency of disclosure.

$$flow_{j,t+1} = \delta_{c,t} + \beta_1 Active_{j,t} + \beta_2 Freq_{j,t} \cdot Active_{j,t} + \beta_3 Performance_{j,t} + X_{j,t}\Gamma + u_{j,t+1}$$

The coefficient of interest is β_2 which captures the increasing flow-activeness sensitivity with more frequent portfolio disclosure. The regression results are presented in Table 9. β_2 is positive and highly significant for institutional flows but not for non-institutional flows. This result makes sense because Morningstar data is not available for free to all investors. Usually only institutional investors who manage a large amount of capital has the incentive and resources to pay for Morningstar service. Therefore, I expect and find a stronger effect for institutional flows. When a fund does not disclosure their portfolio over the past 6 months, institutional flows do not significantly respond to the fund activeness (captured by β_1). Compared to a fund that does not disclose its portfolio, the flow-sensitivity increases by 0.024 for a fund that chooses to report monthly. The results are consistent with the information story that investor flows respond to fund activeness, and more so when portfolios are disclosed more frequently.

To further identify the impact of disclosure frequency on investors' capital allocations, I employ a difference-in-differences analysis to examine the change of flow-activeness sensitivity after a fund chooses to voluntarily report its portfolio to Morningstar more frequently. First, I divide my sample into two sub-samples at 2004. In the first half of the sample (i.e., between 1991 and 2004), I define the treated funds as those that used to report their portfolio semiannually to Morningstar but decided to voluntarily change the frequency to quarterly at some point during the sample period. The control funds are those that always reported semiannually to Morningstar. I find 230 treated funds and 132 control funds in the first sub-sample. In the second half of the sample (i.e., between 2005 and 2020), I define the treated funds as those that used to report their portfolio quarterly to Morningstar but decided to voluntarily change the frequency to monthly at some point during the sample period. The control funds are those that always reported quarterly to Morningstar. I find 311 treated funds and 604 control funds in the second sub-sample.



An assumption made in the difference-in-differences estimation is that the shocks (i.e., change of disclosure frequency) are exogenous. In this case, it's hard to defend that the decision to report portfolio holdings to Morningstar is exogenous. But since my interest is to study how investors respond to the change of disclosure frequency, what I want to argue is that the change of disclosure frequency is an exogenous shock from the perspective of investors. Investors usually do not take part in the decision making of fund families and could only react to the change of disclosure frequency after it actually happens. Therefore, it's valid to compare how flows respond to the activeness of a fund that changes disclosure frequency (treated) to a fund that does not (control). The full regression specification is:

$$\begin{aligned} flow_{j,t+1} = \delta_j &+ \delta_{c,t} + \beta_1 Treat_j \cdot Post_{j,t} \cdot Active_{j,t} \\ &+ \beta_2 Treat_j \cdot Active_{j,t} + \beta_3 Post_{j,t} \cdot Active_{j,t} \\ &+ \beta_4 Treat_j \cdot Post_{j,t} + \beta_5 Treat_j + \beta_6 Post_{j,t} \\ &+ \beta_7 Active_{j,t} + \beta_8 Performance_{j,t} + X_{j,t}\Gamma + u_{j,t+1} \end{aligned} \tag{12}$$

where $Performance_{j,t}$ is the 4-factor alpha estimated over past 12-month period ending at time t, and $Active_{j,t}$ is the most recent Active Share computed using portfolio holdings. $Treat_j$ is an indicator for whether fund j ever chooses to report portfolio holdings to Morningstar voluntarily at a frequency higher than SEC requirement.[16] $Post_{j,t}$ is an indicator equal to one after fund j chooses to voluntarily report their holdings and zero otherwise. δ_j and $\delta_{c,t}$ are fund fixed effects and category-by-date fixed effects, respectively. Several terms in this specification are redundant and thus dropped.[17] The variable of interest is β_1 which captures the change of sensitivity of fund flows to activeness after a fund reports more frequently. A positive β_1 indicates that investors become more sensitive to a fund's activeness

[16] Before mid 2004, SEC required mutual funds to report their portfolio holdings semiannually. After mid 2004, SEC introduced N-Q form and required funds to report their complete holdings quarterly. But many mutual funds choose to voluntarily report their holdings to data vendors (e.g., Morningstar, Thomson Reuters) at a higher frequency.

[17] $Treat_j$ and $Post_{j,t}$ are redundant because of the fund and category-by-date fixed effects. Since $Post_{j,t}$ equals to one only after a treated fund chooses to voluntarily report holdings, $Post_{j,t} \cdot Active_{j,t}$ is also redundant and dropped because of the term $Treat_j \cdot Post_{j,t} \cdot Active_{j,t}$.

after the fund reports their portfolios more frequently. This setting can be thought of as a standard panel diff-in-diff where the post indicator is interacted with fund activeness. This methodology has been used in various settings to study the effect of a staggered treatment on a slope (see Lel and Miller (2015), Edmans, Jayaraman, and Schneemeier (2017)), and Kang and Pflueger (2015)). To further alleviate the endogeneity concern, the regression specification controls for the level effect of a basic diff-in-diff setting. Specifically, coefficient β_4 captures the change of flows after a fund discloses its portfolio more frequently. A non-zero β_4 could be a result of the signaling effect that investors think the decision of voluntary disclosure itself as a signal and thus invest differently in those funds.

Table 10 and Table 11 present the diff-in-diff estimation for the two subsamples, receptively. We observe a significant positive change of flow-activeness sensitivity for institutional flows after a fund chooses to disclose its portfolio more frequently. The effect is not significant for non-institutional flows potentially because most non-institutional investors do not have access to Morningstar holdings data. In the first subsample between 1991 to 2004, the flow-activeness sensitivity increases by 0.104 after a fund switches from semiannually reporting to quarterly reporting, which indicates a 1.5% increase in annual institutional flows for one standard deviation increase in Active Share. In the second subsample between 2005 to 2010, the flow-activeness sensitivity increases by 0.064 after a fund switches from quarterly reporting to monthly reporting, which indicates a 1% increase in annual institutional flows for one standard deviation increase in Active Share. In summary, I show empirical findings that establish the positive association between the flow-activeness sensitivity and frequency of portfolio disclosure, which supports the third prediction of my model.

6 Extension – Nash Game

In this section, I study a strategic version of the model where managers also compete for investor flows. The goal is to examine whether the link between fund performance and activeness still holds when managers act strategically. One limitation of the baseline model in Section 3 is that managers do not care about investor flows since their compensation is based solely on performance. In this strategic version of the model, managers take a fraction of the terminal fund size as their compensation so that managers' compensation depends on both the fund performance and the amount of flows they attract from investors. When fund managers compete for flows, low-skill managers have an incentive to mimic high-skill managers to make it harder for investors to distinguish them from high-skill managers. As a result, two potential equilibriums could happen. The first equilibrium is a pooling equilibrium that low-skill managers successfully replicate the activeness of high-skill managers so activeness becomes uninformative of skill in equilibrium and investors can rely only on performance to evaluate managers. In this type of equilibrium, both Proposition 1 and Proposition 2 break down since it's no longer true that high-skill managers exhibit higher activeness. The second equilibrium is a separating equilibrium that high-skill managers still trade more actively than low-skill managers in equilibrium and all the intuitions established in the baseline model carry over. The separating equilibrium is plausible because high-skill managers also have an incentive to separate themselves from the low-type to attract more flows.

To study which type of the aforementioned equilibrium holds true in the strategic game, I modify manager's compensation as a fraction of terminal fund size, and leave all other settings of the baseline model unchanged. Now, the objective of the young manager j at time t is to maximize the expected utility over his compensation, $E_t^j[U(f_j \cdot S_{j,t+1})]$, where $S_{j,t+1} = S_{j,t}(1+R_{j,t+1})+Flow_{j,t}$. Therefore, managers not only care about the performance of their own funds, but also compete investor flows with each other. The game becomes strategic in the sense that managers internalize the effect of their behavior on future flows and also take into account the choice of the other type of managers when they make their choice. As discussed above, low-skill managers now have an incentive to mimic high-skill managers, whereas high-skill managers have an incentive to separate themselves from low-skill managers. Given the complexity of investors' inference problem, I use numerical methods to solve the model. First, I conjecture a functional form of the best responses for low-skill and high-skill managers given the choice of the other party. Second, by iterating the best responses given the choice of the other party, I show that the conjectured best responses do converge to a steady state that they become mutually optimal so a Nash Equilibrium is obtained. The conjectured best response to invest in stock i by manager j at time t is in the form of:

$$w_{i,t}^j = \frac{k^j \eta_{i,t}^j}{\rho + (\tau_i + \tau^j)\lambda_i S_{j,t}} \ , \quad i \in \{1, ..., n\} \tag{13}$$

The only difference between the best response in Equation (13) and the optimal choice in Equation (4) of the baseline model is that managers can deviate from the "first best" and choose the aggressiveness of their trading given signals. The variable k^j is detached from manager skill τ^j and becomes a choice variable that measures the activeness of manager j.[18] Next, I describe equilibrium conditions.

[18]In the baseline model, when managers care only about performance, they optimally choose $k^j = \tau^j$ which produces the best performance given skill.

Equilibrium Definition A Nash equilibrium at time period t is defined jointly by a strategy of high-skill managers k^H, a strategy of low-skill managers k^L, and a vector of fund size $S_t = \{S_{1,t}, ..., S_{M,t}\}$ such that the following three conditions hold:

1. For high-skill managers h, the strategy k^H maximizes the expected utility over next period compensation given fund size $S_{h,t}$ and the strategy of low-skill managers k^L. $E_t^h[U(f_h S_{h,t+1})|S_{h,t}, k^H, k^L] \geq E_t^h[U(f_h S_{h,t+1})|S_{h,t}, k^{H\prime}, k^L]$, $\forall k^{H\prime} > 0$
2. For low-skill managers l, the strategy k^L maximizes the expected utility over next period compensation given fund size $S_{l,t}$ and the strategy of high-skill managers k^H. $E_t^l[U(f_l S_{l,t+1})|S_{l,t}, k^H, k^L] \geq E_t^l[U(f_l S_{l,t+1})|S_{l,t}, k^H, k^{L\prime}]$, $\forall k^{L\prime} > 0$
3. Zero net alpha condition holds for all funds:
   $S_{j,t}$ is the solution to the equation $G(\phi_{j,t}, S_{j,t}) = 0$, where $\phi_{j,t} = Prob_t^I(\tau^j = \tau^H)$, and

$$G(\phi_{j,t}, S_{j,t}) = \phi_{j,t}\alpha(\tau^H, k^H, S_{j,t}) + (1-\phi_{j,t})\alpha(\tau^L, k^L, S_{j,t}) - \frac{f_j}{1-f_j}$$

   where $\alpha(\tau^j, k^j, S_j)$ is the alpha earned by manager j with fund size S_j.

