Investment Policies and Risk Sharing by Corporate Pensions

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Abstract

Using a corporate risk management model, we show that the investment risk sharing features of defined benefit corporate pensions help explain their aggressive investment policies as well as their diminishing popularity over time. For reasonable parameter values, the model successfully captures key empirical patterns including pension asset allocation and the relations among pension investment risk, corporate bankruptcy probability, and pension funding. Consistent with the observed trend, we find that switching from defined-benefit plans to defined-contribution plans enhances risk transfer and reduces firms' pension funding costs.

Keywords: defined benefit pensions; pension investment

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I Introduction

Defined-benefit corporate pension plans (hereafter the DB plans, or simply pensions) are trust entities established by companies to provide post-retirement income to employees. In the United States, these pension plans manage approximately three trillion dollars of assets. They collectively adopt an aggressive investment approach, with over 50% of their assets allocated to stocks and other high-risk investments. Their risky investment policies are puzzling, as they seem to mismatch the fixed-income nature of pension benefits promised to corporate employees; see, e.g. Bodie (1988), Bodie (1990), Merton (2006), and Cocco (2014).

Meanwhile, the importance of defined-benefit plans in corporate-sponsored retirement savings has been declining (e.g., Munnell and Soto, 2007; Rauh and Stefanescu, 2009). Since mid-1990s, defined-contribution (DC) plans have progressively surpassed DB plans both in terms of the number of participants and the total assets managed, with the gap between the two continuously widening. The shift of popularity from DB plans to DC plans adds another layer to the puzzle. Since employees in the DC plans bear all the investment risk, the popularity of DC plans seems to suggest firms' aversion to such risk (Cobb, 2015). If so, why do DB plans continue to invest so aggressively? Further, letting employees bear all the investment risk of retirement savings is in sharp contrast to the typical risk sharing within a firm, where employees receive safe wages and shareholders bear most of the risk from the firm's operations. In this study, we aim to understand the dual puzzles surrounding the aggressive investment strategies of corporate DB plans and their diminishing appeal.

The existing literature has identified two reasons for defined-benefit pensions to invest conservatively and avoid risky securities. First, as Black (1980) and Tepper (1981) point out, there is a tax advantage for pensions to invest in fixed-income securities. Second, perhaps more importantly, in the context of corporate risk management (e.g., Froot and Stein, 1998), any risky corporate investment lacking a positive alpha does not enhance firm value, while a negative shock from the risky investment can deplete firms' capital, potentially necessitating costly external financing. Therefore, unless pensions can consistently generate superior performance from risky investments, they should primarily hold fixed-income securities that offset the interest rate risk of the pension obligations.[1]

Existing studies have also contemplated on the reasons for the risky pension investment behavior. Sharpe (1976) and Sharpe and Treynor (1977) point out that pension insurance by Pension Benefit Guaranty Corporation (PBGC) creates a moral hazard problem, leading pensions to adopt a high-risk investment approach. However, it is unlikely that pension insurance alone drives risky pension investments. Historical evidence shows that corporate pensions already exhibited aggressive investment behaviors in the 1950s and 1960s, predating the establishment of PBGC. Morevoer, studies by Bodie, Light, Morck, and Taggart (1985), Rauh (2008), and An, Huang, and Zhang (2013) find contrary evidence to the moral hazard hypothesis. They demonstrate that pensions sponsored by firms with higher bankruptcy risk, which indicates a stronger incentive for risk shifting, actually take lower investment risk. Rauh (2008) suggests that risk management considerations, rather than moral hazard, play a dominant role in pension investment decisions.[2]

Our study examines corporate pension investments and pension design from a risk-sharing perspective. Risk sharing is an important element of labor contacts, of which pension is a significant component. In conventional wage contracts, the risk shared between a firm and its employees is typically firm-specific. In the absence of asymmetric information or moral hazard, optimal contracts aim to provide risk-averse employees with safe wages, while allowing well-diversified shareholders to bear firm-specific risks (e.g., Holmstrom, 1983; Harris and Holmstrom, 1982). By contrast, pensions typically maintain well-diversified portfolios; thus pension investment risk is systematic instead of firm-specific. This distinction yields

[1]This risk management rationale provides support to a well-known investment approach, liability-driven investing, which has been advocated by both researchers and investment professionals (Leibowitz, 1986c). However only a small fraction of pensions has actually implemented liability-driven investing.

[2]In addition to the moral hazard perspective, Bodie (1990) discusses three conjectures on aggressive equity investments by defined-benefit (DB) plans, all based on potential misperceptions by corporate managers. First, corporate managers may manage DB plans as if they were DC plans. Second, they may hold the belief that equity investments yield superior performance. The third conjecture involves a misperception that equities serve as an effective hedge against inflation. It is worth noting that empirical evidence does not support the second conjecture. Studies by Lakonishok et al. (1992) and Busse et al. (2010), among others, find that actively managed equity portfolios on behalf of pension funds, on average, fail to generate positive alphas. Furthermore, the motivation to seek alpha does not explain why pensions significantly invest in index portfolios. According to French (2008), during the period of 2000-2006, approximately 30% of DB plans' equity investments are passively managed.

somewhat unexpected implications on how pensions should invest and how pension investment risk should be shared between a firm and its employees. We use a model to illustrate these implications.

In a typical equilibrium with a frictionless capital market, all investors hold their optimal portfolios and agree on the fair risk-return trade-off. However, a crucial aspect of our model is that a firm's employees, constrained by their wealth, perceive the risk-return trade-off differently from the broader capital market. To illustrate this intuition, consider a pension that shifts its investment allocation from riskfree bonds to an equity index fund. Such a reallocation between publicly-traded assets, unless driven by new information, does not inherently impact the firm's valuation by the capital market. Nevertheless, the risky equity investment may yield a negative return, which may trigger costly external financing and result in a negative impact on firm value (Froot and Stein, 1998). Thus, from the perspective of corporate risk management, pensions should avoid such risky investments. On the other hand, the employees of the firm may be under-exposed to the stock market due to the safe wages they receive (because of their aversion to firm-specific risk). If pension benefits represent their only non-wage wealth (i.e., they are wealth-constrained), they may find it desirable to have some systematic risk exposure in their pension payoffs.[3]

Our model further draws on the observation that despite the fixed-income-like promised benefits, employees are nevertheless exposed to the pension investment risk. Employees' pension risk exposure comes from two channels. The first is the bankruptcy channel – when a firm goes bankrupt and in the absence of pension insurance, the value of pension benefits to employees is essentially the net pension asset value. The second is the pension

[3]The preference of employees for investment risk exposure in their retirement savings is evident in the asset allocations of defined contribution plans, where employees make investment decisions based on their own preferences. According to VanDerhei et al. (2018), in 2016, 67% of the aggregate 401(k) assets are invested in equities. The substantial risky assets in tax-shielded retirement saving accounts also suggest significant wealth constraints faced by employees. Studies by Dammon et al. (2004) and Huang (2008) show that, due to different tax rates for interest income and capital gains, if employees have sufficient wealth outside their retirement saving accounts, they should optimally put bonds into tax-deferred retirement accounts and put equities in taxable accounts. The perception that employees have sufficient non-retirement wealth is also inconsistent with the optimality of safe-wage contracts, because unconstrained employees could hold well-diversified portfolios and no longer need safe wages to insure against the firm-specific risk.

surplus sharing channel, whereby employees are entitled to a portion of the excess assets when the value of pension assets exceeds the promised pension benefits.[4] In legal studies, the issue of pension surplus sharing is a subject of debate, as the U.S. pension law, ERISA of 1974, lacks clarity on the ownership of corporate pension surplus. In economics research, a few prominent studies (e.g., Miller and Scholes, 1981 and Bulow and Scholes, 1983) have argued that from the optimal labor contracting perspective, employees own pension surplus. Practically, perhaps the tax code has played the most significant role in promoting pension surplus sharing. Since 1990, the conversion of pension surplus back into firm assets is subject to a punitive excise tax rate of 50%. However, if at least 20% of the surplus is distributed to employees, such as in the form of increased pension benefits, the exercise tax rate is reduced to 20%. This gives a strong incentive for firms to share pension surplus with employees.[5]

In our model, a firm maximizes the firm value—same as in Froot and Stein (1998)—-subject to the participation of wealth-constrained employees. The firm's optimal pension investment decision balances the trade-off between its risk management (financing cost) concern and the preference for investment risk exposure by its employees. Under reasonable specifications for the financing cost function and employees' utility function, along with realistic parameter calibrations, the model's outcomes match key empirical observations on pension investments. For example, in a baseline analysis, we consider a 30-year retirement horizon and an annual bankruptcy probability of 0.5% for the firm (typical for BBB-rated firms), with employees' share of pension surplus at a modest 20%.[6] Other parameters such as market risk premium, market volatility, and financing costs are either calibrated to historical data or taken from estimates by existing studies. The optimal weight on the risky assets in

[4]Bodie (1990) is among the early studies acknowledging the pension surplus sharing channel. He notes that pension contracts are by nature "participating annuities" that guarantee a minimum level of benefits but do not cap the upside of benefits.

[5]Section II of the paper discusses the institutional background of the pension surplus sharing channel and provides data from the Department of Labor to demonstrate the substantial frequency and magnitude of surplus sharing at the termination of defined benefit plans.

[6]Despite that the tax code favors pension surplus sharing, it is worth noting that employees' overall share of the pension surplus may still be modest due to the effective control rights that firms hold over pensions. Nevertheless, our analysis reveals that even a modest level of pension surplus sharing can lead to significant risk-taking in pension investments.

this case is approximately 55%, consistent with the observed asset allocation of corporate pensions.

The model provides several additional insights on corporate pension decisions. First, a firm can give its employees a desired level of risk in the pension payoffs either by adjusting the pension's investment risk or by adjusting empolyees' share of the investment risk (via both the bankruptcy channel and the surplus sharing channel). Our analysis shows that the former approach is more costly to the firm than the latter. Intuitively, this is because increasing the pension's investment risk also increases the firm's expected funding cost. This result turns out to be important for understanding firms' preferences between DB and DC plans (elaborated below). Second, the model provides a prediction on the relation between corporate bankruptcy probabilities and pension decisions. For firms with low default risk, employees' exposure to pension investment risk is mainly through the pension surplus sharing channel. Such firms find it optimal to maintain a high funding ratio, combined with risky investments, to increase the chance of pension surplus.[7] Conversely, firms with high default risk would opt for a low funding ratio. This prediction is consistent with empirical findings in existing studies (e.g., Bodie et al., 1985; Rauh, 2008; An et al., 2013).

From the risk sharing perspective, there is inefficiency in the design of DB plans. Given the divergent preferences between the firm and its employees, it is more cost-efficient to allocate a greater portion, or the entirety, of pension investment risk to employees. However employees' share of the investment risk is limited in typical DB plans. Our analysis indicates that due to such limited risk sharing, DB plans may invest even more aggressively than what employees would do in the DC plans. Furthermore, our analysis demonstrates that, all else equal (e.g., maintaining employees' utility levels), firms' funding costs for DC plans could be significantly lower compared to DB plans. This underscores that inefficient investment risk

[7]It is worth noting that many of our results hold with or without pension insurance. Intuitively, this is because pension insurance turns off the bankruptcy channel but keeps open the pension surplus sharing channel.

sharing could be one of the contributing factors to the declining popularity of DB plans.[8]

The risk sharing perspective is also relevant for evaluating and understanding several recent issues in pension regulations and pension decisions. First, investment risk sharing becomes less effective if pensions are required to maintain full funding, which is mandated by the Pension Protection Act of 2006. In addition to increased funding cost, the full-funding requirement can lead pensions of high-risk firms to take on additional investment risk. Second, our analysis suggests that the pension "de-risking" practice observed in recent years is not a panacea. De-risking involving shifting pension assets into safe annuities or insurance contracts. When employees have a preference for investment risk, "de-risking" a DB plan is apparently more costly relative to conversion into a DC plan. De-risking only makes sense when employees no longer have preference for investment risk – for example when most beneficiaries of a DB plan are already retired and no longer have safe wages, and when plan benefits have already been frozen (thus no pension surplus channel). This helps explain why de-risking transactions mostly happen to such plans. Lastly, although our study primarily focuses on corporate pensions, the framework can be extended to understand public pension decisions, including their aggressive investment policies and the adoption of the "viable-benefits" feature by certain public pensions.

Related Literature

Our study are related to a few streams of literature. First, there is a debate on the optimal pension investment policies and on why pensions tend to invest aggressively. Black (1980) and Tepper (1981)) suggest that for tax reasons, DB plans should invest primarily in fixed income securities. The corporate risk management concern (e.g., Froot and Stein, 1998) also suggests that pensions should avoid risky investments such as equities. Other studies

[8]It is important to recognize that employees' preference for investment risk may have evolved over time. In the 1870s, when corporate pensions were initially introduced, labor productivity was low. Wages and pension benefits combined could merely support a subsistence level of living. During this period, the defined benefit nature of pensions was sensible, and there was likely little inclination for investment risk exposure within pension plans. However, in today's economy, labor productivity has significantly increased, enabling wages and pension incomes to support well above the subsistence level of living. This gives rise to a preference for investment risk by employees.

have contemplated on reasons for observed pension investment practices. Sharpe (1976) and Sharpe and Treynor (1977) argue that pension insurance causes pensions to take excessive risk. Lucas and Zeldes (2006) and Sundaresan and Zapatero (1997) point out that pensions may invest in stocks to hedge against a component of pension obligations related to wage growth. Bergstresser et al. (2006) find that companies use risky pension investments to justify a high expected return on pension assets, in an effort to manipulate corporate earnings. Relative to the existing studies, our paper provides a new perspective based on (lack of) risk sharing; that is, employees' preference for investment risk and the limited investment risk sharing in typical DB plans combined cause pensions to invest aggressively in risky securities. The effect of risk sharing proposed by our model are consistent with empirical observations that existing hypotheses cannot explain – for example, pension investment risk is positively related to a firm's credit rating (e.g., Rauh, 2008).