The equilibrium conditions guarantee that no agents have incentive to deviate from their current best strategy. Fund managers have chosen their optimal activeness to trade their signals, and investors have invested optimally based on their beliefs. The formation of investors' belief follows the same process in the baseline model. Investors compute the likelihood ratio given the portfolio holdings they observe, taking into account that managers now can fake activeness to mislead investors. To find an equilibrium, I derive the best response of low-skill and high-skill managers sequentially given the choice of the other party and solve for fund sizes that satisfy zero net alpha condition in each round of iteration until the best responses converge to a mutually optimal point. The starting point for the search is the solution given by the baseline model (i.e., $k^H = \tau^H$ and $k^L = \tau^L$), but the equilibrium is robust to other starting points.

Figure 11 presents the strategies chosen by fund managers in equilibrium. It turns out that both low- and high-skill managers choose to trade more actively in this strategic game and a separating equilibrium is reached. Low-skill managers choose to trade more actively because they want to mimic high-skill managers and make it harder for investors to distinguish them from high-type managers. In the meantime, knowing that low-skill managers are trying to mimic them, high-skill managers also choose to trade more actively to separate themselves from low-type managers and send a strong signal to investors. As shown in Figure 11, both types of managers choose to trade aggressively at a cost of performance, indicating that managers deviate from the "first best" to compete strategically for flows. In equilibrium, the intuition of Proposition 1 still holds true that a more skilled managers trade more actively to produce better performance. In summary, by studying the strategic game, I show that the link between fund alpha and risk holds even when managers act strategically and compete for investor flows. All the intuitions established in the baseline model carry over to this strategic version of the model.

7 Conclusions

In this paper, I study how investors process information to evaluate fund managers and make their investment decisions. By modeling managers' portfolio construction and investors' inference about manager skill simultaneously in an overlapping generation model, I show that investors care about both fund performance and activeness when they evaluate fund managers. The behavior of investors explains why investor flows are convex in fund performance. In addition, my model demonstrates that portfolio holdings provide more useful information to investors than fund returns because holdings reveal manager activeness. The simulation experiment shows that investors who observe lagged portfolio holdings are able to identify the types of managers more than twice as fast as investors who observe only fund returns.

My model offers three testable empirical predictions that separate my theory from existing studies. I show empirical evidence that supports all of the three predictions. First, investor flows respond to both fund performance and activeness. Second, investor flows are more sensitive to the returns of illiquid holdings. Third, investor flows become more sensitive to activeness after a fund chooses to voluntarily disclose its portfolios more frequently.

My paper sheds light on the capital allocations of rational investors. Following my model's predictions, the empirical analysis in my paper proves that there at least exists a group of investors in the market who evaluate fund mangers and allocate their money rationally. Moreover, this paper brings new insights on the benefits of mandating portfolio disclosure to the public, since portfolio holdings reveal useful information to investors that is not fully captured by fund returns.

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A Proofs

Manager's Problem

Following the set up in Section 3, manager j's posterior expectation and variance on the idiosyncratic returns at time t are:

$$\hat{\mu}_t^j \equiv E_t^j[z_{t+1}] = (\Sigma^{-1} + \tau^j I)^{-1} \tau^j \eta_t^j$$
$$\hat{\Sigma}_t^j \equiv V_t^j[z_{t+1}] = (\Sigma^{-1} + \tau^j I)^{-1}$$

The objective of manager j at time t with power utility is

$$\max_{\{w_{m,t}^j, w_t^j\}} E_t^j[\frac{(f_j S_{j,t}(1 + R_{j,t+1}))^{1-\gamma}}{1-\gamma}]$$

Since the return of fund $R_{j,t+1}$ is normally-distributed with $Prob(R_{j,t+1} \leq 0) > 0$, the utility function is not monotone and everywhere differentiable. To obtain a solution, I approximate the gross fund return $1 + R_{j,t+1}$ as $e^{R_{j,t+1}}$ which is log-normally distributed. Once the approximation is made, the objective function can be written equivalently as

$$\max_{\{w_{m,t}^j, w_t^j\}} E_t^j[R_{j,t+1}] - \frac{\gamma}{2} V_t^j[R_{j,t+1}]$$

Take the first derivatives w.r.t. $w_{m,t}^j$ and w_t^j, we can obtain Equation (3) and Equation (4).

The reason I assume power utility and rely on an approximation to get solutions is to maintain a constant relative risk aversion. Under CRRA, managers choose the same weights given their signals regardless of the size of funds they are managing (assume no trading costs), which I believe is an important property when modeling fund managers. Alternatively, I can simply assume CARA utility and get exact solution for active weights,

$$w_{i,t}^j = \frac{\tau^j \eta_{i,t}^j}{\rho S_{j,t} f_j + (\tau_i + \tau^j) \lambda_i S_{j,t}} \quad , \quad i \in \{1, ..., n\}$$

where ρ is the coefficient of absolute risk aversion. Since both fund size and expense ratio are positive and finite, and always observable to investors, using CARA does not change any of the results in the paper. Under CARA, however, managers become more relative risk averse when they manage larger funds. Given a signal, the chosen active weight eventually converges to zero as fund size goes to infinity even without trading costs. To avoid this unrealistic property of CARA, I assume power utility for all managers.

Proof of Proposition 1

Plug in Equation (3) and Equation (4) into fund return dynamics,

$$R_{j,t+1} = w_{m,t}^j r_{m,t+1} + (w_t^j)' z_{t+1} - \frac{1}{2} S_{j,t} \sum_{i=1}^{n} \lambda_i (w_{i,t}^j)^2 = R_{p,t+1} + \delta_{j,t+1}$$

where $\delta_{j,t+1}$ is the abnormal return, $\delta_{j,t+1} = \sum_{i=1}^{n} w_{i,t}^j z_{i,t+1} - C_{j,t} = w_{i,t}^j z_{i,t+1} - \frac{1}{2} S_j \lambda_i (w_{i,t}^j)^2$. Fund alpha is the expectation of fund abnormal return given manager skill and fund size,

$$\begin{aligned}
\alpha_j \equiv E[\delta_{j,t+1}|\tau^j, S_j] &= E[\sum_{i=1}^{n} \frac{\tau^j \eta_{i,t}^j z_{i,t+1}}{\gamma + (\tau_i + \tau^j)\lambda_i S_j} |\tau^j, S_j] - E[\frac{1}{2} S_j \sum_{i=1}^{n} \lambda_i (w_{i,t}^j)^2 |\tau^j, S_j] \\
&= \sum_{i=1}^{n} \frac{2\gamma \tau^j \tau_i^{-1} + S_j \lambda_i ((\tau^j)^2 \tau_i^{-1} + \tau^j)}{2(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^2}
\end{aligned}$$

which is a function of manager skill and fund size, $\alpha_j = f_1(\tau^j, S_j)$. Computing the derivatives w.r.t. skill and size, we get:

$$\begin{aligned}
\frac{\partial \alpha_j}{\partial \tau^j} &= \frac{1}{2} \sum_{i=1}^{n} \frac{2\gamma^2 \tau_i^{-1} + 3\gamma \lambda_i S_j + S_j^2 \lambda_i^2 (\tau_i + \tau^j)}{(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^3} > 0 \\
\frac{\partial \alpha_j}{\partial S_j} &= -\frac{1}{2} \sum_{i=1}^{n} \frac{\lambda_i \tau^j \tau_i^{-1} (\tau_i + \tau^j)(3\gamma + (\tau_i + \tau^j)\lambda_i S_j)}{(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^3} < 0
\end{aligned}$$

The variance of abnormal return is

$$\begin{aligned}
V[\delta_{j,t+1}|\tau^j, S_j] = \sigma_{u_j}^2 &= V[\sum_{i=1}^{n} w_{i,t}^j z_{i,t+1} - \frac{1}{2} S_j \sum_{i=1}^{n} \lambda_i (w_{i,t}^j)^2 |\tau^j, S_j] \\
&= \sum_{i=1}^{n} \frac{\tau_i^{-1}\tau^j + \frac{1}{2}(\tau^j)^2 \tau_i^{-2}}{(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^2} + \sum_{i=1}^{n} \frac{\frac{1}{2}\gamma (\tau^j)^2 \tau_i^{-2}}{(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^3} + \sum_{i=1}^{n} \frac{\frac{1}{2}\gamma^2 (\tau^j)^2 \tau_i^{-2}}{(\gamma + (\tau_i + \tau^j)\lambda_i S_j)^4}
\end{aligned}$$

which is a function of manager skill and fund size, $\sigma_{u_j}^2 = f_2(\tau^j, S_{j,t})$. Computing the derivatives, we obtain that $\frac{\partial \sigma_{u_j}^2}{\partial \tau^j} > 0$ and $\frac{\partial \sigma_{u_j}^2}{\partial S_{j,t}} < 0$.

Proof of Proposition 2

From the perspective of investors, the net return of investing in fund j at time t is

$$R_{j,t+1}^I = (1 + R_{j,t+1})(1 - f_j) - 1 = R_{j,t+1}(1 - f_j) - f_j$$

Based on the standard results in asset pricing (e.g., Cochrane (2005)), fund returns can be priced using investors' stochastic discount factor m_{t+1}:

$$\begin{aligned} E_t^I[m_{t+1}(R_{j,t+1}(1-f_j)-f_j)] &= 0 \\ E_t^I[m_{t+1}(R_{p,t+1}+\delta_{j,t+1})] &= \frac{f_j}{1-f_j} \\ E_t^I[m_{t+1}\delta_{j,t+1}] &= \frac{f_j}{1-f_j} \\ E_t^I[\alpha_j] &= \frac{f_j}{1-f_j} \end{aligned}$$

where $E_t^I[\cdot]$ denotes the investor's expectation conditional on all information up to time t. The second equation above follows from the assumption that $r_{m,t+1}$ is unpredictable based on time t information. Finally, the investor's expectation of fund alpha can be written as the probability-weighted alpha, $\phi_{j,t}\alpha_H + (1-\phi_{j,t})\alpha_L$, and Proposition 2 is obtained.

According to implicit function theorem,

$$\frac{\partial S_j}{\partial \phi_t} = -\frac{\partial G/\partial \phi_t}{\partial G/\partial S_j} = \frac{Alpha(\tau^H, S) - Alpha(\tau^L, S)}{\phi_{j,t}\frac{\partial \alpha_H}{\partial S_j} + (1-\phi_{j,t})\frac{\partial \alpha_L}{\partial S_j}} > 0$$

where the numerator is positive since alpha increases with manager skill, and the denominator is negative since fund alpha decreases with fund size.

Investor's Problem

In the model, investors are rational and update their beliefs in a Bayesian way by evaluating the conditional probabilities that the observed information is generated by a low- or high-skill manager. Denote $\mathcal{F}_{j,t}$ as all the information on fund j available to the investors up to time t. According to Bayes's rule, the posterior beliefs are given by

$$\begin{aligned} \phi_{j,t} &= \frac{f_t(\mathcal{F}_{j,t}|\tau^j = \tau^H)\phi_{j,t-1}}{\phi_{j,t-1}f_t(\mathcal{F}_{j,t}|\tau^j = \tau^H) + (1-\phi_{j,t-1})f_t(\mathcal{F}_{j,t}|\tau^j = \tau^L)} \\ &= \frac{\phi_{j,t-1}}{\phi_{j,t-1} + (1-\phi_{j,t-1})\frac{f_t(\mathcal{F}_{j,t}|\tau^j=\tau^L)}{f_t(\mathcal{F}_{j,t}|\tau^j=\tau^H)}} \end{aligned}$$

Define the likelihood ratio as $\mathcal{L}_t(\mathcal{F}_{j,t}) \equiv \frac{f_t(\mathcal{F}_{j,t}|\tau^j=\tau^L)}{f_t(\mathcal{F}_{j,t}|\tau^j=\tau^H)}$, which tells us whether it is more or less likely that outcome $\mathcal{F}_{j,t}$ is associated a low-skill manager as opposed to a high-skill manager.

Learning from Portfolio Holdings

At time period $t+1$, investors observe portfolio holdings at time t. From investors' perspective, each portfolio weight follows a normal distribution:

$$w^j_{i,t}|\tau^j, z_{i,t+1} \sim \mathcal{N}(\frac{\tau^j z_{i,t+1}}{\gamma + (\tau_i + \tau^j)\lambda_i S_{j,t}}, \frac{\tau^j}{(\gamma + (\tau_i + \tau^j)\lambda_i S_{j,t})^2}) \tag{14}$$

where $z_{i,t+1}$ is stock i's idiosyncratic return at time $t+1$ observable by investors. The likelihood ratio of observing portfolio weight $w^j_{i,t}$ can be written as:

$$\begin{aligned}
\mathcal{L}_{t+1}(w^j_{i,t}) &= \frac{f_{t+1}(w^j_{i,t}|\tau^L)}{f_{t+1}(w^j_{i,t}|\tau^H)} \\
&= a^j_{i,t} exp\{-(\tau^H - \tau^L)[S_{j,t}\lambda_i \delta^j_{i,t+1} + b^j_{i,t}(w^j_{i,t})^2 - \frac{1}{2} z^2_{i,t+1}]\}
\end{aligned}$$

where $a^j_{i,t} = \frac{\gamma+(\tau_i+\tau^L)\lambda_i S_{j,t}}{\gamma+(\tau_i+\tau^H)\lambda_i S_{j,t}}\sqrt{\frac{\tau^H}{\tau^L}}$, $b^j_{i,t} = \frac{1}{2}\frac{(\gamma+S_{j,t}\lambda_i\tau_i)^2}{\tau^H\tau^L}$, and $\delta^j_{i,t+1} = w^j_{i,t} r_{i,t+1} - \frac{1}{2} S_{j,t}\lambda_i (w^j_{i,t})^2$ is the abnormal return of investing in stock i.

Proof of Lemma 1

To prove that the total likelihood ratio of multiple independent signals equals the product of likelihood ratios from each signal, it's sufficient to prove the case of two signals:

$$\mathcal{L}(s_1, s_2) = \mathcal{L}(s_1)\mathcal{L}(s_2)$$

When s_1 and s_2 are independent, we have

$$\mathcal{L}(s_1, s_2) = \frac{f(s_1, s_2|L)}{f(s_1, s_2|H)} = \frac{f(s_1|L)f(s_2|L)}{f(s_1|H)f(s_2|H)} = \mathcal{L}(s_1)\mathcal{L}(s_2)$$

where $f(\cdot)$ is the probability density function.

Fund Flows

Following Equation (9), I take first-order Taylor expansion on the equilibrium fund size $S_{j,t+1}(\phi_{j,t+1})$ at prior believe $\phi_{j,t}$:

$$\begin{aligned}
flow_{j,t+1} &\approx \frac{1}{S_{j,t}}(S_{j,t} + \frac{\partial S_{j,t}}{\partial \phi_{j,t}}(\phi_{j,t+1} - \phi_{j,t}) - S_{j,t}) \\
&= \frac{1}{S_{j,t}}\frac{\partial S_{j,t}}{\partial \phi_{j,t}}(\frac{\phi_{j,t}}{\phi_{j,t} + (1-\phi_{j,t})\prod_{i=1}^n \mathcal{L}(w^j_{i,t})} - \phi_{j,t})
\end{aligned}$$

where $\mathcal{L}(w^j_{i,t})$ is the likelihood ratio of observing portfolio weight $w^j_{i,t}$. Following Equation (7), we can think of investor flows as a function of performance and activeness. The functional form of flows w.r.t performance and activeness are presented in Figure 3.

Take total differential of flows, we get

$$
\begin{aligned}
\mathrm{d}flow_{j,t+1} &= \sum_{i=1}^{n} \frac{\partial flow_{j,t+1}}{\partial w^j_{i,t}} dw^j_{i,t} \\
&= \sum_{i=1}^{n} \frac{1}{S_{j,t}} \frac{\partial S_{j,t}}{\partial \phi_{j,t}} \frac{-\phi_{j,t}(1-\phi_{j,t}) \prod_{s\neq i}^{n} \mathcal{L}_s}{(\phi_{j,t} + (1-\phi_{j,t}) \prod_{i=1}^{n} \mathcal{L}_i)^2} \frac{\partial \mathcal{L}_i}{\partial w^j_{i,t}} dw^j_{i,t} \\
&= g^j_t \sum_{i=1}^{n} \lambda_i (\underbrace{\mathrm{d}\delta^j_{i,t+1}}_{\text{performance}} + \frac{S_{j,t}\lambda_i}{2} \frac{(\frac{\gamma}{S_{j,t}\lambda_i} + \tau_i)^2}{\tau^H \tau^L} \underbrace{\mathrm{d}(w^j_{i,t})^2}_{\text{activeness}}))
\end{aligned}
$$

where $g^j_t = \frac{\partial S_{j,t}}{\partial \phi_{j,t}} \frac{(\tau^H - \tau^L)\phi_{j,t}(1-\phi_{j,t}) \prod_{i=1}^{n} \mathcal{L}_i}{(\phi_{j,t} + (1-\phi_{j,t}) \prod_{i=1}^{n} \mathcal{L}_i)^2}$ which is invariant given manager type and time. From the last equation above we can see that the change of flows are determined jointly by the change of performance and the change of activeness. Moreover, the sensitivity of flows to the holdings-level performance is proportional to the illiquidity of the underlying stock (λ_i). Since we do not observe the transaction costs of trading each stock in the data, I rewrite the last equation as

$$
\mathrm{d}flow_{j,t+1} = g^j_t \sum_{i=1}^{n} \lambda_i (\underbrace{\mathrm{d}w^j_{i,t} r_{i,t+1}}_{\text{hypothetical return}} + h^j_{i,t} \underbrace{\mathrm{d}(w^j_{i,t})^2}_{\text{activeness}}))
$$

where $h^j_{i,t} = \frac{S_{j,t}\lambda_i}{2} \frac{(\frac{\gamma}{S_{j,t}\lambda_i} + \tau_i)^2}{\tau^H \tau^L} - \frac{1}{2} S_{j,t}\lambda_i$, and $w^j_{i,t} r_{i,t+1}$ is the hypothetical return of investing $w^j_{i,t}$ in stock i at time t, which does not take into account trading costs but can be calculated empirically in the data (Kacperczyk et al. (2008)).

Learning from Fund Returns

At time period $t+1$, investors observe the realized asset returns and fund returns at time $t+1$. By construction, fund abnormal return is a function of portfolio weights.

$$
\delta_{j,t+1} = \sum_{i=1}^{n} w^j_{i,t} z_{i,t+1} - \frac{1}{2} S_{j,t} \sum_{i=1}^{n} \lambda_i (w^j_{i,t})^2
$$

where $z_{i,t+1}$, $S_{j,t}$, and λ_i are all observable to investors at time $t+1$. Equation (14) shows that the portfolio weight is normally distributed conditional on manager skill and stock returns. Therefore, fund abnormal return $\delta_{j,t+1} | \tau^j, z_{i,t+1}$ follows a distribution of a normal distribution minus a non-central χ^2 distribution. The likelihood ratio can not be solved analytically with this non-standard distribution, so I rely on numerical methods to solve the investor's inference problem.

Table 1. Evolution of Investors' Beliefs under Different Information Sets

This table presents the evolution of investors' beliefs over time. The economy is populated with 1000 high-skill managers and 1000 low-skill managers. Each manager observes private signals on 10 stocks each period. Investors start with a flat prior that each manager is equally likely to be high-skill or low-skill. Investors update their beliefs on managers at each period based on the new information they observe. Information Set 1 ($\mathcal{F}_1$) means that investors can observe portfolio holdings. Information Set 2 ($\mathcal{F}_2$) means that investors observe only fund returns.

	Time Periods	**5**	**10**	**20**	**30**	**50**	**100**
$\mathcal{F}_1$	Perceived High	130	362	733	850	959	995
	True High	128	353	725	845	958	995
	Perceived Low	134	359	735	856	959	998
	True Low	131	354	729	853	954	998
	Power	12.95	35.35	72.7	84.9	95.6	99.65
	False Positive	1.89	1.94	0.95	0.47	0.31	0
$\mathcal{F}_2$	Perceived High	2	32	269	464	746	877
	True High	1	23	226	404	651	774
	Perceived Low	33	107	427	634	933	1080
	True Low	26	96	360	522	752	874
	Power	1.35	5.95	29.3	46.3	70.15	82.4
	False Positive	22.86	14.39	15.8	15.66	16.44	15.79

Table 2. Summary Statistics

This table reports the summary statistics for a sample of 3725 U.S. equity funds from January 1991 to December 2020. The sample includes 175,265 fund-quarter observations and 141,494 portfolio disclosures. Panel A reports the summary statistics on the key fund-level variables. Panel B presents the correlation matrix of three flow measures.