Second, several studies have observed the decline of DB plans relative to DC plans in the recent decades and have conjectured on the reasons behind this trend. Munnell and Soto (2007) and Rauh and Stefanescu (2009) discuss a range of social, economic, and regulatory reasons. Recently, Rauh et al. (2020) report that firms realize substantial cost savings by freezing DB plans. They point out that firms may have used pension benefit freezing to renege their implict contracts with employees on the un-accrued benefits. Our study finds that DB plans have substantially higher funding costs than DC plans. Such a funding cost disadvantage, which originates from inefficient investment risk sharing, may also be a contributing factor to the relative decline of DB plans.

Third, several existing studies derive optimal asset allocations based on pre-defined utility functions for pensions; e.g., Sharpe and Tint (1990) and van Binsbergen and Brandt (2016). In these studies, the objective of the pension investment decision is either a mean-variance or a power utility function on pension surplus. The derived optimal investment policies typically include a component of portfolio hedging the interest rate risk of liabilities, and a component taking advantage of the attractive return-risk trade-off by risky assets. However, it is unclear what gives rise to the utility function, or the investment objective, of pensions in the first place. In our model, the investment objective of a pension plan is more directly related to the risk preferences of the firm (or its shareholders) and the employees. In a way, the approach taken in our study can be viewed as a structural model that supports the utility-function assumptions in the prior studies. One apparent advantage of our approach is that the model parameters can be more realistically calibrated to the data on firms' financial costs and investors' (employees') risk preferences.

Finally, our paper is related to Love, Smith, and Wilcox (2011), who examine the effect of pension insurance provided by PBGC on pension risk-taking. In their model, employees are not wealth-constrained and value pension investment risk the same way as investors in the financial market. Further, there is no pension surplus sharing. As a result, their model has a strong prediction that pensions should invest risk-free as long as PBGC insurance is fairly priced. By contrast, our model builds on the observation that employees covered by DB plans tend to be wealth-constrained and their preference for investment risk differs from that of investors. This gives rise to the role of risk sharing, and leads to very different predictions on pension investment policies.

The rest of the paper is organized as follows. Section II provides a discussion on the evolution of defined benefit pension plans and pension investments, and the institutional details relevant for understanding pension investment risk sharing. In Section III, we introduce a one-period model of pension investments and provide some analytical results that help understand the key intuitions of the model. Section IV reports the numeric solutions of the one-period model, perform comparative statistic analysis, and examine various extensions of the model. Section V examines a dynamic model of pension investments and pension funding, to address issues left open by the one-period model. Finally, Section VI provides concluding remarks.

II Background: Pension Investments and Risk Sharing

In this part, we discuss the institutional background of pension investments and the risk sharing features of corporate pensions. Section II.1 briefly describes the evolution and statistics of corporate pensions. Section II.2 discusses the legal aspects of pension investment and risk sharing. Section II.3 discusses empirical evidence on pension surplus sharing in the existing literature, as well as recent examples and our empirical analysis of pension surplus sharing at pension terminations.

II.1 DB vs. DC Plans, and Asset Allocation by DB Plans

The origin of corporate-sponsored pensions in the United States can be traced back to the 1880s when companies in the flourishing railroad industry began utilizing pension benefits as a means to attract and retain workers. In 1875, American Express, which was a railway company at the time, established the first pension plan incorporating defined benefit features. The popularity of such plans grew significantly, and by 1929, approximately four hundred corporate defined benefit (DB) pension plans were in operation, sponsored by numerous large corporations of that era (Munnell, 1982). Corporate pensions took a hit during the Great Depression, but recovered afterwards and grew rapidly after World War II, covering 25%, 41%, 45%, 46%, and 43% of all private-sector employees in 1950, 1960, 1970, 1980, and 1990, respectively (McDonnell, 1998).

In recent decades, with the emergence of defined contribution (DC) plans and individual retirement accounts (IRAs), the significance of defined benefit (DB) pensions in the overall landscape of retirement savings has diminished. Figure 2 plots the total assets of DB plans and DC plans annually from 1975 to 2018.[9] Around the mid-1990s, DC plan assets surpassed those of DB plans, and this asset gap has continued to widen. As of 2018, DB plan assets amount to $3.0 trillion, whereas DC plans hold $6.3 trillion in assets.[10] The decline in popularity of corporate DB plans relative to DC plans has been extensively discussed in prior research, such as Munnell and Soto (2007) and Rauh and Stefanescu (2009). These studies have identified various factors that may have contributed to this trend, ranging

[9]The data are obtained from the Private Pension Plan Bulletin, Department of Labor, available at: https://www.dol.gov/agencies/ebsa/researchers/statistics/retirement-bulletins/private-pension-plan.

[10]Additionally, IRAs account for $9.3 trillion in assets as of mid-2018. See "The Role of IRAs in US Households' Saving for Retirement, 2018", by Investment Company Institute and available at https://www.ici.org/pdf/per24−10.pdf

from pension funding costs, job mobility, stock market performance, changes in technology and business structures, the decline of the unionized manufacturing industry, increased life expectancy, to regulatory burdens.

Prior to the stock market boom of the 1950s, U.S. corporate pensions primarily invested in safe assets, including bank deposits and government bonds. It was not until 1950 when General Motors' defined benefit (DB) plan became the first to venture into the stock market (McDonnell, 1998). Over time, pensions have shifted toward substantial risky investments. According to data from the Federal Reserve Board's Flow of Funds report (Z.1 Statistical Release), by the mid-1960s, aggregate corporate pension allocations to stocks surpassed those to fixed income assets. Therefore, the practice of risky pension investment predates the Employee Retirement Income Security Act (ERISA) of 1974 and cannot be solely attributed to the moral hazard problem associated with pension insurance. Recent statistics and surveys indicate that corporate pensions allocate approximately 50% to 60% of their investments to risky assets, including stocks, hedge funds, private equities, and real estate (e.g., Stockton, 2012; Andonov et al., 2012; and Panis and Brien, 2015).

In Figure 1, we present the annual asset allocations by corporate pensions from 2008 to 2018, based on data from the Department of Labor. The figure includes five asset classes: Fixed Income, Insurance, Equity, Real Estate, and Other.[11] The plot illustrates relatively stable weightings across the five asset classes over the period. Fixed Income and Insurance represent relatively safe assets, while Equity and Real Estate are considered relatively risky. If we divide the "Other" category (mainly mutual funds) based on the reported proportions

[11] Corporate DB plans report their asset allocations to the Department of Labor in Schedule H of Form 5500. The reported investments may include externally delegated portfolios in the forms of master trust, common/collective trust, pooled separate account, and 103-12 investment entity. These delegated portfolios further report their investments as "direct filing entities" (DFEs). Starting from 2008, the Department of Labor provides asset allocation statistics that "look through" the asset allocations of DFEs. The data is available at `https://www.dol.gov/agencies/ebsa/researchers/statistics/retirement-bulletins/direct-filing-entity`. We aggregate various subcategories of assets reported in the Department of Labor statistics into the five asset classes. Equity includes preferred and common stocks, as well as partnership/joint venture interests. Fixed income includes cash, receivables, debt securities, and loans. Real estate encompasses both properties used by the plan and those held for investment purposes. Insurance includes assets held in insurance companies' general accounts. Other primarily includes investments in registered investment companies (e.g., mutual funds), but also encompasses other unspecified assets.

of fixed income and equity, corporate DB plans allocate approximately 50% to risky assets.

The concept of aligning the interest rate risk of pension assets with that of liabilities emerged in the 1980s (Leibowitz, 1986a; Leibowitz, 1986b; Leibowitz, 1986c). Initially referred to as cash flow matching, portfolio dedication, or interest rate immunization, this approach is now commonly known as asset-liability management or liability-driven investing (LDI) (e.g., Ang, 2014). Researchers such as Sharpe and Tint (1990), Sundaresan and Zapatero (1997), Rudolf and Ziemba (2004), Ang et al. (2013), and van Binsbergen and Brandt (2016) have examined asset allocation decisions under LDI. In general, this strategy emphasizes the use of fixed income investments to hedge against the interest rate risk associated with pension liabilities.[12] This approach is consistent with the prescription of the corporate risk management theory; e.g., Froot and Stein (1998).

Liability-driven investing has gained significant traction among pension managers in recent years (e.g., Cooper and Bianco, 2003; Leibowitz and Ilmanen, 2016). A more recent and radical approach within this framework is pension "de-risking" or "risk transfer." This strategy involves freezing pension benefits to reduce uncertainty in pension obligations, making additional contributions to address funding shortfalls, and converting pension assets into annuities or insurance policies to eliminate investment risk entirely.[13] In recent years, notable pension de-risking transactions have been carried out by companies such as FedEx, Verizon, UPS, United Technologies, and GM. We discuss the effectiveness of the pension de-risking practice from the risk-sharing perspective in Section IV.5.

[12]However, the prevailing models of liability-driven investing assume that the plan sponsor's objective is to maximize a concave utility function (or a mean-variance utility function) of the pension asset-liability ratio or the surplus ratio (i.e., the ratio of plan surplus to liabilities). Consequently, they do not explicitly consider the impact of differing risk preferences between the firm and its employees when determining pension investment decisions.

[13]In a "buy-in" de-risking deal, companies continue to sponsor the DB pensions while purchasing annuities or insurance contracts from insurers to secure pension obligations. In a "buy-out" deal, insurers assume responsibility for both pension obligations and assets, replacing pension investments with insurance policies and/or annuities.

II.2 Pension Investment Risk Sharing: Legal and Tax Factors

Prior to the Employee Retirement Income Security Act (ERISA) of 1974, corporate pension plans are primarily governed by trust laws. ERISA introduces a comprehensive set of standards for pension vesting, funding, termination, and disclosure. It establishes explicit fiduciary duties for pension trustees and asset managers and creates the Pension Benefit Guaranty Corporation (PBGC) to provide insurance coverage for pension beneficiaries in the event of plan failures. Furthermore, court precedents and tax codes have also exerted significant influence on corporate pension decisions. In the following, we discuss how laws and tax codes shape pension investment decisions and the features of risk sharing within pension plans.

To begin, there is a separation of beneficiary rights and control rights within pension plans. Under ERISA, corporate pension plans must be established as trusts, with qualified employees (both current and retired) serving as beneficiaries. The trustees are responsible for pension administration and are expected to act in the best interests of the beneficiaries. Section 403(a) of ERISA explicitly states that "a fiduciary shall discharge his duties with respect to a plan solely in the interest of the participants and beneficiaries...". Additionally, Section 403(c) stipulates that "the assets of a plan shall never inure to the benefit of any employer and shall be held for the exclusive purposes of providing benefits to participants in the plan...". These provisions indicate that pension decisions, including investment decisions, should prioritize the interests of the plan beneficiaries rather than the plan sponsors. However, ERISA does not provide specific requirements regarding the appointment of plan trustees or the investment of pension assets. In practice, plan trustees are typically appointed by the firms sponsoring the pension plans (except for multi-employer plans, where laobor unions also play a role in appointing trustees). This arrangement grants sponsoring firms effective control over pension decisions.

Acknowledging the influence of plan sponsors' control rights, our model operates under the assumption that pension decisions aim to maximize firm value (or shareholder value) subject to the participation constraint of employees.

Second, employees of a firm may share the downside of pension investment risk via the bankruptcy channel. Due to the separation of DB plan assets in a dedicated trust, they are typically shielded from the demands of a company's creditors in the event of bankruptcy. In this sense, pension assets can be viewed as collateral for fulfilling pension obligations. However, if the pension assets prove inadequate to cover the obligations, the courts treat the unfunded portion of pension liabilities as general unsecured debt of the company, which holds a lower priority in bankruptcy proceedings. Consequently, in situations of bankruptcy, employees with underfunded pensions may receive less than the initially promised benefits. Furthermore, plan sponsors facing bankruptcy proceedings can utilize the distress termination procedure outlined in Section 4041(c) of ERISA, which places limitations on the claims of pension beneficiaries beyond the available pension assets.

The Pension Benefit Guaranty Corporation (PBGC) is established under ERISA to mitigate the downside risk faced by plan beneficiaries in the event of plan sponsor failures. PBGC operates by collecting premiums from plan sponsors in exchange for providing insurance and guarantees on pension benefits. In cases where a company initiates distress termination of its pension plan or when PBGC initiates an involuntary plan termination, PBGC assumes responsibility for the pension assets and fulfills pension obligations to the beneficiaries. However, it is important to note that PBGC's coverage of pension benefits has certain limitations. As of 2019, the maximum covered benefit for a 65-year-old retiree in a single-employer plan is set at $67,295 per year. Additionally, PBGC does not provide coverage for unvested benefits and certain types of vested benefits.[14] Therefore, while PBGC insurance partially mitigates the pension risk associated with the bankruptcy channel, it does not provide complete coverage. It is worth noting that PBGC insurance creates a "pension put" for firms, incentivizing them to increase pension investment risk. This effect has been extensively discussed in existing literature (e.g., Sharpe, 1976; Sharpe and Treynor, 1977).

To demonstrate the bankruptcy channel of pension risk sharing, our baseline model does

[14] An analysis by PBGC (2008) on 125 healthy pension plans with 525,000 participants revealed that 16% of the participants experienced reductions in their (vested) benefits when PBGC assumes control, with an average reduction of 28%.

not include PBGC insurance. We introduce pension insurance as an extension to the model at a later stage. As it turns out, since pension insurance does not affect the pension surplus channel of risk sharing, the key results of the model retain even under full pension insurance.

Third, pension surplus are shared between the sponsoring firm and the employees. Pension surplus refers to the part of pension assets in excess of pension obligations. Whether the firms or the employees have ownership of pension surplus is perhaps the most vague and uncertain legal issue regarding corporate pensions. And the extent of pension surplus sharing has fluctuated largely over time, depending on the law and tax code.