Panel A: Summary Statistics on Fund Variables

	Mean	Std	25%tile	Median	75%tile
Fund Flows (% per qtr)	0.53	10.02	-4.14	-1.12	3.01
Institutional Flows (% per qtr)	1.35	11.95	-3.99	-0.39	4.58
Non-institutional Flows (% per qtr)	0.52	10.43	-4.40	-1.44	2.83
Net Return (% per qtr)	2.50	10.14	-1.88	3.23	8.00
DGTW Return (% per qtr)	0.20	2.53	-1.14	0.12	1.43
4-Factor Alpha (% per annum)	-0.53	6.41	-3.66	-0.62	2.44
Active Share	0.76	0.14	0.67	0.78	0.88
TNA (million $)	1,430	5,787	56	214	871
Age (years)	12.21	8.63	5.17	10.33	17.42
Expense Ratio (% per annum)	1.20	0.44	0.94	1.16	1.43
Number of holdings disclosure (per fund)	55.50	26.87	33	57	78
Number of stocks (per disclosure)	118	197	49	73	115

Panel B: Correlation Matrix of Three Flow Measures

	Fund Flows	Institutional Flows	Non-institutional Flows
Fund Flows	1	0.804	0.910
Institutional Flows	0.804	1	0.387
Non-institutional Flows	0.910	0.387	1

Table 3. Double Sort: 5-by-5 Performance and Activeness Portfolios

The table reports the average next-quarter flows (Panel A) and alpha (Panel B) for the 5-by-5 performance and activeness portfolios for all equity funds from 1991 to 2020. At the end of each quarter, funds are sorted by fund performance and activeness (sequentially and in that order). Fund performance is measured as past 12-month 4-factor alpha. Fund activeness is measure as the Active Share defined as the percentage of a fund's portfolio holdings that differ from the fund's benchmark index. T-statistics (in parentheses) are based on White's standard errors.

Alpha quintile	Active Share quantile					
	Low	2	3	4	High	High-Low
	Panel A: Subsequent quarter fund flows					
Low	-0.020	-0.021	-0.025	-0.031	-0.030	-0.010
	(-5.74)	(-5.36)	(-7.15)	(-9.04)	(-7.52)	(-1.7)
2	-0.003	-0.005	-0.007	-0.001	-0.007	-0.004
	(-0.89)	(-1.6)	(-1.99)	(-0.32)	(-1.58)	(-0.78)
3	0.007	0.005	0.007	0.019	0.015	0.008
	(2.46)	(1.66)	(2.49)	(6.21)	(3.52)	(1.72)
4	0.016	0.028	0.027	0.029	0.031	0.016
	(5.35)	(5.43)	(7.86)	(8.01)	(6.64)	(2.62)
High	0.039	0.052	0.067	0.074	0.073	0.037
	(9.73)	(9.91)	(13.52)	(12.01)	(11.29)	(4.54)
High-Low	0.060	0.075	0.092	0.105	0.107	
	(12.1)	(10.8)	(14.3)	(14.23)	(12.78)	
	Panel B: Subsequent quarter four-factor alpha (annualized)					
Low	-0.024	-0.023	-0.030	-0.026	-0.021	0.004
	(-9.33)	(-5.63)	(-5.92)	(-4.83)	(-3.16)	(0.54)
2	-0.015	-0.017	-0.016	-0.012	-0.007	0.008
	(-6.49)	(-5.3)	(-3.88)	(-2.3)	(-1.22)	(1.3)
3	-0.011	-0.014	-0.012	-0.006	0.003	0.013
	(-5.36)	(-4.93)	(-3.08)	(-1.35)	(0.5)	(2.28)
4	-0.010	-0.010	-0.009	0.000	0.008	0.018
	(-4.88)	(-3.62)	(-2.57)	(0.02)	(1.57)	(3.19)
High	-0.003	0.001	0.004	0.014	0.018	0.021
	(-0.86)	(0.29)	(0.85)	(2.53)	(3.07)	(3.15)
High-Low	0.022	0.024	0.034	0.040	0.039	
	(5.57)	(3.93)	(4.79)	(5.41)	(4.47)	

Table 4. Investor Flows and Fund Activeness

This table reports the effect of manager activeness on future fund flows for U.S. actively managed equity mutual funds from 1991 to 2020. All estimates are from the regression of quarterly mutual fund flows on prior-quarter activeness measure, fund returns, and controls. Stars denote standard statistical significance ($^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, respectively).

Dependant Var:	Fund Flows		Institutional Flows	
	(1)	(2)	(3)	(4)
Active Share	0.0106^{***}		0.0080^{*}	
	(0.0026)		(0.0045)	
Active Share (t20)		0.0039^{**}		0.0015
		(0.0018)		(0.0033)
4-factor alpha	0.3197^{***}	0.2980^{***}	0.3888^{***}	0.3575^{***}
	(0.0086)	(0.0091)	(0.0137)	(0.0135)
Lagged flows	0.4877^{***}	0.4879^{***}	0.4035^{***}	0.4026^{***}
	(0.0062)	(0.0061)	(0.0068)	(0.0066)
$log(TNA)$	-0.0006^{***}	-0.0005^{***}	0.0044^{***}	0.0043^{***}
	(0.0002)	(0.0002)	(0.0003)	(0.0003)
$log(age)$	-0.0095^{***}	-0.0093^{***}	-0.0134^{***}	-0.0131^{***}
	(0.0004)	(0.0004)	(0.0008)	(0.0007)
Expense ratio	-0.4482^{***}	-0.3398^{***}	0.6867^{***}	0.9044^{***}
	(0.0863)	(0.0869)	(0.1599)	(0.1575)
Observations	103,460	108,177	56,037	58,846
R^2	0.3912	0.3885	0.2675	0.2655

Table 5. Predictive Regression for Fund Flows (Amihud Ratio)

This table presents the regression results of future flows on the return of the most illiquid subportfolio and the portfolio DGTW return controlling for past flows and a set of fund characteristics. Stock illiquidity is measured as Amihud Ratio. The sample period is from 1991 to 2020. Standard errors are clustered by category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance (***$p < 0.01$, **$p < 0.05$, *$p < 0.1$, respectively).

Dependant Var:	Fund Flows		Institutional Flows	
	(1)	(2)	(3)	(4)
$\tilde{R}^{illiquid}_{j,t}$ (top 10%)	0.403***		0.207***	
	(0.035)		(0.057)	
$\tilde{R}^{illiquid}_{j,t}$ (top 20%)		0.373***		0.186***
		(0.030)		(0.048)
$\tilde{R}_{j,t}$	0.396***	0.395***	0.432***	0.431***
	(0.021)	(0.022)	(0.033)	(0.033)
$R^b_{j,t}$	0.262***	0.261***	0.220***	0.219***
	(0.035)	(0.035)	(0.049)	(0.049)
Lagged flows	0.503***	0.503***	0.376***	0.376***
	(0.009)	(0.009)	(0.010)	(0.010)
$log(TNA)$	−0.010***	−0.009***	−0.003***	−0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
$log(age)$	−0.017***	−0.017***	−0.030***	−0.030***
	(0.002)	(0.002)	(0.004)	(0.004)
Expense ratio	−1.500***	−1.487***	0.530	0.529
	(0.291)	(0.289)	(0.577)	(0.576)
Category×date fixed effects	Yes	Yes	Yes	Yes
Observations	77,866	77,876	44,825	44,829
R^2	0.424	0.425	0.308	0.309

Table 6. Predictive Regression for Fund Flows (PIR)

This table presents the regression results of future flows on the return of the most illiquid portion of the portfolio and the portfolio return controlling for past flows and a set of fund characteristics. Stock illiquidity is measured as Price Impact Ratio. The sample period is from 1991 to 2020. Standard errors are clustered by funds and category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance (***$p < 0.01$, **$p < 0.05$, *$p < 0.1$, respectively).

Dependant Var:	Fund Flows		Institutional Flows	
	(1)	(2)	(3)	(4)
$\tilde{R}^{illiquid}_{j,t}$ (top 10%)	0.391***		0.200***	
	(0.038)		(0.059)	
$\tilde{R}^{illiquid}_{j,t}$ (top 20%)		0.408***		0.250***
		(0.033)		(0.049)
$\tilde{R}_{j,t}$	0.396***	0.395***	0.431***	0.431***
	(0.021)	(0.021)	(0.033)	(0.033)
$R^b_{j,t}$	0.260***	0.258***	0.220***	0.218***
	(0.035)	(0.035)	(0.049)	(0.049)
Lagged flows	0.503***	0.503***	0.376***	0.376***
	(0.009)	(0.009)	(0.010)	(0.010)
$log(TNA)$	−0.010***	−0.009***	−0.003***	−0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
$log(age)$	−0.017***	−0.017***	−0.030***	−0.030***
	(0.002)	(0.002)	(0.004)	(0.004)
Expense ratio	−1.504***	−1.486***	0.524	0.532
	(0.291)	(0.290)	(0.577)	(0.576)
Fund fixed effects	Yes	Yes	Yes	Yes
Category×date fixed effects	Yes	Yes	Yes	Yes
Observations	77,811	77,873	44,817	44,829
R^2	0.424	0.425	0.308	0.309

Table 7. Correlation Matrix of Liquidity-based Subportfolios

This table presents the correlation matrix among the portfolio-level DGTW benchmark-adjusted return and the return of the five liquidity-based subportfolios. The sample period is from 1991 to 2020.

Panel A: illiquidity measure: Amihud Ratio

	$\tilde{R}_{j,t}$	$\tilde{R}^{p1}_{j,t}$	$\tilde{R}^{p2}_{j,t}$	$\tilde{R}^{p3}_{j,t}$	$\tilde{R}^{p4}_{j,t}$	$\tilde{R}^{p5}_{j,t}$
$\tilde{R}_{j,t}$	1					
$\tilde{R}^{p1}_{j,t}$ (liquid)	0.562	1				
$\tilde{R}^{p2}_{j,t}$	0.558	0.127	1			
$\tilde{R}^{p3}_{j,t}$	0.549	0.096	0.151	1		
$\tilde{R}^{p4}_{j,t}$	0.517	0.055	0.119	0.163	1	
$\tilde{R}^{p5}_{j,t}$ (illiquid)	0.443	0.017	0.079	0.127	0.185	1

Panel B: illiquidity measure: Price Impact Ratio

	$\tilde{R}_{j,t}$	$\tilde{R}^{p1}_{j,t}$	$\tilde{R}^{p2}_{j,t}$	$\tilde{R}^{p3}_{j,t}$	$\tilde{R}^{p4}_{j,t}$	$\tilde{R}^{p5}_{j,t}$
$\tilde{R}_{j,t}$	1					
$\tilde{R}^{p1}_{j,t}$ (liquid)	0.576	1				
$\tilde{R}^{p2}_{j,t}$	0.577	0.164	1			
$\tilde{R}^{p3}_{j,t}$	0.535	0.088	0.147	1		
$\tilde{R}^{p4}_{j,t}$	0.512	0.050	0.138	0.161	1	
$\tilde{R}^{p5}_{j,t}$ (illiquid)	0.435	0.013	0.074	0.118	0.175	1

Table 8. Predictive Regression for Fund Flows – Subportfolios (Amihud Ratio)

This table presents the horserace regression results of future flows on the return of the five liquidity-based subportfolio and a set of control variables. Stock illiquidity is measured as Amihud Ratio. The sample period is from 1991 to 2020. Standard errors are clustered by funds and category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance (***$p < 0.01$, **$p < 0.05$, *$p < 0.1$, respectively).

Dependant Var:	Fund Flows	Institutional Flows	Non-institutional Flows
	(1)	(2)	(3)
$\tilde{R}^{p1}_{j,t}$ (liquid)	0.337***	0.307***	0.342***
	(0.035)	(0.058)	(0.037)
$\tilde{R}^{p2}_{j,t}$	0.361***	0.388***	0.368***
	(0.038)	(0.061)	(0.039)
$\tilde{R}^{p3}_{j,t}$	0.458***	0.461***	0.490***
	(0.039)	(0.063)	(0.042)
$\tilde{R}^{p4}_{j,t}$	0.396***	0.506***	0.395***
	(0.039)	(0.073)	(0.040)
$\tilde{R}^{p5}_{j,t}$ (illiquid)	0.505***	0.567***	0.482***
	(0.046)	(0.089)	(0.050)
$R^{b}_{j,t}$	0.271***	0.256***	0.317***
	(0.035)	(0.046)	(0.037)
Lagged flows	0.505***	0.371***	0.516***
	(0.009)	(0.010)	(0.009)
$log(TNA)$	−0.009***	−0.003***	−0.010***
	(0.001)	(0.001)	(0.001)
$log(age)$	−0.016***	−0.030***	−0.015***
	(0.002)	(0.004)	(0.002)
Expense ratio	−1.172***	0.481	−2.038***
	(0.273)	(0.585)	(0.288)
Fund fixed effects	Yes	Yes	Yes
Category×date fixed effects	Yes	Yes	Yes
Observations	83,498	46,539	72,913
R^2	0.426	0.304	0.459
$\beta_{p5} - \beta_{p1}$ (p-value)	0.003	0.015	0.026
MR test (p-value)	0.605	0	0.827

Table 9. Predictive Regression for Fund Flows – Frequency of Disclosure

This table presents the regression results of future flows on fund activeness interacted with the frequency of disclosure. The sample period is from 1991 to 2010. Standard errors are clustered by category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance (***$p < 0.01$, **$p < 0.05$, *$p < 0.1$, respectively).

Dependant Var:	Institutional Flows	Non-institutional Flows	Fund Flows
	(1)	(2)	(3)
Active Share	−0.008	0.011**	0.006
	(0.007)	(0.004)	(0.004)
Freq×Active	0.004***	0.001	0.001
	(0.001)	(0.001)	(0.001)
$\hat{\alpha}$	0.382***	0.312***	0.317***
	(0.014)	(0.009)	(0.008)
Lagged flows	0.401***	0.509***	0.488***
	(0.007)	(0.007)	(0.006)
$log(TNA)$	0.004***	−0.001***	−0.001***
	(0.000)	(0.000)	(0.000)
$log(age)$	−0.013***	−0.007***	−0.009***
	(0.001)	(0.000)	(0.000)
Expense ratio	0.654***	−0.792***	−0.465***
	(0.157)	(0.088)	(0.085)
Freq	−0.003***	−0.001	−0.001*
	(0.001)	(0.001)	(0.001)
Category×date fixed effects	Yes	Yes	Yes
Observations	58,233	92,075	108,628
R^2	0.266	0.429	0.392

Table 10. Predictive Regression for Fund Flows – Diff-in-Diff (1991-2004)

This table presents the results of diff-in-diff estimation from 1991 to 2004. The treated funds are those that used to report portfolio semiannually to Morningstar but decided to voluntarily change the frequency to quarterly at some point during the sample period. The control funds are those that always reported portfolios semiannually to Morningstar during the sample period. Standard errors are clustered by category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance (***$p < 0.01$, **$p < 0.05$, *$p < 0.1$, respectively).

Dependant Var:	Institutional flows	Non-institutional flows	Fund flows
	(1)	(2)	(3)
Treat×Post×Active	0.104*	−0.016	0.003
	(0.060)	(0.025)	(0.022)
Treat×Post	−0.074*	0.007	−0.002
	(0.040)	(0.018)	(0.015)
Treat×Active	−0.142	0.001	0.034
	(0.100)	(0.036)	(0.036)
Active Share	0.082	0.013	−0.008
	(0.072)	(0.034)	(0.031)
$\hat{\alpha}$	0.244***	0.171***	0.209***
	(0.057)	(0.029)	(0.027)
Lagged flows	0.302***	0.479***	0.430***
	(0.043)	(0.027)	(0.026)
$log(TNA)$	−0.006	−0.013***	−0.014***
	(0.007)	(0.003)	(0.003)
$log(age)$	−0.020	−0.018*	−0.029***
	(0.025)	(0.011)	(0.010)
Expense ratio	−2.176	−1.718**	−1.150
	(2.326)	(0.846)	(0.814)
Fund fixed effects	Yes	Yes	Yes
Category×date fixed effects	Yes	Yes	Yes
Observations	1,965	3,671	4,280
R^2	0.532	0.624	0.592

Table 11. Predictive Regression for Fund Flows – Diff-in-Diff (2005-2019)

This table presents the results of diff-in-diff estimation from 2005 to 2020. The treated funds are those that used to report portfolio quarterly to Morningstar but decided to voluntarily change the frequency to monthly at some point during the sample period. The control funds are those that always reported portfolios quarterly to Morningstar during the sample period. Standard errors are clustered by category-by-date groups, and are reported in parenthesis. Stars denote standard statistical significance ($^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, respectively).

Dependant Var:	Institutional flows	Non-institutional flows	Fund flows
	(1)	(2)	(3)
Treat×Post×Active	0.064***	0.014	0.015
	(0.021)	(0.012)	(0.011)
Treat×Post	−0.046***	−0.013	−0.011
	(0.015)	(0.008)	(0.008)
Treat×Active	−0.065*	0.019	0.019
	(0.034)	(0.016)	(0.015)
Active Share	−0.004	−0.005	−0.017
	(0.027)	(0.012)	(0.012)
$\hat{\alpha}$	0.430***	0.346***	0.346***
	(0.025)	(0.016)	(0.015)
Lagged flows	0.299***	0.419***	0.412***
	(0.011)	(0.011)	(0.009)
$log(TNA)$	−0.001	−0.013***	−0.011***
	(0.002)	(0.001)	(0.001)
$log(age)$	−0.041***	−0.023***	−0.025***
	(0.005)	(0.003)	(0.003)
Expense ratio	2.245**	−3.523***	−2.115***
	(0.949)	(0.454)	(0.434)
Fund fixed effects	Yes	Yes	Yes
Category×date fixed effects	Yes	Yes	Yes
Observations	15,821	23,521	27,646
R^2	0.313	0.464	0.439

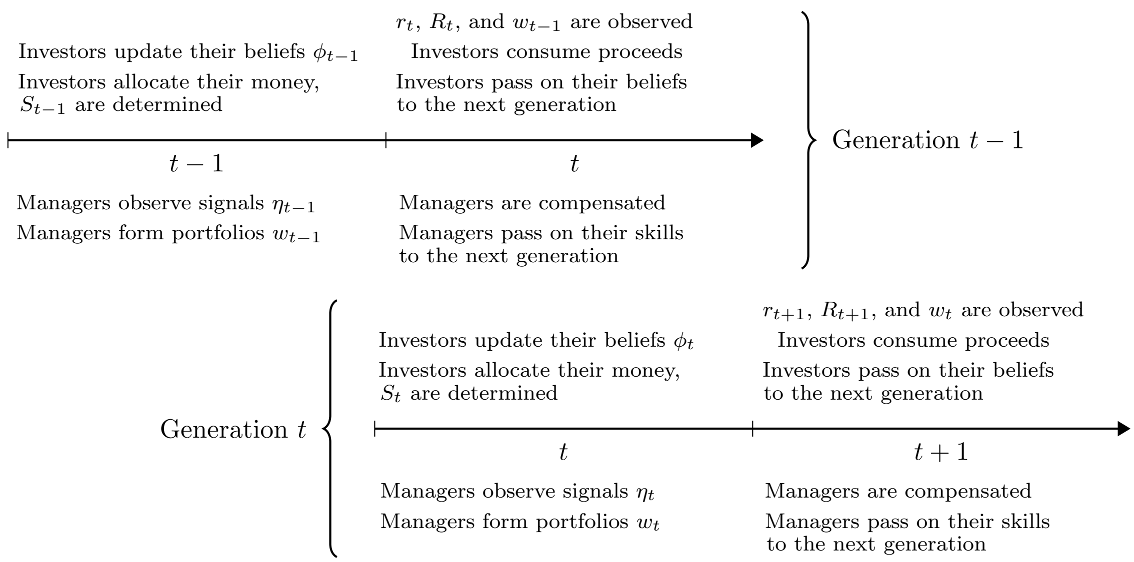
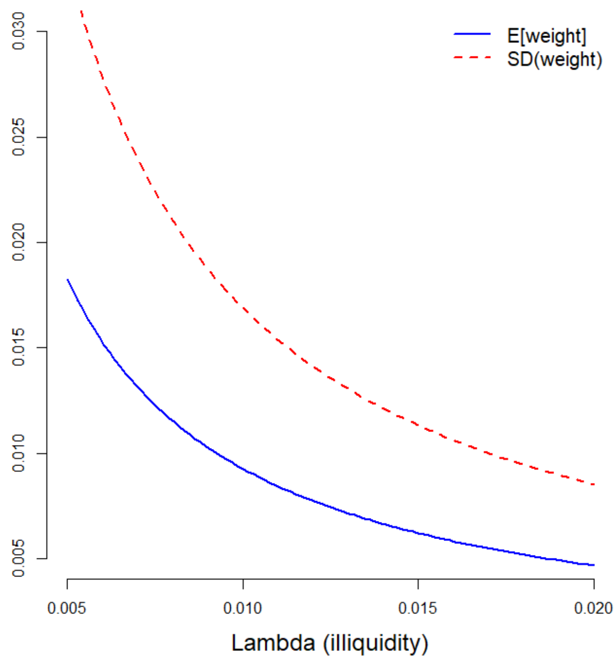


Figure 1. **Timeline of the Overlapping Generation Model** This figure plots the timeline and the actions of the managers and investors from Generation $t-1$ and Generation t. The details are described in Section 3.