To begin with, while ERISA mandates that pension assets be managed solely for the benefit of plan participants and beneficiaries, it does allow firms to reclaim pension surplus as corporate assets through a practice known as "surplus reversion." Specifically, ERISA's standard termination procedure (Section 4041(b)) permits a plan sponsor to terminate a pension plan and retain pension assets, provided that the sponsor establishes alternative arrangements to fully meet plan liabilities, such as offering a lump sum payment to beneficiaries or substituting pensions with annuities.

Since the implementation of ERISA until the late 1980s, firms were not subject to significant taxes on surplus reversion. Furthermore, investment returns generated by pension assets were exempt from corporate income tax. As a result, many companies utilized DB plans as a tax-advantaged corporate savings tool. They would overfund the pension plans, allowing pension investments to grow tax-free, and later reclaim the surplus (without incurring taxes) when funds were needed by the firm. Consequently, in the 1980s, there was a notable frequency of outright terminations of overfunded pensions (e.g., VanDerhei, 1987; Cather, Cooperman, and Wolfe, 1991). Court decisions during this period generally supported this practice (Cocco, 2014).

The practice of surplus reversion was effectively halted by a series of tax code changes staring in mid-1980s. The Tax Reform Act of 1986 established a 10% excise tax rate when firms revert pension surplus into corporate assets. The tax rate was raised to 15% by the Technical and Miscellaneous Revenue Act of 1988. Finally, the Omnibus Budget Reconciliation Act of 1990 increased the rate to 50%, which remains in effect today. However, the tax code allows for a reduction in the excise tax rate to 20% if at least 25% of the surplus reversion is transferred into a qualified replacement plan, such as a defined contribution plan, or if at least 20% of the surplus is utilized to enhance the pension benefits of qualified participants. These rules create a strong incentive for pension surplus sharing between plan sponsors and participants. As a result, surplus sharing has become a prevailing practice. In Section II.3, we present statistics demonstrating that surplus sharing is the norm rather than an exception at DB plan terminations.

It is worth highlighting that firms, given their effective control over pension decisions, have alternative methods to capture pension surplus without relying on pension terminations. One approach is to reduce ongoing pension contributions, which can effectively diminish plan surplus in the face of growing pension liabilities. Another strategy involves merging an over-funded plan with an under-funded one, thereby circumventing the excise tax on surplus reversion. Additionally, regulations concerning health benefits and severance benefits related to pensions are incomplete or ambiguous, enabling firms to utilize pension surplus to cover expenses that would typically be categorized as normal operating costs or restructuring expenses.

Given these legal and tax factors, it is reasonable to expect that both the firm and the employee have a share of the pension surplus.[15] This surplus sharing feature is a key aspect of our model.

II.3 The Pension Surplus Sharing Channel: Existing Studies, Recent Examples, and Empirical Evidence

Since the pension surplus sharing channel is less known, in this part of the paper we discuss related empirical studies and several recent anecdotal examples. We also provide empirical evidence on pension surplus sharing.

[15] The multi-employer pension plans are an exception, where plan participants (via labor unions) have effective control and typically enjoy all the pension surplus.

Pension surplus sharing may take two forms. First, at pension termination, firms may share pension surplus with employees by making additional payments to participants or making additional contributions to the substitute retirement saving plans (e.g., DC plans). Second, in continuing (non-terminating) pension plans, surplus sharing may take the form of ad hoc increases of plan benefits (Bodie, 1990). Allen et al. (1986) report that retired employees of DB plans receive substantial ad hoc benefit increases. They point out that this is consistent with the notion that firms share pension investment returns with employees. Although they do not link benefit increases directly to pension surplus, they report that larger pension plans provide more of such benefit increases, and argue that larger pensions tend to have better investment returns.

There is also empirical evidence that the financial market values pension assets and liabilities in a way consistent with pension surplus sharing by employees; see, e.g., Oldfield (1977), Feldstein and Seligman (1981), Feldstein and Morck (1983), Bodie et al. (1987), Bulow et al. (1987), and Bodie and Papke (1992). As summarized by Jin et al. (2006), these studies find that "[w]hile each dollar increase in [pension] liabilities lowers the market value of the firm by about a dollar, an equal increase in pension assets raises the firm's market value by less than a dollar." They point out that "while an underfunded pension liability should be fully reflected as a corporate liability, overfunded pension assets are not entirely a corporate asset, because of the difficulty of converting an overfunded pension plan's assets into unencumbered corporate assets." Further, Carroll and Niehaus (1998) find that pension deficit reduces credit ratings of corporate debt by more than the pension surplus of the same size increases credit ratings. They explicitly attribute this to "the mandatory sharing of reverted excess assets" (with employees).

In the recent decades, amid the wave of firms shifting DB plans into DC plans, there are ample anecdotal observations of surplus sharing at DB plan terminations. For example, in January 2019, Sherwin Williams announced that the company expected to complete the termination of its over-funded pension plans through cash payouts to plan participants and annuity purchases, and that it "will use the remaining surplus cash for funding future

Company contributions to a replacement defined contribution pension plan."[16] As another example, in June 2019, after terminating its overfunded DB plan, STRATTEC Security Corporation announced to transfer the pension surplus into its employees' 401(k) accounts, "[r]ather than paying punitive taxes for STRATTEC to recapture those excess funds," as noted in the firm's 4th quarter report of fiscal year 2019.[17]

To gauge how systematic the pension surplus is shared with employees at pension terminations, we perform analysis using the Department of Labor filings by private pension plans, known as the Form 5500 data. This dataset has been used in several existing studies on corporate pensions, such as Rauh (2008) and An et al. (2013). For the sample period from 2000 to 2018, in each year we identify terminations of single-employer DB plans from the data. For these plan terminations, we further obtain information on their plan surplus, if any, and whether any plan surplus is reverted back to corporate assets. Specifically, we identify single-employer DB plans by Line E of Schedule SB of Form 5500 after 2008 and Schedule B before 2008. We identity a plan termination event by the first-time appearance of an answer of "Yes" at Line 5a in Schedule H across all the annual filings of the plan. For the period after 2008, plan surplus is calculated as the market value of plan assets (Line 2a of Schedule SB) minus the sum of total funding target (Line 3d of schedule SB) and target normal cost (Line 6 of Schedule SB) during the year the plan termination is first reported. For the period before 2008, plan surplus is calculated as the current value of assets (Line 1b(1) of Schedule B) minus plan benefit liabilities based on the RPA '94 information in

[16] See corporate news release available at https://www.sec.gov/Archives/edgar/data/89800/000119312519023138/d699691dex99.htm.

[17] See, "STRATTEC Security Corporation Reports Fiscal 2019 Fourth Quarter and Full Year Operating Results Including Non-Cash Pension Settlement and Compensation Expense Charges", corporate news release by STRATTEC, available at https://strattec.gcs-web.com/news-releases/news-release-details/strattec-security-corporation-reports-fiscal-2019-fourth-quarter.

Schedule B (Line 1d(2)(a) plus Line 1d(2)(b) minus Line 1d(2)(d)).[18] Plan asset reversion to the sponsor is identified by the value of "Amount" at Line 5a of Schedule H. We sum up the amount of reverted assets during and after the plan termination year.[19]

Table 1 shows that from 2000 to 2018, the number of single-employer DB plans shrinks from 11,085 to 6,042, with 115 to 375 plans terminated each year.[20] Among the terminated plans, on average 50.3% have a surplus at termination. Further, among plans terminated with a surplus status, the ratio of surplus to plan liability ranges from 2.8% to 50% across years, with an average of 13.8%. Thus, both the frequency and magnitude of surplus at plan termination are substantial.

The table also shows the number of terminated plans that have any plan assets reverted back to the sponsors during the termination year or afterwards. Since 2002, only 1 to 4 terminated plans each year have such assets reversion. The number of terminated plans with asset reversion is 26 and 13 for 2000 and 2001, higher than other years but still a small fraction of plans terminated during those two years (375 and 223 respectively). Finally, the last column of the table reports the average ratio of the amount of reverted assets (during and after the plan termination year) to the amount of surplus across all the terminated plans for a given year. This ratio is typically small, with an all-year average of 1.4%. The surplus of a terminated DB pan may be paid outright as cash to the employees, or as contribution to a DC plan, or paid to a insurance company in a de-risking deal. Regardless of the use of the surplus at plan termination, employees (not the firms) effectively benefit from such surplus.

[18]The funding target reported in Form 5500 is the present value of all benefits accrued or earned as of the reporting year (ERISA Section 303(d)(1)), akin to the Accumulated Pension Obligation (ABO) defined by FASB and reported in COMPUSTAT. By contrast, the Projected Benefit Obligation (PBO) in COMPUSTAT includes the present value of additional future benefits not accumulated or earned yet at the time of reporting, but are expected based on the predicted future employee services and compensation levels. At pension termination, plan sponsors are typically no longer responsible for such additional projected future benefits. Thus using the pension liability reported in Form 5500 yields a better estimate of pension funding status at plan termination, relative to using PBO.

[19]Note that due to a reporting format change, there are no data available on Schedule SB or Schedule B from Department of Labor for the year of 2008. See `https://www.dol.gov/agencies/ebsa/about-ebsa/our-activities/public-disclosure/foia/form-5500-datasets`. Therefore we do not produce statistics for that year.

[20]For several years such as 2002 and 2018, the number of terminations is lower than the decrease in the number of plans filing Form 5500. This is because a plan may stop filing for reasons not related to termination – for example, when the number of the plan participants drops below 100.

Thus it is fair to say that at plan terminations, complete surplus reversion to sponsors is very rare and surplus sharing with employees has become the norm.

III One-period Model

In this part of the paper we introduce a one-period model of corporate decisions on pension benefits, pension funding, and pension investment.[21] The model is an extension of the corporate risk management framework of Froot and Stein (1998). Further, we provide several key analytical results to highlight the model intuitions.

III.1 Model Setup

Consider a firm with a representative employee. The firm maximizes it's share value, which is determined by the capital market that values cash flows with a risk-neutral probability Q. Such a risk-neutral valuation is supported by the assumption that shareholders already hold optimal portfolios. The employee is wealth-constrained and values her cash flows according to a utility function specified below. The firm has a defined benefit pension plan for the employee. To focus on the risk-sharing effect, we make the following simplifying assumptions. First, wage is exogenously determined in labor market. Second, the retirement benefit is in the form of a lump-sum payment.[22] The employee stays with the firm from time 0 to T, and the promised amount of lump-sum payment is known at time 0.[23] Third, there is no pension

[21]In the online appendix, we extend our one-period model to a multi-period dynamic model and inspect how pension investments and contributions vary with the investment horizon. The dynamic model predicts similar relations among the pension investment policy, the pension funding policy, and other parameters.

[22]The retirement benefits of DB plans may be in the form of annuities that the employee receives until death. Therefore the DB plan insures the employee against the longevity risk. We do not consider longevity risk in the model. Alternatively, our assumption here can be understood as that the lump sum payment can be used to purchase annuities with longevity risk insurance.

[23]This assumption abstracts away from the debate on whether the Accrued Benefit Obligation (ABO), or the Projected Pension Obligation (PBO), better represents the firm's pension liability. The former is based on the current salary level of the employee while the latter depends on expected salary level prior to retirement.

insurance in our baseline model.[24]

Timeline. In the one-period model, there are only two dates: 0 and T. At time 0, the firm decides on a promised level of lump sum retirement payment, F, payable at time T, makes a pension contribution, W_0, which constitutes time-0 value of pension assets, and makes investment allocation decision for the pension assets. At time T, pension investments are liquidated to pay the retirement benefit in lump sum.

Investment Opportunity and the Capital Market. The pension can invest in two assets: a risk-free asset with gross return R_f, and the risky asset with gross return R_m. Given the time-0 portfolio weight ω on the risky asset, the pension asset value at time T, W_T, is:

$$W_T = W_0(\omega R_m + (1 - \omega)R_f). \tag{1}$$

The risky asset is fairly priced such that under the a risk-neutral probability Q, $E^Q(R_m) = R_f$. This assumption can be further understood as that the firm's investors (shareholders) hold well-diversified portfolios and the the risk-neutral probability Q is supported by their risk preference.

Time-T Retirement Benefit Payment. At time T, the employee receives a lump sum retirement payment, denoted by S. Note that this actual payment may differ from the promised payment, F, because what the retiree gets depends on two additional factors: 1) whether the firm is bankrupt, which we denote by the indicator variable D, and 2) whether the pension has a surplus, i.e., pension assets W_T exceeding the promised benefit F.

Consider the following two scenarios: 1) In the first scenario, the pension is underfunded at time T, i.e., $W_T < F$. If the firm is not bankrupt, the firm has to make a contribution

[24] In the baseline model we do not consider pension insurance, for two reasons. The first is that the baseline model is used to highlight the effect of the bankruptcy channel. In Section IV.6, we additionally examine the effect of pension insurance. A second reason is that it is important to understand factors affecting risky pension investment without pension insurance. As noted in Section II.1, corporate pensions' aggressive investments in the equity market started in the 1950s, predating the pension insurance by PBGC.

to fill the shortfall, so that the employee receives the full promised payment F. If the firm is bankrupt, we assume that the employee receives the value of pension asset W_T, without further recourse to the firm's other assets. 2) In the second scenario, the pension has a surplus, which is shared between the employee and the firm. Specifically, the employee receives the full promised payment F plus a fraction (α) of the pension surplus $W_T - F$, while the firm retains the reminder $1 - \alpha$ fraction, with $\alpha \in (0, 1)$. Parameter α may be influenced by various factors, including the employee's bargaining power and exercise tax rate when the firm reverts pension surplus into corporate assets.[25] Let D be an indicator variable taking the value of 1 if the firm is bankrupt and 0 otherwise. Then the time-T pension payment to the employee, S, is:

$$S = (1 - D)F + D \min(W_T, F) + \alpha \max(W_T - F, 0) \tag{2}$$

Further, we assume that firm bankruptcy at time T is an idiosyncratic event with a probability of p, and is independent of the firm's pension investment return. With this assumption, the bankruptcy risk does not command risk premium. Thus $E_0^Q(D) = p$.