Panel A: Portfolio Weights and Stock Illiquidity



Panel B: Investors' Inference and Stock Illiquidity

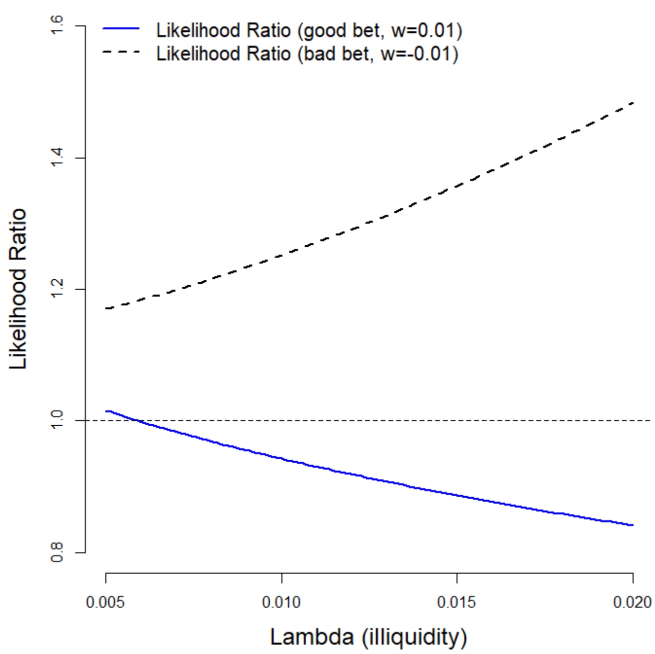


Figure 2. The Effects of Holdings Illiquidity Panel A plots how the expectation and volatility of portfolio weight changes w.r.t. the illiquidity of the underlying stock, given a 10% profit opportunity and holding other things constant. Panel B plots the relation between the underlying stock illiquidity and the likelihood ratio of observing a successful bet ($w = 0.01$) or observing a unsuccessful bet ($w = -0.01$).

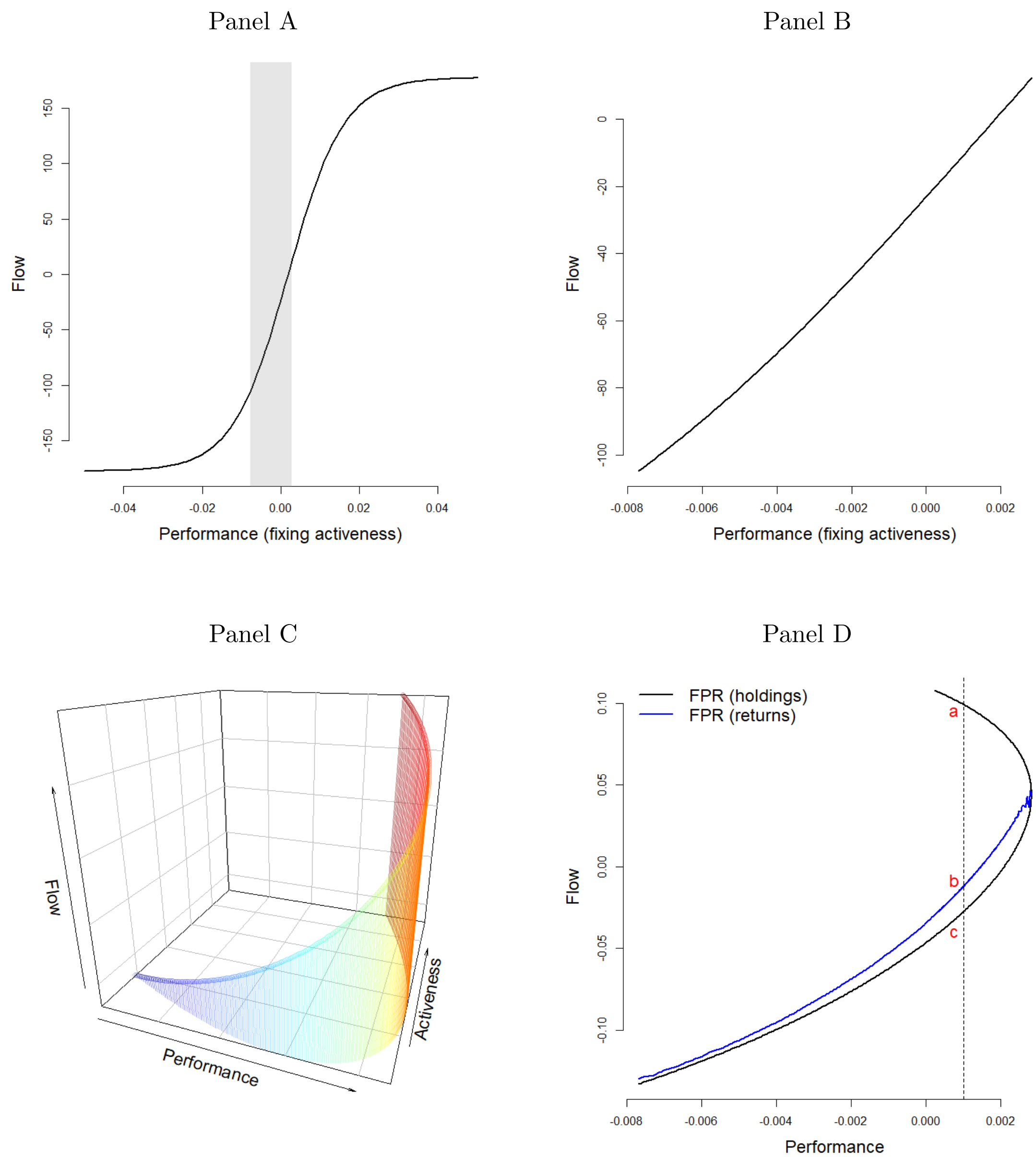
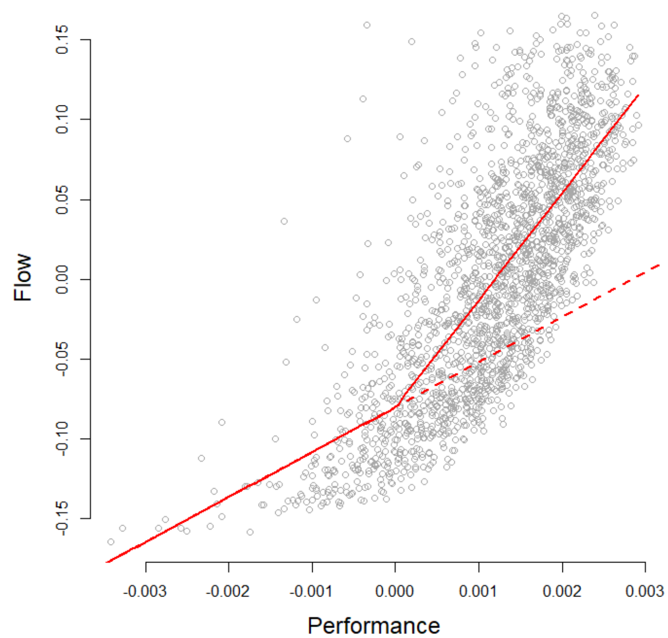


Figure 3. Flow-Performance Relation (Theory) This figure plots the model-implied flow-performance relation (FPR) under the simplest case that there is only one holding in the portfolio. Panel A plots the FPR when the activeness is fixed, where the gray area represents the performance range between 1^{th} and 99^{th} percentile. Panel B zooms in the gray area in panel A. Panel C plots how flows are determined jointly by performance and activeness. Panel D plots the FPR by allowing activeness to vary with performance. Performance is measured as the abnormal return (net of trading costs). Activeness is measured as the squared weight. Except for the blue curve in panel D, all plots are created by assuming investors can observe portfolio holdings.

Panel A: Flow-Performance Relation (Portfolio Holdings)



Panel B: Flow-Performance Relation (Fund Returns)

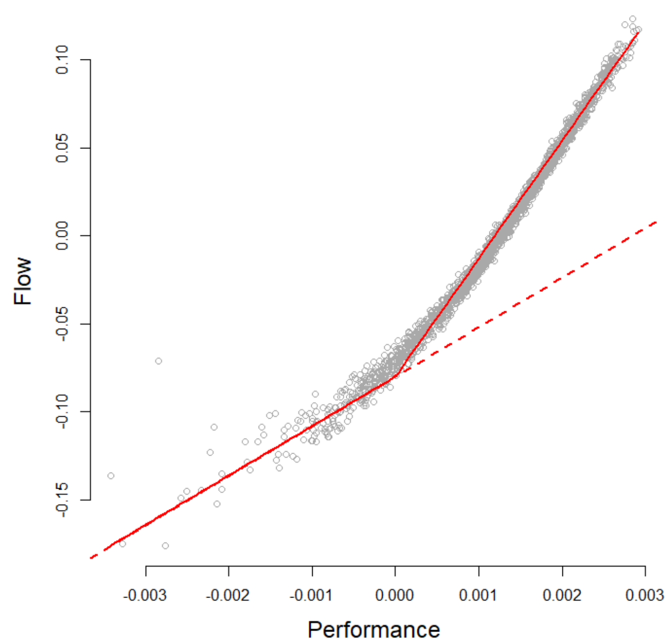


Figure 4. Flow-Performance Relation (Simulation) This figure plots the simulated flow-performance relation (FPR) with ten holdings in the portfolio. Panel A plots the FPR when investors observe portfolio holdings. Panel B plots the FPR when investors do not observe portfolio holdings so they infer skill from fund returns. The red lines are fitted lines from regression $flow_{j,t+1} = \alpha + \beta_1 \delta_{j,t} + \beta_2 \delta_{j,t} \mathbf{1}_{\{\delta_{j,t} \leqslant 0\}} + u_{j,t+1}$.