Employee's Utility. As mentioned above, the employee is wealth-constrained—she does not have wealth outside the pension payment S. Thus her utility is defined over her time-T wealth, $U(S)$, where $U(.)$ is an increasing and concave function. In later analysis we assume that the employee has power utility, with a constant relative risk aversion coefficient γ:

$$U(S) = \frac{S^{1-\gamma}}{1-\gamma} \tag{3}$$

Firm's Financing Cost. The firm incurs financing cost associated with pension contribution at time 0 and time T. The financing cost function at time 0 and T is $C(.)$, an increasing and strictly convex function. At time 0, the firm has an initial cash holding of H_0. Pension contribution can be made out of internal cash H_0 without external financing cost. However

[25]See the online appendix for a more formal exploration of this dependence within a Nash Bargain framework.

the part of contribution in excess of internal cash, $W_0 - H_0$, is associated with financing cost. Thus the firm's time-0 external financing cost is $C(h_0)$, where $h_0 = \text{Max}(W_0 - H_0, 0)$. At time T, if the firm is bankrupt, no further pension contribution is required. But if the firm is not bankrupt and $W_T < F$, the firm has to make a pension contribution $F - W_T$. Thus the external financing cost at time T is $C(h_T)$, where $h_T = (1 - D)\text{Max}(F - W_T, 0)$ and D is an indicator variable taking a value of 1 if the firm is bankrupt and 0 otherwise, as defined earlier. Equivalently, $h_T = \text{Max}(S - W_T, 0)$.[26]

Firm's Cash Flows and Objective. The firm's objective is to maximize its share value subject to the employee's participation. The firm's pension-related cash flow at time 0 is $CF_0 = -W_0 - C(h_0)$. Its time-T pension-related cash flow is $CF_T = W_T - S - C(h_T)$. We assume that the pension-related cash flows are independent of the firm's other cash flows. Since the firm's investor values cash flows in accordance with the risk neutral probability Q, the firm's objective function can be expressed as:

$$\max_{F, W_0, \omega} CF_0 + \frac{1}{R_f} E_0^Q(CF_T). \tag{4}$$

$$\text{subject to:} \quad E_0(U(S)) \geq \mathbb{U}. \tag{5}$$

where E_0^Q denotes the time-0 risk-neutral expectation and E_0 denotes time-0 expectation under the objective probability. $\mathbb{U}$ is the employee's reservation utility.

One can see from the above setup that our model is in essence the corporate risk management model of Froot and Stein (1998) plus the participation constraint of an important group of corporate stakeholders—employees. As it turns out, the model offers a reconciliation between the traditional firm-value (or shareholder value) maximization view and an alternative view on how pension decisions are made. Given the difficulty to reconcile risky pension

[26] The financing cost for pension contribution can be understood as external financing cost as in Froot and Stein (1998), or the opportunity cost if the firm has divert internal funds from its operations to pay for pension contribution. Existing studies have reported that such opportunity cost may be substantial. For example, Rauh (2006) shows that firms have to cut capital investments to pay for pension contributions.

investment with the traditional firm-value maximization objective, Bodie (1990) conjectures that pension decisions may have been made to maximize employees' utility. This alternative view implicitly assumes that a DB plan should invest in a way similar to what employees are to do with their own portfolios—a mean-variance efficient portfolio, for example. However, it does not take into account the convoluted cash flows a DB plan offers to employees, which may substantially alter the investment decisions. We show that by introducing the employee's utility as a participation constraint into the firm value maximization problem, the model goes a long way in explaining observed corporate pension decisions.

III.2 Analytical Results

We start by showing that when the employee is not wealth constrained, the pension should always avoid the risky asset. This result serves to highlight the role of employee wealth constraint in the following analysis. We then describe how the employee and the firm shares pension investment risk in our model, which provides intuition to understand some of the important outcomes of the model.

III.2.1 When the Employee Faces No Wealth Constraint

Our model modifies the standard corporate risk management model of Froot and Stein (1998) by including a participation constraint of the wealth-constrained employee. As a starting point, let's consider the case when the employee faces no wealth constraint—that is, the employee has sufficient outside wealth to "undo" the risk (or the lack of risk) in pension payment provided by the firm. Once the employee is able to achieve the unconstrained optimal risk allocation for her total wealth, her marginal utility would be aligned with the market's pricing kernel, and she would value cash flows under the same risk-neutral probability as the capital market. Thus the employee's participation constraint becomes $E_0^Q(S) = \mathbb{U}$.

Since $E_0^Q(S) = \mathbb{U}$ and $E_0^Q(W_T) = W_0 R_f$, the firm's objective function in Eq. (4) becomes:

$$\begin{aligned}\max_{F,W_0,\omega} CF_0 + \frac{1}{R_f} E_0^Q(CF_T) &= \max_{F,W_0,\omega} -W_0 - C(h_0) + \frac{1}{R_f}(E_0^Q(W_T - C(h_T)) - \mathbb{U}) \\ &= \max_{F,W_0,\omega} -C(h_0) - \frac{E_0^Q(C(h_T)) - \mathbb{U}}{R_f} \qquad (6)\end{aligned}$$

It can be seen that once pension benefit F and initial pension contribution W_0 are chosen, the investment decision w matters only via affecting the expected time-T financing cost $E_0^Q(C(h_T))$, which is an increasing function of w. Therefore, the optimal investment decision is simply to minimize expected financing cost by avoiding any investment risk, i.e., by setting $\omega = 0$. This result is consistent with the prediction by Froot and Stein (1998) in a general model of corporate risk management and by Love et al. (2011) in a model specific to corporate pensions (without the moral hazard problem induced by pension insurance). With the wealth constraint, the employee values pension cash flows differently from the investor. The optimal investment is the result of a trade-off between reducing the firm's financing cost and offering the employee an attractive risk-return trade-off in her retirement payment.

III.2.2 Employee's Share of Pension Investment Risk

The optimization problem in our model is to maximize the firm's value while keeping the employee at the reservation utility. Alternatively, from the social-planner perspective, it is to maximize an appropriately weighted average of both the firm's value and the employee utility. It is thus important to recognize a pension arrangement's impact on both the firm and the employee. To help illustrate the intuition, we introduce a quantity that captures how the employee and the firm jointly share of pension investment risk. We use the sensitivity of pension investment returns W_T/W_0 to the return of the risky asset R_m to measure pension investment risk, which is simply the portfolio weight w:[27]

$$\frac{\partial W_T/W_0}{\partial R_m} = \omega \qquad (7)$$

[27] The partial derivatives in the paper are to be understood in Lebesgue sense.

The employee's exposure to investment risk is the sensitivity of pension payment received by the employee (scaled by initial value of pension assets) to the return of the risky asset:

$$\frac{\partial S/W_0}{\partial R_m} = \frac{\partial W_T/W_0}{\partial R_m}\frac{\partial S}{\partial W_T} = \omega \frac{\partial S}{\partial W_T} = \omega\Delta \tag{8}$$

where $\Delta = \frac{\partial S}{\partial W_T}$ is the employee's share of pension investment risk.

From Eq. (2), we can express Δ as:

$$\Delta = \frac{\partial S}{\partial W_T} = D\mathbb{1}_{W_T<F} + \alpha\mathbb{1}_{W_T>F}. \tag{9}$$

The first component of the expression, $D\mathbb{1}_{W_T<F}$, represents the bankruptcy channel. Specifically, it describes the employee's share of the downside risk of pension investment due to bankruptcy since this terms is non-zero only when $W_T < F$. The second component, $\alpha\mathbb{1}_{W_T>F}$, represents the pension surplus sharing channel. It describes the employee's share of the upside risk due to pension surplus. Note that Δ is itself random, varying with W_T. The following expression provides the average value of Δ.

$$E_0(\Delta) = pE_0(\mathbb{1}_{W_T<F}) + \alpha E_0(\mathbb{1}_{W_T>F}) \tag{10}$$

We note two important effects related to Δ. First, from Eq. (8), the firm can deliver a given level of investment risk exposure to the employee via a combination of w and Δ. In this sense, pension investment risk w and the employee's risk share Δ are mutual substitutes to deliver the employee's risk exposure. With a high Δ, the firm can choose a low w while keeping the employee's risk exposure fixed. We refer to this as the w-Δ substitution effect.

Second, between a high-Δ-low-w plan and a low-Δ-high-w plan that keep the employee's investment risk exposure the same, the former is more cost efficient. This is because high-w has the additional effect of increasing the volatility of time-T pension assets W_t, which in turn increases the likelihood of W_T falling short of pension liability F, and thus increases the likelihood of costly external financing. By contrast, with high Δ and low w, the employee shoulders a larger share of the investment risk and the firm's expected financing cost is lower. Both effects are desirable. Overall, with high Δ, the pension investment risk is allocated more efficiently toward the employee. We refer to this as the Δ-efficiency effect.

One immediate implication from the above discussion is that, when feasible, it is optimal for the firm to offload all the pension investment risk to the employee, i.e., setting Δ to 1. This is essentially a feature of a defined contribution plan, which we will discuss in Section IV.5. However, within the realistic arrangement of a defined benefit plan, the feasible choice of Δ is constrained and it may be much lower than 1. In the optimal pension decision, the firm chooses within feasible ranges of F, W_0, and w to achieve high average Δ while balancing other considerations. We will subsequently use these two effects to understand several key model outcomes.

III.2.3 Positive Portfolio Weight on Risky Assets

In this section, we formalize the proposition showing that the optimal pension portfolio has a strictly positive weight on the risky asset in the presence of employee wealth constraint. Detailed proof is provided in the Appendix.

Proposition 1. *Assume that the firm's time 0 cash H_0 is not sufficient to fully fund the pension.*[28] *If the probability of bankruptcy is nonzero ($p > 0$), the optimal pension portfolio weight ω on the risky asset is strictly positive.*

That is, as long as the employee is exposed to the pension investment risk through the bankruptcy channel, the firm finds it optimal for the pension to put a positive weight on the risky asset.

[28] The precise condition is that $H_0 < \frac{S_r}{R_f}$ where $S_r \equiv [(1-\gamma)\mathbb{U}]^{1/(1-\gamma)}$ is the time-T riskfree payoff to satisfy the employee's participation constraint.

IV Numerical Solutions

In this section, we numerically solve the one-period model, with model parameters calibrated either to the data or to values reported by existing empirical studies. We then perform a variety of comparative analysis to examine the effect corporate bankruptcy probability, pension surplus sharing, pension funding restriction, and pension insurance on pension decisions. We also compare pension decisions and pension funding costs between DB plans and DC plans.

IV.1 Parameter Calibration and Baseline Results

We assume that the return to the risky asset is log-normally distributed, with risk premium and volatility equal to those of the CRSP value-weighted index returns from 1926 to 2016. The risk-free rate is assumed to be constant, equal to the average one-year Treasury yield from 1926 to 2016. Specifically, the annual log return of the risky asset has a mean of 9.4% and volatility of 18.5%, and the annual log risk free rate is 3.3%.

The retirement horizon of the employee, T, is set to 30 years. The annual bankruptcy probability of the firm, p, is set to that of a BBB-rated firm, at 0.5% (from Berk and DeMarzo, 2016). This means that over a 30-year horizon, the probability of bankruptcy is 13.93% (=$1 - (1 - 0.5\%)^{30}$). We assume a power utility function for the employee with the relative risk averse coefficient $\gamma = 6$, based on Constantinides (1990).

We set the employee's reservation utility to the expected utility she would otherwise obtain by optimally investing \$1 billion in her defined contribution plan for 30 years, without interim portfolio rebalancing. We choose an initial wealth level of \$1 billion to be close to the average firm-level present value of pension obligations in the data—\$1,016.25 million as reported by Chen, Yao, Yu, and Zhang (2014).

Following Hennessy and Whited (2007), we assume a quadratic financing cost function for the firm: $C_t(h) = c_a + c_b h + c_c h^2$, where $t = 0,\ T$. We take the estimates of c_a, c_b and c_c from Hennessy and Whited (2007). Specifically, $c_a = 5.98 \times 10^{-4}$, $c_b = 0.091$, and $c_c = 0.4$, per billion dollar of financing. In the baseline model we assume that the firm's initial discretionary cash H_0 is zero, but consider variations of H_0 values in extended analysis. Given this assumption, the extra cost of making \$1 billion of pension contribution is \$0.491 billion, a quite substantial concern for the firm.[29]

We set the pension surplus sharing parameter $\alpha = 0.2$. That is, the employee receives 20% of the pension surplus while the firm keeps 80%. This choice is motivated by the fact that when a firm terminates an over-funded pension, the firm has to offer the plan participants a minimum of 20% of the surplus to avoid the punitive exercise tax rate of 50%.

The above baseline parameter choices are summarized below.

Parameters for the Baseline Case

μ	r_f	σ	c_a	c_b	c_c	p	γ	H_0	α
0.094	0.033	0.185	5.98×10^{-4}	0.091	0.4	0.005	6	0	0.2

Based on the parameter values above, we solve the firm's optimization problem and report the results below. The firm's promised pension payment F is \$3.10 billion, which has a present value ($PV_0(F)$) of \$1.24 billion at the risk-free discount rate. This present value can be interpreted as either the Projected Benefit Obligation (PBO) or the Accumulated Benefit Obligation (ABO) since they are the same in our model. The initial (time-0) pension contribution W_0 is \$1.08 billion. Thus, with a funding ratio $W_0/PV_0(F)$ of 0.87, the pension is slightly underfunded. Further, the optimal portfolio weight on the risky asset, w, is 55%, close to the observed risky asset allocation by aggregate corporate DB plans (above 50%; see discussions in Section II.1). Finally, the financing cost for the initial pension contribution at time 0 is $C(h_0)$=\$0.5689 billion. With relatively high initial funding ratio, the expected time-T net pension contribution $E_0(S - W_T)$ is relatively low, at \$0.4252 billion, and the expected financing cost for time T is $E_0(C(h_T))$ =\$0.1131 billion. The total financing costs are quite substantial. Thus, the balance between the financing cost and the employee's preference for investment risk is a serious concern to the firm, and the solution does not merely tip toward one side of the trade-off.