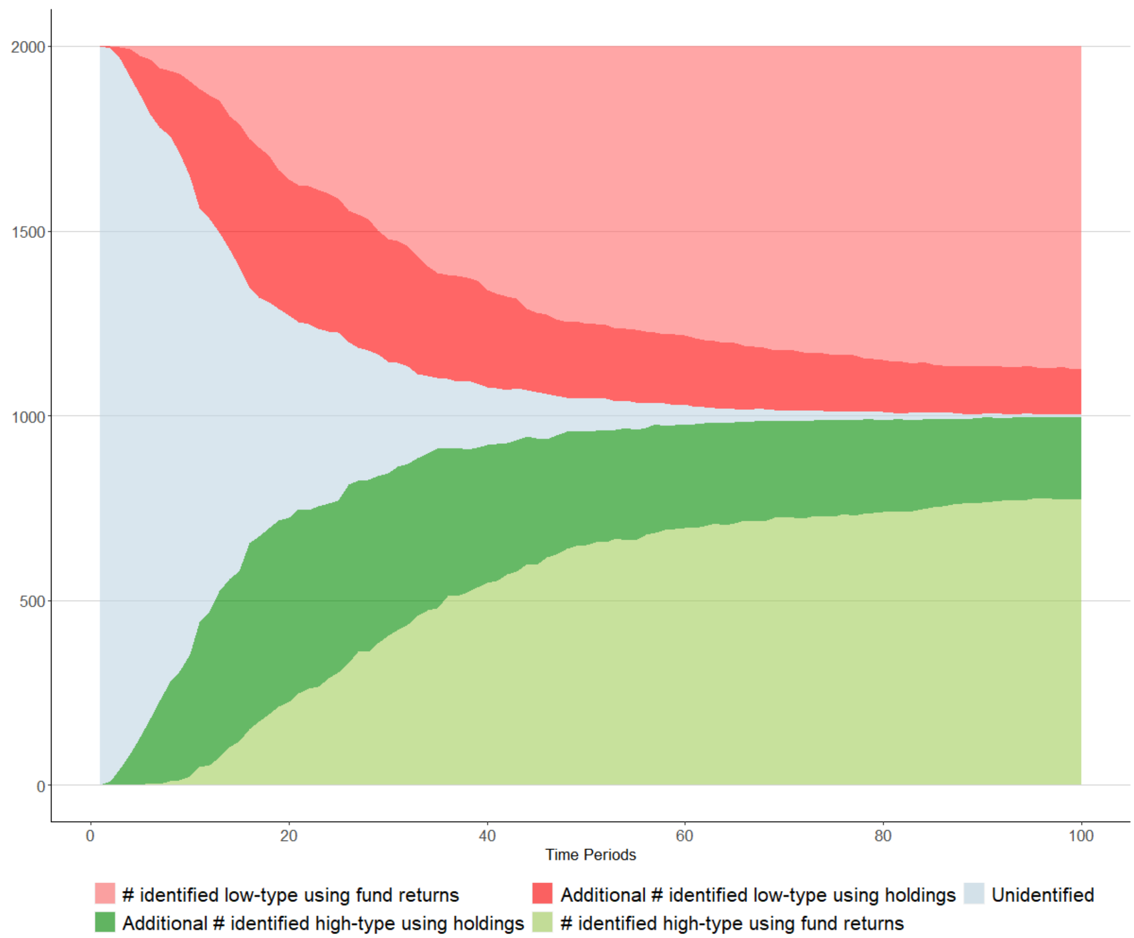
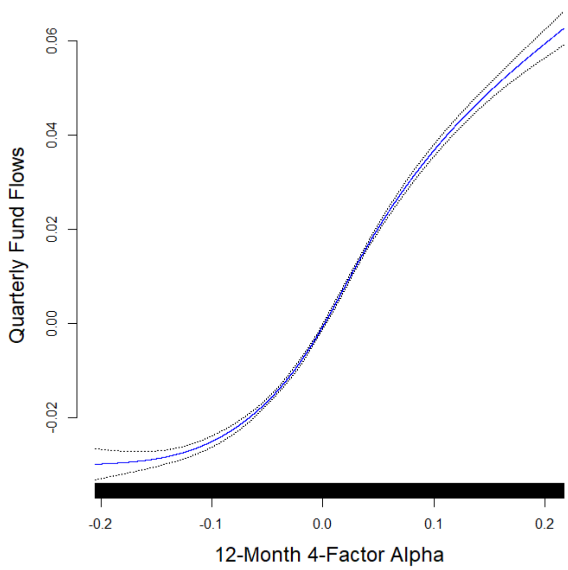
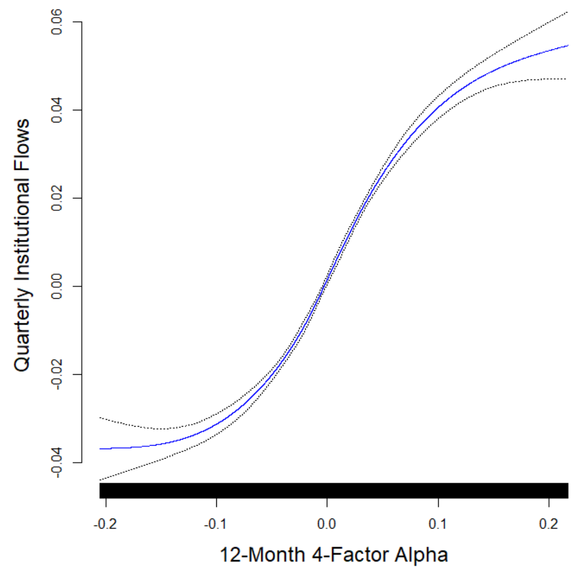


Figure 5. Simulation: Investors' Beliefs Over Time under Different Information Sets The graph plots the number of identified managers using fund returns and portfolio holdings over time. The light green and red areas denote the number of identified high-type and low-type managers using fund returns, respectively. The entire green and red areas denote the number of identified high-type and low-type managers using portfolio holdings, respectively. The darker green and red areas represent the advantage of observing portfolio holdings over fund returns.

Panel A: FPR (Fund Flows)



Panel B: FPR (Institutional Flows)



Panel C: FPR (Non-institutional Flows)

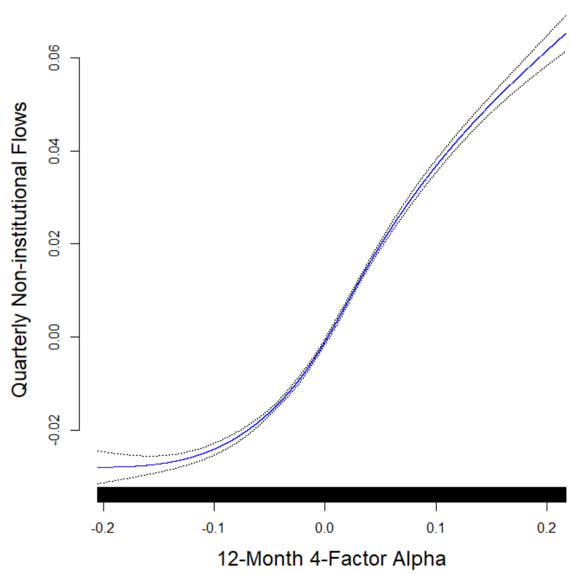


Figure 6. **Flow-Performance Relation (Natural Splines)** This figure plots the fitted natural splines (with 5 df) of the flow-performance relation in the data. Performance is measured as past 12-month 4-factor alpha estimated using daily returns. Three panels use fund flows, institutional flows, and non-institutional flows as the response variable, respectively. Dashed curves represent twice-standard-error curves. Univariate histogram (i.e., rugplot) is displayed along the base of each plot.

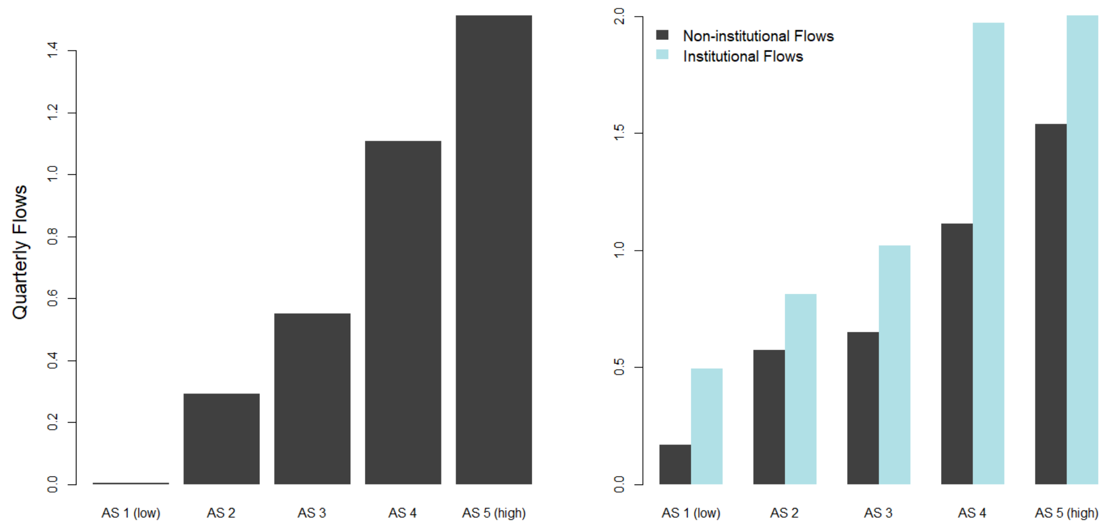


Figure 7. Flow-Activeness Relation The figure plots the flow-activeness relation for all actively managed equity funds from 1991 to 2020. At the end of each quarter, funds are sorted into five quintile portfolios based on their Active Share, and next-quarter flows are computed for each portfolio. The plots presents the time-series average of flows for each of the five portfolios.

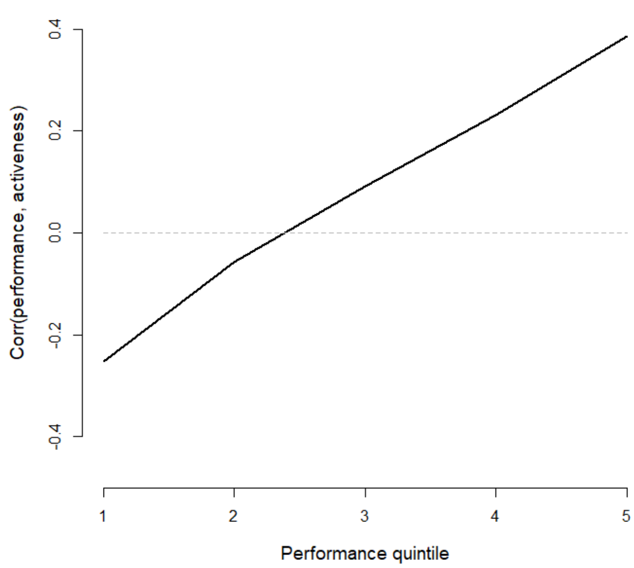


Figure 8. The Correlation between Performance and Activeness within Performance Groups The figure plots the correlation between fund performance and activeness within each of the five performance groups. At the end of each quarter, funds are sorted into five quintile portfolios based on their past 12-month 4-factor alpha. Activeness is measured as the Active Share. The sample period is from 1991 to 2020.