[29] The typical size of financing in the sample of Hennessy and Whited (2007) is much lower than \$1 billion. Thus the \$0.491 billion extra financing cost for \$1 billion of financing appears to be quite high. Given this possible over-estimation of financing cost, we show that the other side of the trade-off – risk sharing with employees – still has a very strong effect on firms' pension decisions.

IV.2 Comparative Results

In this section, we examine how the firm's optimal solution varies with key model parameters. We start by examining two parameters directly related to the two risk sharing channels—the bankruptcy probability p and the the surplus sharing parameter α. Other parameters examined include the initial free cash H_0, the funding cost parameters c_b and c_c, the risk-free rate r_f, the expected return of the risky asset μ, and its volatility σ.

Panel A of Figure 3 examines pension decisions by varying p, while keeping other parameters the same as the baseline case. The decision variables plotted are the pension's allocation to the risky asset (w), the present value of promised pension payment ($PV_0(F)$) and the initial contribution (W_0). We choose bankruptcy probabilities roughly corresponding to those for the six credit rating categories—AAA, AA, A, BBB, BB, B, and F (sure-bankruptcy). Their annual bankruptcy probabilities are 0, 0.1%, 0.2%, 2.2%, 5.5%, and 100% (according to Berk and DeMarzo, 2016). The corresponding bankruptcy probabilities compounded over a 30-year span are 2.96%, 5.82%, 12.93%, 48.31%, 80.80% and 100% respectively.

Plot A.1 shows that the portfolio weight w decreases as the bankruptcy probability p increases, from over 60% for the AAA-rated firm to around 40% for the F-rated firm. The negative relation between investment risk and bankruptcy probability is consistent with the empirical findings in existing studies; e.g., Bodie et al. (1985), Rauh (2008), and An et al. (2013).[30] Plot A.2 shows that the present value of pension liabilities $PV_0(F)$ increases, while initial pension contribution W_0 slightly decreases, with the bankruptcy probability p. As a result, the initial funding ratio, $W_0/PV_0(F)$, decreases with p. Such a negative relation between the funding ratio and bankruptcy probability has been documented by empirical studies (see, e.g., Bodie et al. (1985)).

The concept of Δ, i.e., the employee's share of the investment risk discussed in length in Section III.2.2, can be utilized to understand the firm's choices of F, W_0, as well as w.

[30] An et al. (2013) show that among financially distressed firms, there is a positive relation between bankruptcy probability and pension investment risk. Our baseline analysis here does not reflect such a pattern. However, we show later that once we introduce initial full funding requirement or PBGC pension insurance, our model delivers a positive relation between bankruptcy probability and pension investment risk for firms with credit rating of B or lower.

To reiterate the two main points in that section, there are a w-Δ substitution effect and a Δ-efficiency effect. Recall from Eq. (9) that $\Delta = D\mathbb{1}_{W_T<F} + \alpha\mathbb{1}_{W_T>F}$. Consider the extreme case of zero bankruptcy probability. In this case Δ is 0 in the region $W_T < F$ (an underfunded pension) and $\Delta = \alpha$ in the region $W_T > F$ (an overfunded pension). The average Δ is thus between 0 and α, which is relatively low since $\alpha = 0.2$. Thus it takes a high portfolio weight w to give the employee a desirable risk exposure in her pension payment, due to the w-Δ substitution effect. Furthermore, as shown in Plot A.2, the firm sets a relatively low level of promised benefit F, and chooses W_0 to be close to full funding, which helps enlarge the relatively high-Δ region (i.e., the region with $W_T > F$) and reduce the 0-Δ region (i.e, the region with $W_T < F$), thereby increasing the average Δ to take advantage of the Δ-efficiency effect. Also, since there is no bankruptcy risk, it takes a relatively low level of promised benefit F to keep the employee at her reservation utility.

At the other extreme is the case that the bankruptcy probability is 1. In this case Δ is 1 in the region of $W_T < F$, and α in the region of $W_T > F$. The average Δ is thus somewhere in between α and 1, relatively high compared with the first case discussed above (i.e., $p = 0$). Therefore, the firm needs only to choose a modest w to get close to the employee-desired level of investment risk for S, due to the w-Δ substitution effect. Further, to benefit the most from the Δ-efficiency effect, the firm would start the pension with severe initial underfunding. Doing so would increase the average Δ because severe initial underfunding increases the chance of terminal underfunding (i.e., the region of $W_T < F$), which is also the region of Δ being 1, the highest Δ value possible. The firm implements severe initial underfunding by choosing a high F and a low W_0. A high level of promised benefit F is needed to keep the employee at her reservation utility. Furthermore, with sure bankruptcy, the firm never faces future financing cost from pension underfunding. The firm's decisions for other bankruptcy probabilities fall naturally in between the two extreme cases discussed above.[31]

[31]Rauh (2008) suggests that the negative relation between bankruptcy probability and pension investment risk is due to the dominating effect of corporate risk management over the risk shifting incentive. Interestingly, our model provides an additional explanation. In our model, this negative relation is the result of efficiently providing desirable investment risk exposure to the employee.

In Panel B of Figure 3, we examine the effect of varying the pension surplus sharing parameter, α, while keeping other parameters the same as the baseline case. Plot B.1 shows an inverse-U shaped relation between the employee's share of pension surplus α and portfolio weight w. Plot B.2 shows that the pension turns from underfunding to overfunding as the employee's share of pension surplus increases, indicating a shift from relying on the bankruptcy channel to the surplus sharing channel to deliver risk exposure to the employee's pension payment S. The non-monotonic relation between α and portfolio weight w is to be expected in our model, because the optimal w is reached by balancing the firm's financing cost concern and the employee's desire for investment risk exposure. As α increases from 0, the firm begins to shift the upside risk exposure to the employee. On the one hand, when α increases, the employee would prefer an decrease in w to maintain a comparable level of effective risk exposure in W, due to the w-Δ substitution effect. This is the reason behind the negative relation of w and α in much of the region of high α values, where the employee's risk preference plays an important role in determining w. On the other hand, as α increases, for the same magnitude of change in w, the resulting change in the firm's financing cost is diminished while the impact on the employee's effective risk exposure is magnified due to the fact that the firm now takes on a smaller share of the risk exposure. That is, as α increases, the effect of changing α on the optimal w manifests itself more through the impact on the employee's benefit from desirable risk exposure and less through its impact on the firm's financing cost. Such a shift is related to the inherent inefficiency of the DB plan to pass the pension's investment risk to the employee. This shift away from the firm's financing cost consideration as α increases relieves the downward pressure on the optimal w. This effect is particularly strong when α is low, resulting in an increasing optimal w. In the region with low α, the firm shifts away from the surplus sharing channel, keeps the pension funding ratio low to utilize the bankruptcy channel, and relies on a alternative way to keep the employee's reservation utility—by offering a high promised pension benefit F, as shown in Plot B.2. The high F, and thus high cost to the firm, is also a symptom of the loss of risk-sharing efficiency due to lowered overall Δ (the Δ-efficiency effect).[32]

In Figure 4, we examine the effects of several additional model parameters on the optimal pension decisions. Panels A, B, and C illustrates the effect of three parameters related to the firm's funding cost—the initial cash available H_0 and the linear and quadratic funding cost parameters c_b and c_c. In general, the firm's marginal and average funding cost becomes lower for higher H_0, and lower c_b and c_c. In the trade-off between reducing the firm's financing cost concern and giving the employee her desired investment risk, this tips the firm's choice toward the latter. Consistent with this effect, we see in Plots A.1, B.1, and C.1 that the pension invests more aggressively (with a higher w under higher H_0, lower c_b and lower c_c. Cheaper funding further makes the surplus sharing channel more cost effective relative to the bankruptcy channel, as the former does not involve any funding cost at time T. This in turn gives the firm a stronger incentive to keep a high initial funding ratio, since time-T pension surplus is more likely under higher time-0 funding ratio. This explains the negative relation of the initial funding ratio $W_0/PV_0(F)$ with the two funding cost parameters c_b and c_c, as illustrated in Plots B.2 and C.2. Finally, as shown in Plots A.2, B.2 and C.2, low financing cost is also associated with low promised benefit F. This is related to the high investment risk by both the firm and by the employee in this case. Given a higher risk exposure, the employee is willing to accept a lower promised benefit while keeping the same utility. The firm also needs more compensation, in the form of lower F, for bearing higher investment risk.

Panels D, E, and F of Figure 4 illustrate the effects of three parameters related to the pension investment opportunities—the risk-free rate r_f, the expected return μ and volatility σ of the risky asset. As shown in Plots D.1, E.1 and F.1, the pension invests more aggressively (with a higher w) when the risky asset becomes more attractive to the employee—lower r_f, higher μ and lower σ. Further, higher risk-free rate, higher expected return and lower volatility represent better investment opportunity. To take advantage such investment op-

[32] We also consider the case where the firm shares no surplus with the employee in the event of bankruptcy. We find similar patterns, shown in the Online Appendix, with a few exceptions when the bankruptcy risk is very high.

portunity, the firm will make more pension contribution at time 0. Thus W_0 is positively related to r_f and μ, and negatively related to σ, as illustrated in Plots D.2, E.2, and F.2. Finally, with higher initial pension funding, the firm benefits more from the surplus sharing channel than the bankruptcy channel. Thus the pension funding ratio $W_0/PV_0(F)$ is also positively related to r_f and μ, and negatively related to σ, as illustrated in the same plots.

IV.3 Endogenous Relation between Pension Investment and Pension Funding Ratio

The relation between pension funding ratio and pension investment risk-taking is of interest to researchers. However, empirical studies have produced somewhat mixed evidence on this relation so far. Bodie et al. (1985) and Panis and Brien (2015) report that the funding ratio is negatively related to pension investment risk-taking, while Rauh (2008) and An et al. (2013) report a positive relation. Our model offers an insight on why it is possible to observe a mixed relation. In our model, both the initial funding ratio and asset allocation are endogenous decisions. Thus, the empirically observed cross-sectional relation between the funding ratio and risky asset allocation should be interpreted with care. In this subsection, we examine various cross-sectional relations that can result from the model.

We first examine the endogenous relation between initial funding ratio and asset allocation that is driven by the bankruptcy probability. We vary the bankruptcy probability from 0 to 1 while keeping other parameters the same under the baseline case. The resulting pairs of (w, $W_0/PV_0(F)$) are plotted in Panel A of Figure 5. The relation between the two is positive. This is consistent with what we observe in Panel A of Figure 3—as the bankruptcy probability increases, F increases and W_0 decreases, while w decreases.

We next generate the w-$W_0/PV_0(F)$ pairs by varying the surplus sharing parameter α. They are plotted also in Panel A of the figure. As the initial funding ratio increases, risky investment w first increases (for α below 0.25, approximately) and then starts to decrease afterwards (for α above 0.25, approximately). This is consistent with what we observe in Panel B of Figure 3—as α increases, the funding ratio increases while the portfolio weight increases initially and decreases in the end. Overall, there is a negative relation between the funding ratio and the risky allocation when the employee's share of the surplus is high, but when the employee's share of the surplus is low, the relation is positive.

If a firm has less free cash at time 0, the pension is more likely to be underfunded, ceteris paribus. In Panel B of Figure 5, we present the endogenous cross-sectional relation between the initial funding ratio and the risky allocation as the result of varying the initial free cash H_0. The relation is positive. As the initial funding ratio becomes higher due to higher initial free cash, the pension's risky allocation is also higher. Also plotted in the panel are the results of varying the two financing cost parameters c_b and c_c—The endogenous relation between initial funding ratio and investment risk-taking is positive.

To summarize, under different causes of the cross-sectional variations, the relation between the funding ratio and the risky allocation can be either positive (in most cases) or negative (in some cases).

IV.4 Pension Decisions with Initial Full-Funding Restriction

The Pension Protect Act of 2006, among other things, tightens pension funding requirements for DB plans. It requires firms with underfunded pensions to make additional pension contributions each year until full funding is reached. The purpose of this requirement is to enhance the safety of retirement benefits and reduce moral hazard problems associated with severe pension underfunding. In this part of the paper, we examine how the full pension funding requirement affects pension decisions by solving the model while imposing an additional exogenous restriction that the pension must be fully funded at time 0, i.e., $W_0/PV_0(F)$=1. We find that this requirement may have an unintended consequence of driving the pension to make more risky investment, especially for low credit-rating firms.

For easy comparison to the analysis in Section IV.2, we vary a set of model parameters including p, α, H_0, c_b and c_c, r_f, μ, and σ. The results on the optimal portfolio weight w and present value of promised pension benefit $PV_0(F)$ are plotted in Figure 6. Below, we focus our discussion on parameters p and α.

Compared with the unconstrained results in Figure 3, Plot A.1 of Figure 6 shows that under the full-funding restriction, pension investment becomes more aggressive for any given credit rating. This is particularly the case when the bankruptcy probability is very high—for credit ratings of B and lower, the portfolio weight w increases with the bankruptcy probability. As discussed in Section III.2.2, when there is no initial full-funding restriction, an important leeway for a firm with a noticeable chance of bankruptcy to enlarge Δ (in order to benefit from the Δ-efficiency effect) is to have severe initial underfunding. The full-funding constraint eliminates this mechanism. As a consequence, the pension has to invest more aggressively due to the w-Δ substitution effect. This effect is especially strong for firms with high bankruptcy probability. For such a firm, there is an additional incentive for risky investment, because in bankruptcy the firm does not face any external financing cost. Our result is consistent with empirical evidence reported by An et al. (2013) for near-bankrupt firms.