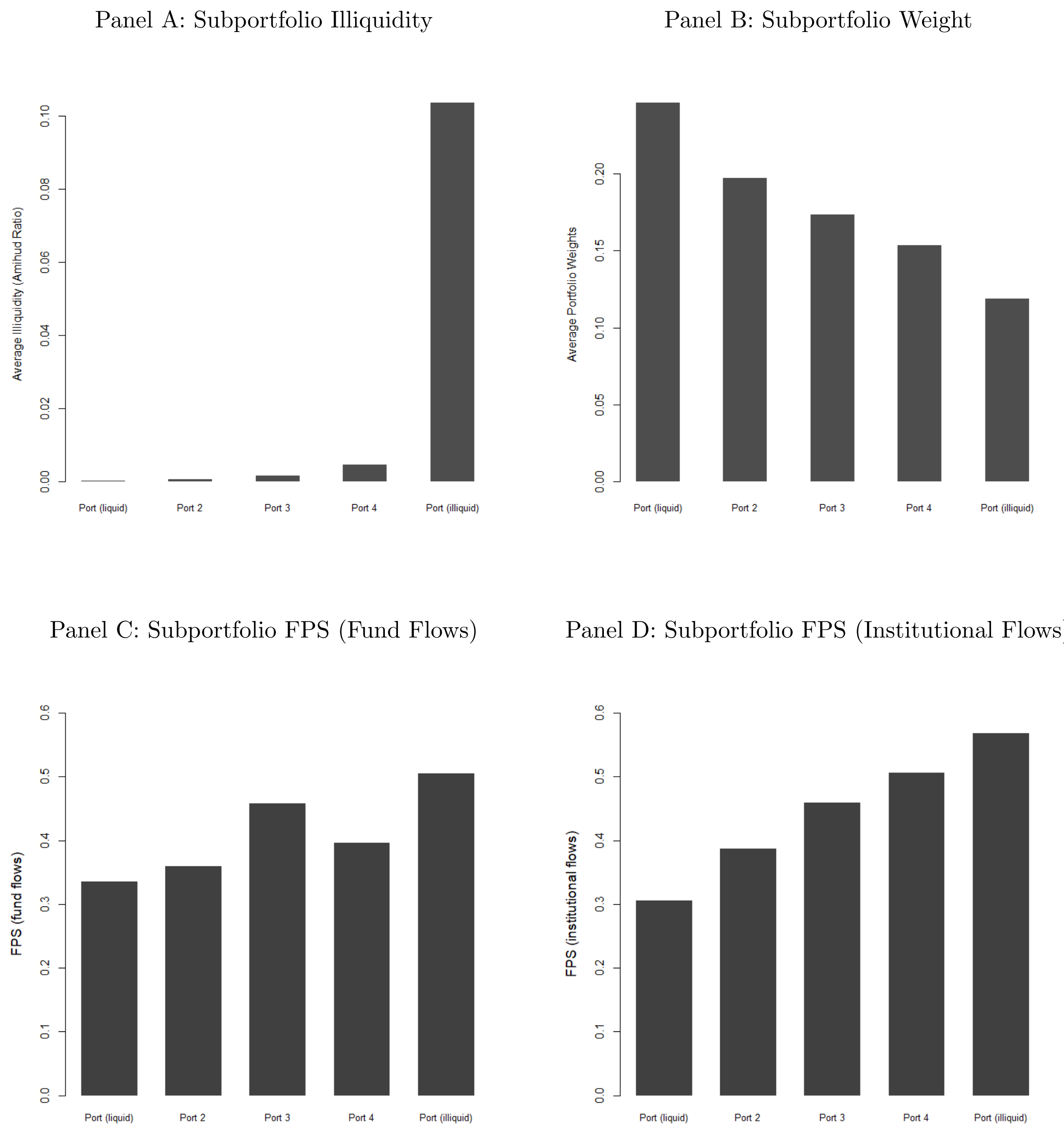


Figure 9. Flow-Performance Sensitivity (FPS) across Liquidity-based subportfolios This figure plots the characteristics and the flow-performance sensitivity of the five liquidity-based subportfolios. Panel A plots the average Amihud Ratio for each of the five subportfolios. Panel B plots the portfolio weights of the five subportfolios. Panel C plots the coefficients of FPS for the five subportfolios from regression (11). Panel D plots the coefficients of FPS from regression (11) using only institutional flows.

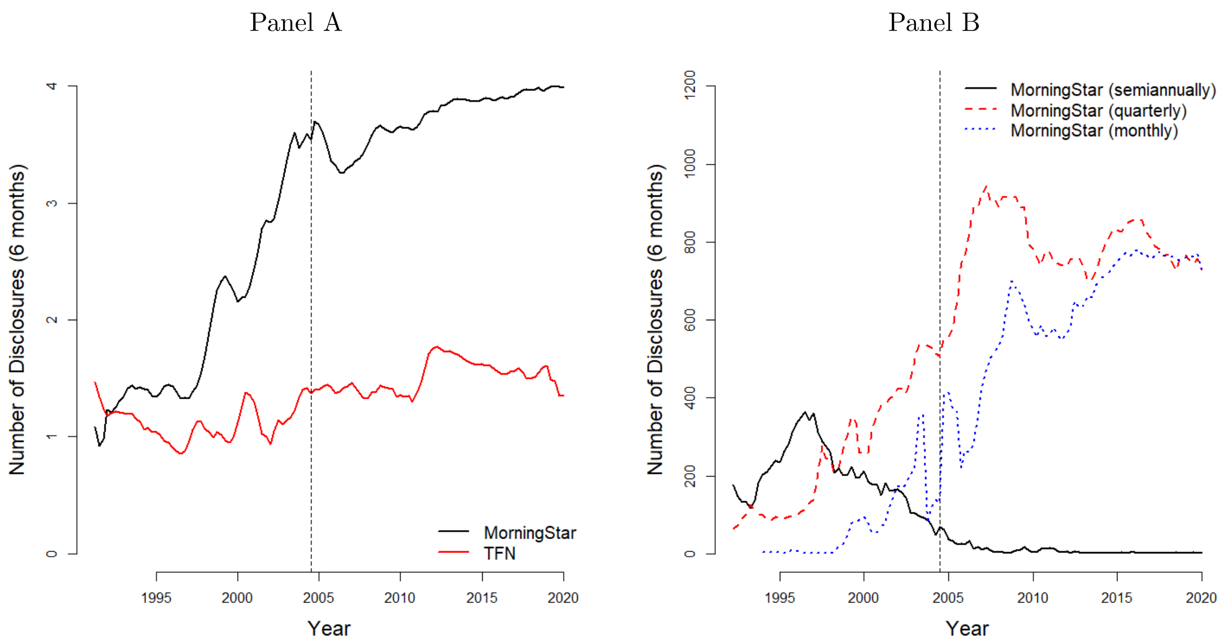


Figure 10. Frequency of Portfolio Disclosure The figure plots the frequency of portfolio disclosure in Morningstar holdings data and Thomson Reuters mutual funds holdings data. Panel A compares the average number of disclosures per fund over the past 6-month period between Morningstar and Thomson Reuters. Panel B plots the number of funds that report semiannually, quarterly, and monthly to Morningstar.

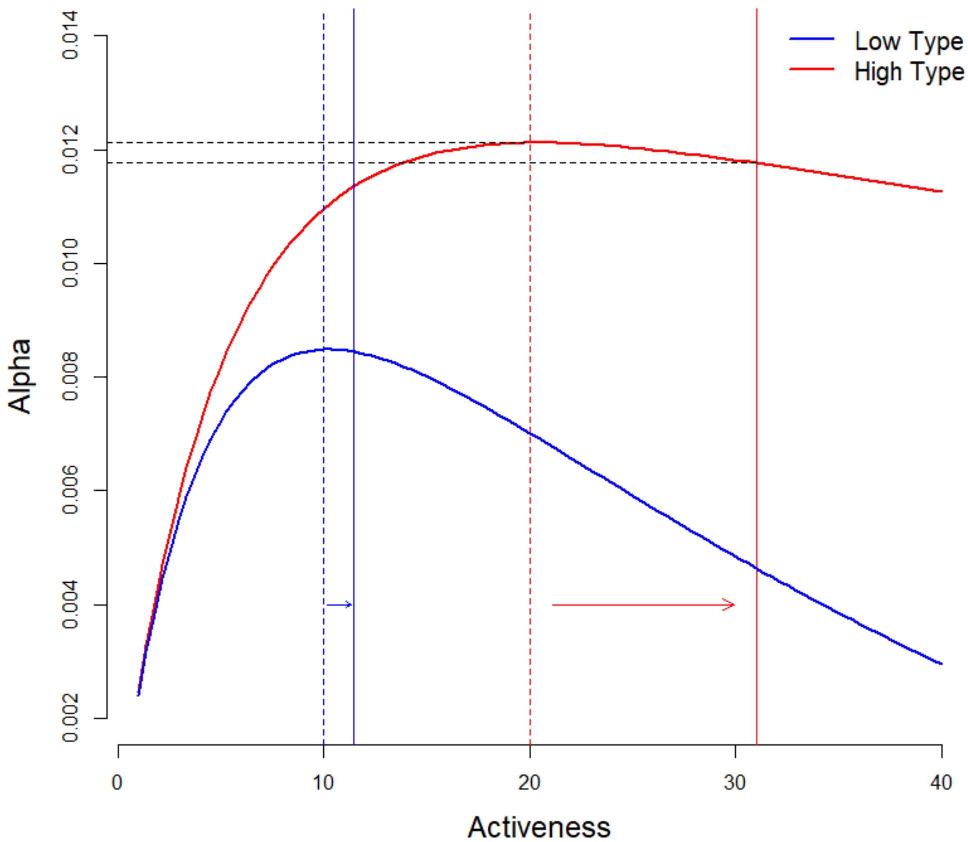


Figure 11. Nash Equilibrium of the Strategic Game The figure plots Nash Equilibrium for the strategic version of the model where managers compete for investor flows. Strategies of low-type managers are denoted in blue and strategies of high-type managers are denoted in red. Dashed vertical lines represent the "first best" solution in the baseline model. The solid vertical lines represent the Nash Equilibrium strategies in the modified model.