In Plot A.2, we see that promised pension benefit $PV_0(F)$ stays relatively flat as the bankruptcy probability increases, in stark contrast to the results in Plot A.2 of Figure 3, where the firm sharply raises $PV_0(F)$ and reduces the initial contribution (shown in Plot A.2 of Figure 3) as the bankruptcy probability increases. Without the full-funding constraint, the firm implements a severe under-funding strategy in order to fully benefit from the bankruptcy channel with high Δ. Such a strategy is eliminated by the full-funding restriction.

Overall, our analysis suggests that despite its purpose of enhancing the safety of pension benefits and reducing moral hazard, the requirement for full pension funding may make pension investment decisions suboptimal by reducing the effectiveness of pension investment risk sharing.

IV.5 Comparison with A "Variable Benefit" Plan: Asset Allocation and Funding Cost

Our model shows that it is optimal for the employee to bear a large fraction of pension investment risk (i.e., the Δ-efficiency effect). The typical defined benefit plan, however, severely limits the employee's exposure to investment risk by having a low average Δ.[33] This leads naturally to the question: what would happen if the pension lets the employee bear full investment risk?

We address this question by comparing the typical pension outlined by the model in Section III.1 with one that lets the employee bear all pension investment risk, which is a key feature of DC plans. Note that we only focus on the investment risk sharing feature, while ignoring other differences between DB and DC plans, such as the longevity risk insurance of DB plans and potential behavioral bias of individual investors making investment decisions for the DC plans. For this reason we label such a comparable DC plan a "variable-benefit" (VB) plan. For the VB plan, we assume that the firm makes a one-time contribution at time 0. At time T, the employee receives the full value of pension assets W_t, with no further firm contribution, thus $\Delta \equiv 1$.

Our comparison focuses on the pension investment decision (w) and the present value of the firm's total expected pension funding costs (simply referred to as the funding cost below). The funding cost for the DB plan is calculated as $-CF_0 - \frac{1}{R_f} E_0^Q(CF_T)$. The total funding cost for the VB plan is simply the firm's time-0 contribution plus the financing cost, $W_0 + C(h_0)$. W_0 in the VB plan is set to keep the employee at the same reservation utility as the DB plan. This set-up makes the comparison on the firm's pension funding cost equivalent to a comparison on the combined welfare of the firm and the employee.

We consider the following variations for the DB plan in comparison. First, we set the initial pension funding ratio $W_0/PV_0(F)$ to be an exogenous value from 0.5 to 1.5. This range is wide enough to include the optimal, endogenously-determined, funding ratio for realistic parameter values of the model. Further, we consider bankruptcy probability corresponding to three credit ratings—AA, BBB, and B, while keeping the surplus sharing parameter α at 20%. Finally, we consider three values of the surplus sharing parameter α: 0.1, 0.5, and 1.0. In this case the firm's bankruptcy probability is kept at the level for the BBB firms. Given

[33] For example, consider a plan with $p = 14\%$ (matching a BBB-rated firm over a 30-year horizon), $\alpha = 20\%$, and a 50% probability of over or underfunding at time T. The average Δ is 17% according to Eq. (10).

these specifications, the time-0 choice variables are reduced to the promised pension benefit F and pension portfolio weight on the risky asset w. The other parameters of the model are kept the same as those in Section IV.1. Note that these variations only affect the investment decision and expected pension funding cost of the DB plan, and not those of the comparable VB plan.

Plots A.1 and B.1 of Figure 7 display the portfolio weights on the risky asset for the DB plan and the VB plan. The portfolio weight w for the VB plan is 0.3659. For most of the parameter values in these two plots (except for very low funding ratios), w of the DB plan is higher than that of the VB plan. That is, the DB plan tends to invest more aggressively than the VB plan (except for the case of α=100% and with a high funding ratio). This is largely due to the w-Δ substitution effect, given that Δ in the VB plan reaches the upper bound of 1.

Plots A.2 and B.2 of the figure show that the total pension funding cost of the DB plan is always higher than that of the VB plan under all parameter variations. The funding cost of the VB plan is \$1.4916 billion, which consists of \$1 billion time-0 contribution and \$0.4916 billion of financing cost. By contrast, in Plot A.2, for a BBB-rated firm with a funding ratio of 1, the total funding cost of the DB plan is \$1.8537 billion, about 24% higher than the VB plan. Plot B.2 further shows that the funding costs of DB plans with various surplus sharing parameters are generally higher than that of the VB plan. The only exception is that when the exogenous funding ratio becomes very high and when α=1, the funding cost of the DB plan converges to that of the VB plan. The funding cost difference between the two plans can be viewed as the shadow cost of inefficient risk sharing by the DB plan, which is strikingly large and highlights the importance of investment risk sharing in pension design.

As discussed in Section II.1, in recent decades, firms increasingly shift away from DB plans and toward DC plans. An oft-cited reason by firms is the high funding cost of the defined benefit plans (Munnell and Soto, 2007). The results we obtain suggest that inefficient risk sharing may be a cause of the cost disadvantage of DB plans, among other factors.

The analysis can further help us understand and evaluate an emerging trend of public pensions. DB plans still dominate the public-sector retirement saving. Public DB plans invest aggressively and many are underfunded. Despite the difference in plan sponsors (states and local governments vs. for-profit companies owned by investors), public-sector employees should also have a preference for investment risk, just like corporate employees. Thus, the funding cost of public pensions may potentially be reduced if plan participants share the investment risk. Indeed, in recent years a few state pensions have included a variable-benefit component that is partially tied to pension investment performance.[34]

The analysis also has an implication for the pension "de-risking" strategy popularly contemplated by some firms in recent years. As discussed in Section II.1, "de-risking" typically involves freezing pension benefits and shifting pension assets into safe annuities or insurance contracts. This eliminates investment-risk sharing by employees. Given the risk preference of employees, de-risking may be suboptimal and leads to high funding costs for firms (e.g., the lump-sum payment to insurers in the de-risking transactions). Conversion into a DC plan, by comparison, is a less costly alternative. The cost disadvantage of de-risking may be less significant for plans with mostly retired employees, who no longer receive sage wages and thus reduced preference for investment risk. This helps explain why de-risking transactions mostly happen to such plans but not plans dominated by employees in active service.[35]

IV.6 The Effect of Pension Insurance

PBGC pension insurance provides guarantee to pension benefits in the case of distress termination of a corporate DB plan. As a consequence, PBGC pension insurance has the potential to shut off the bankruptcy channel of pension investment risk sharing. However, it leaves the

[34]See "Proactive Pension Management: An Elected Official's Guide to Viable Benefit and Contribution Arrangements," by Center for State & Local Government Excellence, AARP. The document is available at `https://www.slge.org/assets/uploads/2019/09/proactive-pension-management.pdf`. The states with variable-benefit components of pensions related to investment performance include Wisconsin, South Dakota, Utah, Iowa, Virginia, and Colorado.

[35]There are additional factors, beyond our model, that give retirement saving plans with DB features advantages over DC plans. One such factor is that a DB plan, or a hybrid plan that gives pension surplus to employees and yet offers a promised minimum pension payoff, offers mortality risk sharing among employees. Another factor is the existence of a subsistence level of wealth, below which an employee cannot survive after retirement. The guaranteed minimum pension payoff can be used to satisfy the subsistence requirement.

pension surplus sharing channel intact. Here we examine the effect of pension insurance on pension decisions by taking into account the following features of pension insurance. First, as discussed in Section II.2, PBGC coverage has limits. PBGC's coverage excludes any unearned pension benefits at the time of bankruptcy, and has an upper limit—in 2019, \$67,295 a year for a retiree at age of 65. Thus for highly paid employees (e.g., airline pilots), PBGC insurance provides only a partial coverage of their pension benefits. Second, PBGC is thinly "capitalized", in that its net position (i.e., equity) is very low relative to the benefit coverage it provides.[36] Third, as a "quasi-government" institution, PBGC's financial obligations do not have an explicit guarantee by the federal government; but rather, it relies on an implicit guarantee. Thus, it is fair to assume that PBGC faces a shadow cost of external financing. Fourth, PBGC invests its assets in stocks, bonds, private equities and real assets. Thus we assume that it values cash flows in a way similar to well-diversified investors, i.e., using the same risk-neutral probability. Finally, PBGC's insurance premium could be either overpriced, fairly priced, or underpriced (e.g., Love, Smith, and Wilcox (2011)), and the pricing of pension insurance may affect corporate pension decisions.

We introduce pension insurance into our model in the following way. First, we assume that in the case of bankruptcy by a firm, PBGC provides the employee a guarantee of total pension payment θF, where F is the promised pension benefit, and $\theta \in (0, 1)$ is the coverage factor.[37] We set a baseline case of $\theta = 0.7$, and allow θ to vary in additional analysis. Thus,

[36]According to its annual report for the fiscal year of 2019, PBGC's single-employer pension insurance program has a small positive net position of \$8.6 billion relative to a total liabilities of \$113 billion. Its multi-employer program has a negative net position of -\$65 billion, bring PBGC's total net position to a negative -\$56.5 billion. Without government bailout, the multi-employer program is projected to be insolvent by FY2025.

[37]As discussed earlier, PBGC provides a maximum coverage on each individual plan participant's benefit. Suppose there are N participants in a DB plan and the maximum PBGC coverage is M per participant. Let individual i's promised benefit be B_i, where B_i for the first K individuals is no more than M and the remaining N-K individuals' promised benefits exceed M. The total PBGC payment is then $\sum_{i=1}^{K} B_i + (N-K)M$. Now define $\theta = 1 - (\sum_{i=K+1}^{N}(B_i - M))/F$, where $F = \sum_{i=1}^{N} B_i$ is the total promised benefits of all participants. Then it is easy to see that the total PBGC payment is $\sum_{i=1}^{K} B_i + (N-K)M = \theta F$.

the employee's time-T pension payment becomes:[38]

$$S = F(1 - D) + \min(F, max(W_T, \theta \times F))D + \alpha \times \max(W_T - F, 0). \tag{11}$$

Second, we assume that PBGC faces a convex shadow financing cost in the following quadratic form:

$$C_B(x) = c_{ins,a} + c_{ins,b}x + c_{ins,c}x^2 \tag{12}$$

where x is the amount that PBGC has to pay out at time T. We assume that PBGC's financing cost is comparable to that of large publicly traded companies, and set the coefficients of the financing function accordingly to those estimated by Hennessy and Whited (2007) for large firms: $c_{ins,a} = 3.89 \times 10^{-4}$, $c_{ins,b} = 0.053$, and $c_{ins,c} = 0.2$, where parameter values are adjusted such that the unit is in billion dollars. The values we choose for $c_{ins,a}$, $c_{ins,b}$, and $c_{ins,c}$ are much lower than those for the average firms calibrated in Section IV.1.

Third, we assume that the fairly priced insurance premium by PBGC is one that makes PBGC break even on its insurance. To model the over-/under-pricing of pension insurance, we follow the literature (i.e., Doherty and Schlesinger, 1990) and set the insurance premium as a multiplier m of the fair premium:

$$I = mD\frac{1}{R_f}E^Q\left[\max(\theta \times F - W_T, 0) + C_B(\max(\theta \times F - W_T, 0))\right] \tag{13}$$

The insurance is overpriced if $m > 1$, underpriced if $m < 1$, and fairly priced if $m = 1$.

In the analysis, we start with the parameter values in the baseline case described in Section IV.1. Further, as the baseline case we set θ=0.7 and m=1. Then, we vary the following parameters one at a time: the bankruptcy probability p, the surplus sharing parameter α, the linear and quadratic components of the shadow financing cost of PBGC, $c_{ins,b}$ and $c_{ins,c}$, the underpricing/overpricing parameter for insurance premium, m, and the pension insurance coverage parameter θ.

[38]PBGC only takes over underfunded pensions in bankruptcy. It does not take over a pension if it decides that the pension asset value is sufficient to cover pension obligations.

Figure 8 displays the pension investment and funding decisions in the presence of pension insurance, under variations of parameter values. In Plot A.1, the solid line is for pension portfolio weight under PBGC insurance, while the doted line is for the case without pension insurance (for comparison purpose). It shows that the pension's risky investment w initially decreases with the bankruptcy probability p, but when credit rating drops below BB (30-year bankruptcy probability above 50%), w becomes increasing in p. Relative to the case without pension insurance (i.e., Plot A.1 of Figure 3), the pension invests more aggressively, especially at very low credit ratings. This is because pension insurance reduces the effectiveness of the bankruptcy channel of risk sharing by the employee. As a result, the average Δ of the pension is lower, and thus w has to be higher due to the w-Δ substitution effect. Furthermore, with a higher p, pension investment becomes more aggressive to take advantage of the lower financing cost of PBGC relative to that of the firm.

Plot A.2 shows that the pension funding ratio $W_0/PV_0(F)$ decreases with the bankruptcy probability. However, compared with the case without pension insurance (Plot A.2 of Figure 3), the present value of pension benefit $PV_0(F)$ increases at a slower pace as p increases. This is because pension insurance reduces the component of F that compensates the employee for bankruptcy.

Plots B.1 and B.2 of the figure show that under pension insurance, the effects of the surplus sharing parameter α on w, F, and W_0 are largely similar to those without insurance (corresponding plots in Figure 3). Intuitively, this is because the pension insurance does not significantly affect the surplus sharing channel of investment risk sharing. Further, the impact of the bankruptcy channel on the BBB-rated firm in the plot is limited without pension insurance.

In Panels C and D, we plot pension decisions under alternative values of the two parameters for PBGC's funding cost, $c_{ins,b}$ and $c_{ins,c}$. The plots show that pension investment, benefit and contribution decisions are not sensitive to PBGC's financing cost parameters. This is because the effect of pension insurance is limited for firms with relatively low bankruptcy probability—the BBB-rated firm in the plot.

Further, as shown in Plot E.1, when pension insurance premium is underpriced, the pension is more aggressive in risk taking to take advantage of insurance underpricing. Further, variations in pension pricing m affect the funding cost to the firm, but does not directly affect the employee's utility. Therefore, in Plot E.2 we see that the promised pension benefit F is not sensitive to m. The firm chooses lower initial funding in the presence of underpricing, to take advantage of cheap pension insurance. Plots F.1 and F.2 show that when pension insurance coverage is low (θ below 50%), the pension decisions are not much different from the case without pension insurance. This is because there is only about a 1.16% probability for a 25% pension shortfall at time T and almost 0% probability for a shortfall of 50%. Thus the insurance coverage has limited effect on pension decisions. However, as shown in Plot F.1, once insurance coverage increases above 50%, the portfolio weight w starts to increase with θ. This is because with higher coverage, Δ via the bankruptcy channel is reduced, which is then compensated by an increase in w, due to the w-Δ substitution effect. From Plot F.2, as the coverage increases beyond 50%, further increase in θ is associated with a decrease in F. This can be interpreted as that with higher pension insurance coverage, the employee demands less promised benefit.

V A Dynamic Model

While the one-period model goes a long way to deliver the key intuitions, it restricts the firm from adjusting pension investments or making pension contributions in the interim. In this section, we consider a dynamic model that relaxes such restrictions.

V.1 Model Setup

In a discrete-time dynamic model, there are $T+1$ dates, $t = 0, ..., T$. At time 0, the firm hires a representative employee and sets up a DB plan that promises a time-T retirement payment F. The investment opportunity of the pension consists of a risk-free asset with a constant periodical gross return of R_f, and a risky asset with gross return of $R_{m,t}$ for the t-th period. The firm adjusts portfolio weights (denoted by ω_t) and makes pension contribution (denoted by X_t) each period. The contribution X_t is non-negative prior to time T. That is, before time T, while the firm can make pension contribution, due to tax penalty it cannot revert pension assets.

Given the pension's investments and contributions, pension asset W_t evolves as:

$$W_{t+1} = (W_t + X_t)((1 - \omega_t)R_f + \omega_t R_{m,t+1}) \tag{14}$$

with $W_0 = X_0$. The firm's investors value pension-related cash flows with a risk neutral probability Q, with $E_t^Q(R_{m,\tau}) = R_f$ for $t < \tau$.

Bankruptcy is determined by the first arrival of a Bernoulli process with per period probability of bankruptcy p. We denote by τ the stopping time of the first arrival (i.e., the random bankruptcy time). When the firm goes bankrupt, it stops future pension contribution and liquidates pension asset. The employee receives a pension payment of $S_\tau = \min(R_f^{T-\tau}F, W_\tau) + \alpha \max(W_\tau - R_f^{T-\tau}F, 0)$. In the expression, the time-T promised pension benefit of F is discounted at the risk-free rate, and the employee takes α share of the pension surplus.

The employee has a power utility function on time-T retirement wealth S_T: $U(S_T) = \frac{S_T^{1-\gamma}}{1-\gamma}$. Her reservation utility is $\mathbb{U}$. In the event of bankruptcy, we assume that the employee transfers the pension payment S_τ into a DC plan and invests optimally onward.

We continue to assume a quadratic financing cost function for the firm: $C(X_t) = c_a + c_b X_t + c_c X_t^2$. The firm's pension-related cash flow at time t is thus $CF_t = -X_t - C(X_t)$. We assume that such cash flows are additive to the firm's other cash flows. The firm's objective is to maximize the expected present value of all pension-related cash flows under the risk neutral probability, subject to the employee's participation constraint, which is valued under physical probability:

$$\max_{F;\{X_t\};\{\omega_t\}} E^Q \left[\sum_{t=0}^{T} \frac{CF_t}{R_f^t} \right] \tag{15}$$

$$\text{subject to:} \quad E(U(S_T)) = \mathbb{U}. \tag{16}$$

We numerically solve the dynamic model, using fine discrete grids of the asset returns in time. Between adjacent grid points, we use log linear approximation for the portfolio allocation decision to get closed-form utility valuation under the physical probability for the employee and the continuation utility under the risk neutral probability for the firm, and then aggregate across grids at each period to reach an approximation of the expected values. The utility valuations outside the lower and upper tails of the grids are interpolated and integrated using the Gauss-Laguerre quadrature method. Given the fine grid, the above approximation is with high precision. We follow the dynamic programming principle to solve the optimization problem through backward propagation. Most of the economic parameters stay the same as those in the one-period model (listed in Section IV.1). The exception is that we assume the fixed cost of external financing as zero (i.e., $c_a = 0$) in the dynamic model to smooth the path of the optimized decisions.

V.2 Initial Pension Decisions

Under the dynamic model, we revisit some of the comparative results obtained for the one-period model. In particular, we examine how the firm's initial (time-0) decisions regarding pension benefit, funding, and investment vary when we change two key model parameters—the bankruptcy probability p and the surplus sharing parameter α. The results are plotted in Panels A and B of Figure 9 respectively. The pension decisions plotted include the time-0 portfolio weight ω_0 (Plots A.1 and B.1), the present value of pension benefit $PV_0(F)$, and initial pension contribution (W_0) (Plots A.2 and B.2).

Plot A.1 of the figure shows that everything else the same, a higher bankruptcy probability p is associated with less risky portfolio investment ω_0. This pattern is similar to what we find in Figure 3 for the one-period model. However in general ω_0 in the dynamic model is higher than that in the one-period model. For example, ω_0 is 80.5% for an AA-rated firm and 62.0% for a BBB-rated firm. The corresponding numbers for the one-period model in Figure 3 are 58.9% for the AA firm and 54.8% for the BBB firm respectively. The reason for the more aggressive pension investment in the dynamic model is that the firm can spread pension contributions across multiple dates. Such an additional flexibility in the dynamic setting reduces the effect of the convexity in the financing cost, which is the key counterweight that holds back the pension's investment risk-taking.

Plot A.2 of the figure shows that the promised pension benefit $PV_0(F)$ increases with the bankruptcy probability p, consistent with the one-period model. Further, the initial pension funding level W_0 is generally lower than that in the one-period model at any given p. This is again because of the firm's ability to optimally amortize pension contribution in multiple periods to alleviate the impact of convex financing cost.

With varying surplus sharing parameter α, Plot B.1 shows that ω_0 initially increases, and then decreases, with α. The inverse-U shaped relation, similar to that in the one-period model discussed in Section IV.2, except that the peak of the graph is shifted to the right.[39] Plot B.2 further shows that the relation of α with pension benefit $PV_0(F)$ and initial pension funding W_0 is similar to that in the one-period model.

Overall, we find that in the dynamic model, when facing a long horizon, the relation of pension risk taking with some of the key parameters remains qualitatively similar to what we find in the one-period model. With the ability to dynamically adjust pension investments and contributions over time, the firm invests aggressively in the risky asset.

V.3 The Dynamics of Pension Investments and Contributions

We further investigate how pension investments and contributions vary with the investment horizon. Unlike the one-period model where the asset allocation decision and the funding

[39] In the one-period model, the Δ-efficiency effect dominates for α lower than 0.2, resulting in an upward slope in the graph. The two forces reaches a balance at α around 0.2, and the w-Δ substitution effect takes over afterwards, resulting in a downward slope. In the dynamic model, as we later explain in more depth in Section V.3, the downward pressure from the w-Δ substitution effect manifests mainly in the overfunding region while the firm optimally underfunds the plan at time 0 for a wide range of α value (α less than 0.6) to benefit from cost-efficient amortization of pension funding. As a result, the w-Δ substitution effect is muffled till α is high enough such that underfunding is no longer optimal for the firm. Therefore, in the dynamic model, the Δ-efficiency effect dominates in a wider range of α, relative to the one-period model.

decision are made only once at the beginning of the period, in the dynamic model, both investment and funding are decided at every time points, and these decisions depend strategically on the current funding level. In Figure 10, we plot portfolio weights (w) and pension contributions (X_t) against a grid of funding levels at each time t and let time t evolve from year 0 (the start of the pension) to year 30 (the retirement year). In Panel A of the figure, we plot the baseline results where the annual bankruptcy probability is p=0.05% (for a BBB-rated firm) and the employee's share of pension surplus is α=20%. In Panel B, we change p to 0.01% (for a AA-rated firm) while keeping α at 20%. In Panel C, we change α to 40% while keeping p at 0.05%. The time-t funding ratio in the figure is before time-t contribution, i.e., $(W_t - X_t)/PV_t(F)$.

All three cases share some common features. When the horizon is long, the portfolio weight on the risky asset w is relatively high. As time draws closer to retirement, for most range of the funding ratio, w drops. The intuition in Proposition 1 can help us understand this pattern, but there are also important differences due to the added richness of the dynamic model, as we explain below.

The two main counterbalancing factors identified in Proposition 1 for the optimal portfolio weight are: (1) the employee's desire to have access to the risk premium of the risky asset; and (2) the firm's financing cost. In the dynamic model, the pension chooses to have high risk exposure at such times and at such funding levels that investment risk can be most effectively passed onto the employee and that the cost associated with investment risk (due to the convex financing cost) is relatively low.

Along the time dimension, the firm would choose to have high pension investment risk when the retirement time is far away. When the horizon is long, the firm can smooth pension contributions over multiple periods, which substantially reduces the impact of the convexity in the financing cost.[40] Furthermore, when the retirement is far in the future, the firm has the option to choose whether and when to make the contributions, and the option to adjust

[40]Consider this simple thought experiment. For a one-time pension contribution of \$10, the convex component of the financing cost is proportional to $10^2 = 100$. In contrast, if the firm breaks down the contribution into 10 equal pieces, the convex component of the financing cost adds up to an amount proportional to only $10 \times 1^2 = 10$, which is only one tenth of the case of the one-time contribution.

the portfolio weight strategically in all the following years until the retirement year. Such an option gains value from uncertainty, i.e., when the pension's investment risk is high. All these factors point to the direction that the pension plan should have higher investment risk in earlier periods, ceteris paribus.

In addition, at any given time, the firm allocates the pension's investment risk exposure across different funding level states in a strategic way. Let us start with the time that is close to the retirement year T and go backward in time. When the retirement year T is near, the probability of firm bankruptcy before the retirement year becomes very small, and the employee gets the risk exposure mainly through the surplus sharing channel. When there is a higher pension surplus, there is less concern of funding shortfall and the surplus channel is more effective as Δ is approaching α. In this case it is cost-efficient to invest very aggressively. At the extreme, when the funding ratio is very high, there is essentially no concern of pension shortfall and thus no concern of financing cost, Δ conditional on the firm's survival at retirement year is approaching α. Thus, as illustrated in Plot A.1, B.1 and C.1, w is approximately ω^*/α, where ω^* is the optimal weight if the employee is to invest by herself to maximize her own utility.[41] Conversely, when the funding ratio is low, the pension should invest more conservatively. At the extreme, when there is substantial funding shortfall, Δ conditional on the firm's survival at retirement year is approaching 0, and the financing cost is almost inevitable. In this case, risky investment does not benefit the employee due to zero Δ but carries substantial financing cost. With the retirement year imminent, there is little time to amortize pension contributions, and thus the convexity of financing cost has a strong effect. Therefore, as illustrated in the plots, w approaches 0.

As we go backward in time, i.e., with the time to retirement getting longer, the bankruptcy channel weighs more in the consideration. In bankruptcy, underfunding does not come with additional financing cost, and the employee is fully exposed to investment risk ($\Delta = 1$) if the pension is underfunded. In this case, risky pension investment is beneficial to the employee

[41] Thus, even when it approaches the employee's retirement, a sufficiently funded pension continues to invest aggressively. This is consistent with the empirical finding by Lucas and Zeldes (2006) that allocation to equities remains substantially high for pensions with mostly retired employees, which have a low hedging demand for future wage growth.

and meanwhile it does not hurt the firm. Thus, as shown in the plots, we start to see an increase in w at moderate level of underfunding for $t < T$. However, with high level of underfunding, the firm's concern of the convex financing cost becomes the dominating factor, ultimately leading to reduction of w. Further going backward in time, as it gets closer to time 0, the variation of investment risk across funding-level states is muted, resulting in a more flattened surface in the figure. This is because when the horizon to retirement is longer, the correlation of the current funding level and the funding level at the retirement year is weaker. Thus, the current funding level is less correlated with time-T realized Δ, and also less correlated with the firm's overall financing cost. Therefore, the funding level at earlier time points is less relevant for the pension's investment decision.

The patterns we observe from optimal portfolio weights across different bankruptcy probabilities help us gain further intuition on the asset allocation problem in the dynamic setting. When we reduce p from that of a BBB-rated firm in our baseline case in Panel A to an AA-rated firm in Panel B, we see that w becomes higher at any given funding level, in particular when the time to retirement is long. This is consistent with what we find in Panel A of Figure 3. That is, an AA-rated firm has a lower Δ than a BBB-rated firm, and therefore, the pension of the AA-rated firm should choose a higher w to compensate for the reduced Δ. As the time to retirement draws near, the bankruptcy probability becomes less relevant. Therefore, as it gets closer to time T, w by firms with different credit ratings, conditional on the same funding level, starts to converge.

Panel C of the figure illustrates the impact when the employee's share of the surplus, α, is increased. The main result is that w is lower at a higher level of α. This pattern is consistent with what we observe in Panel B of Figure 3 for the one-period model. As α increases, the pension's Δ is higher. Due to the w-Δ substitution effect, the allocation to the risky asset is lower. Further, by comparing Plot C.1 with Plot A.1, we see that the impact of α on w is mostly visible in the overfunding region, because α as a surplus-sharing parameter only matters when the pension is overfunded at time T.

Plots A.2, B.2, and C.2 show that, when the pension is overfunded and when the time is far away from retirement, pension contribution is mostly flat and insensitive to the funding ratio and to time. Changing the bankruptcy probability parameter (Plot B.2) or the surplus sharing parameter (Plot C.2) has only limited impact on this decision. This is because in such cases, the pension contribution decision is mainly driven by amortization of funding costs. However, when the time draws closer to retirement and when there is severe underfunding, the firm is forced to make large pension contributions to avoid an expensive lump-sum contribution at time T.

VI Conclusions

This paper provides a perspective to understand the risky asset allocation policies pursued by corporate pensions. In our model, pension risk taking is driven by employees' preference for systematic risk exposure, while the firm balances employees' preference with its concern of financing cost. The pension investment risk is shared between the firm and its employees. The firm's decisions on pension benefits and pension funding are endogenous to such risk sharing. For a reasonable set of parameter values, the optimal pension investment risk and its relations with a firm's bankruptcy probability and pension funding ratio predicted by the model are consistent with empirical observations.

Our analysis highlights the inefficient risk sharing in typical DB plans and its important consequences. The defined benefit plans, possibly out of a motive to protect employees from firm-specific risk, make employees' pension payoffs relatively insulated from systematic investment risk as well. This makes DB plans suboptimal in allocating the systematic component of pension investment risk between the firm and its employees. A more efficient contract would let employees to shoulder all the pension investment risk while keeping them insulated from firm-specific risks. Interestingly, this arrangement resembles what a defined contribution plan offers. Our analysis shows that such an arrangement may substantially reduce firms' pension funding costs, which may be one of the reasons behind the popularity of DC plans.

Appendix A

Proof of Proposition 1. The firm's decisions include the initial funding W_0, promised payment to the employee at retirement F, and the weight on the risky asset ω. Note that for any fixed W_0 and F, a negative portfolio weight on the risky asset ($\omega < 0$) is suboptimal, because simply reverting the portfolio weight to $-\omega > 0$ does not change the portfolio risk but increases the expected portfolio return. Therefore, we only need to show that $\omega = 0$ cannot be optimal.

First, we show that if $\omega = 0$, the firm will choose to underfund the pension at time 0, i.e., $W_0 < F/R_f$. Note that when $\omega = 0$, the employee's participation constraints dictates the relation between the remainder two decision variables, W_0 and F. In the overfunded case, i.e., $W_0 > F/R_f$, the employee's participation constraint is

$$F + \alpha(W_0 R_f - F) = S_r, \tag{17}$$

where $S_r \equiv [(1-\gamma)\mathbb{U}]^{1/(1-\gamma)}$ is the time-T riskfree payoff to satisfy the employee's participation constraint. The resulting firm's objective, defined in Equation 4, takes the value of $-C(W_0 - H_0)$. We have

$$\frac{d(\textit{Firm Obj})}{dW_0} = -C'(W_0 - H_0) < 0. \tag{18}$$

For the underfunded case, i.e., $W_0 \leq F/R_f$, the employee's participation constraint is

$$(1-p)F^{1-\gamma} + p(W_0 R_f)^{1-\gamma} = S_r^{1-\gamma}. \tag{19}$$

Thus, $dF/dW_0|_{F=W_0R_f=S_r} = -pR_f/(1-p)$. The resulting firm's objective is $-C(W_0 - H_0) - \frac{1-p}{R_f}C(F - W_0R_f)$. Taking the derivative with respect to W_0 at the critical point of just-funded case, where $F = W_0R_f = S_r$, we have

$$\left.\frac{d(\textit{Firm Obj})}{dW_0}\right|_{F=W_0R_f=S_r} = -C'(W_0 - H_0) + \frac{1-p}{R_f}C'(0)\left[\frac{pR_f}{1-p} + R_f\right] = C'(0) - C'\left(\frac{S_r}{R_f} - H_0\right) < 0 \tag{20}$$

where the inequality is due to the fact that function C is strictly convex and thus $C'(S_r/R_f - H_0) > C'(0)$ for $S_r/R_f > H_0$. Therefore, the firm's objective value increases by decreasing W_0. Thus, the optimal W_0 must be less than F/R_f, i.e., the pension is optimally underfunded.

We now turn to the proof of the proposition. To maximize the firm value subject to employees' participation constraint, the firm essentially maximizes the following Lagrange function:

$$\mathbb{L} = CF_0 + \frac{1}{R_f}E_0^Q(CF_T) + \lambda\left[E_0(U(S)) - \mathbb{U}\right]. \tag{21}$$

with $\lambda > 0$.

Denote the triple $\{W_0^*, F^*, \omega = 0\}$ as the optimal choice for the firm among all strategies with $\omega = 0$. From the proved first step above, we have $W_0^* R_f < F^*$ (i.e., the pension is underfunded). We have

$$\begin{aligned}\left.\frac{\partial\mathbb{L}}{\partial\omega}\right|_{W_0^*, F^*, \omega=0} =& \int_{-\infty}^{+\infty}\left\{\frac{1}{R_f}(1-p)\left(1 + C'(F - W_T)\right)f^Q(R_m) + \lambda U'(S_1)pf(R_m)\right\}W_0(R_m - R_f)dR_m \\ =& \lambda pU'(\bar{S}_1)W_0(E(R_m) - R_f) > 0\end{aligned}$$

where $\bar{S}_1 = (1-p)F + pW_0R_f$. This suggests the optimal portfolio weight has to be positive. □

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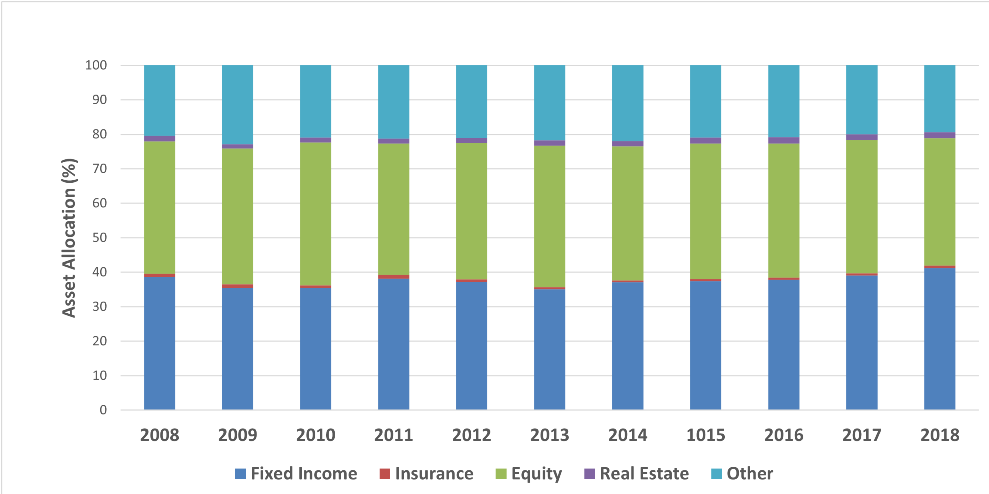
Table 1: Single-employer DB Plan Terminations, Plan Surplus, and Asset Reversion

This table presents the statistics on the terminations of U.S. private-sector single-employer Defined Benefit (DB) plans from 2000 to 2018, as well as the statistics on plan surpluses at plan terminations and subsequent asset reversions. The statistics are obtained from the Form 5500 data provided by Department of Labor. The plan termination year is identified as the first reporting of termination by a plan in its annual filings. The plan surplus and plan benefit liabilities are identified at the plan termination year. Plan asset reversions are the sum of assets reverted to sponsors during and after the plan termination year. The last column reports the average ratio of reverted assets to plan surplus, over all plans terminated with surplus. The last row provides the statistics averaged over the years.

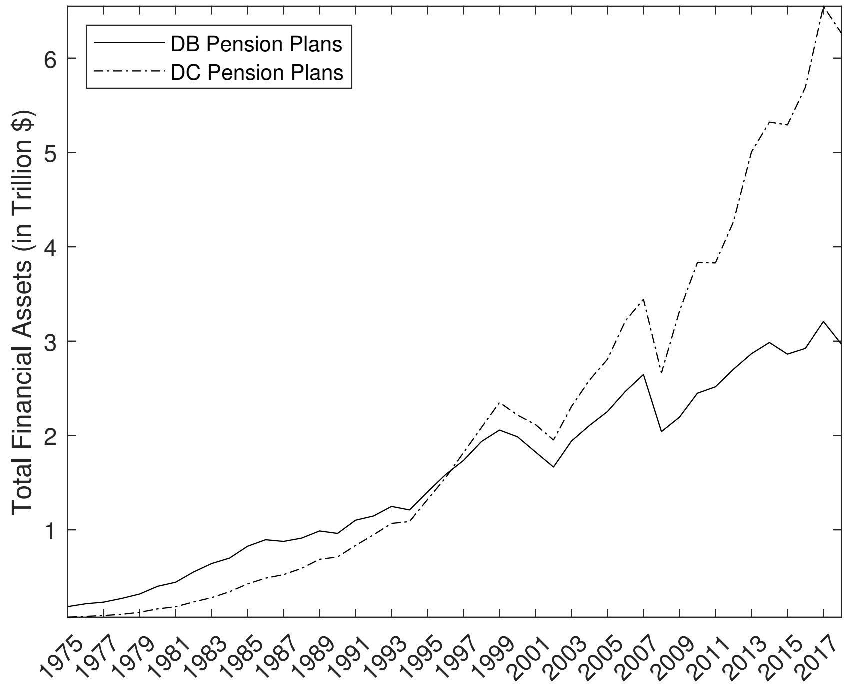
Year	# of Single-employer DB Plans	# of Plan Terminations	% of Plans Terminated	# of Plans Terminated with Surplus	% of Plans Terminated with Surplus	Surplus / Plan Benefit Liabilities at Termination (%)	# of Terminated Plans with Subsequent Asset Reversions	Average Ratio of Reverted Assets to Surplus (%)
2000	11,085	375	3.4%	223	59.5%	2.8%	26	7.5%
2001	10,976	223	2.0%	102	45.7%	50.0%	13	6.5%
2002	10,460	139	1.3%	53	38.1%	6.6%	2	0.1%
2003	10,141	115	1.1%	29	25.2%	9.5%	2	4.2%
2004	9,865	110	1.1%	32	29.1%	6.4%	1	1.6%
2005	9,748	167	1.7%	58	34.7%	15.3%	1	0.0%
2006	9,536	221	2.3%	79	35.7%	7.0%	2	1.8%
2007	9,242	218	2.4%	91	41.7%	12.9%	2	0.1%
2009	8,473	149	1.8%	49	32.9%	3.4%	3	1.2%
2010	8,265	129	1.6%	46	35.7%	9.0%	2	0.5%
2011	7,984	132	1.7%	50	37.9%	3.2%	1	0.2%
2012	7,793	146	1.9%	71	48.6%	7.4%	1	0.0%
2013	7,529	129	1.7%	93	72.1%	20.5%	1	0.0%
2014	7,366	194	2.6%	161	83.0%	21.4%	1	0.1%
2015	7,056	157	2.2%	103	65.6%	27.3%	2	1.1%
2016	6,809	187	2.7%	127	67.9%	12.2%	2	0.1%
2017	6,512	209	3.2%	150	71.8%	9.9%	2	0.0%
2018	6,042	217	3.6%	172	79.3%	23.1%	4	0.3%
Average	8,605	179	2.1%	94	50.3%	13.8%	4	1.4%

Figure 1: Assets Allocation by Defined Benefit Corporate Pensions



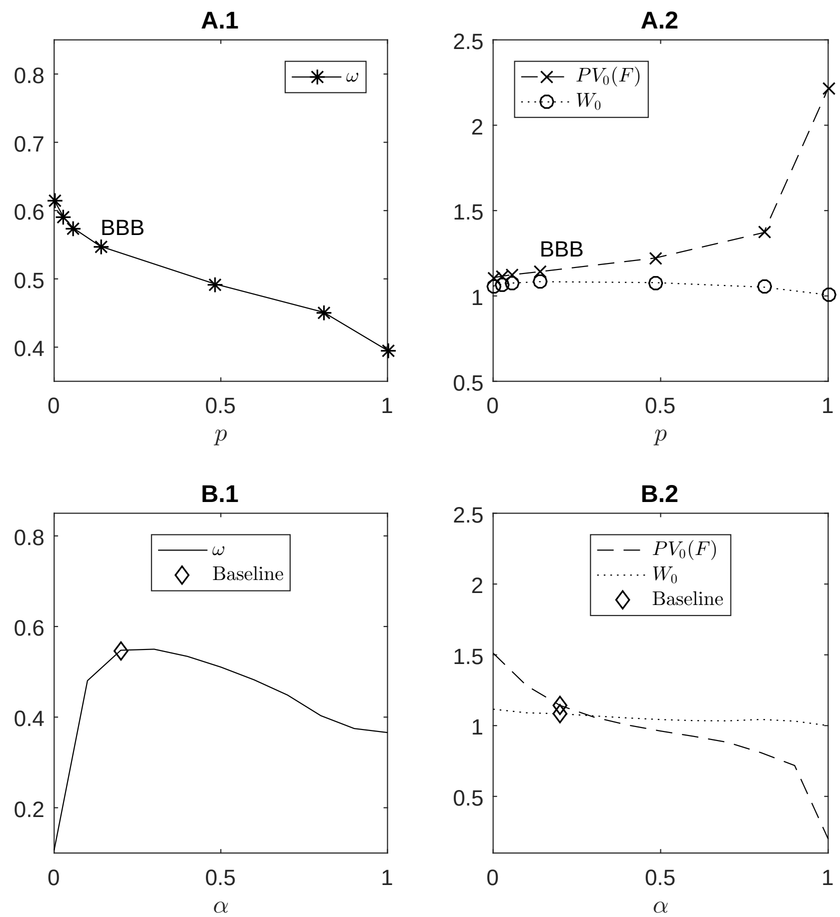
This figure plots the asset allocations of aggregate corporate pensions, from 2008 to 2018. The asset classes include fixed income, insurance, equity, real estate, and other. The data are from Department of Labor.

Figure 2: Total Assets of Private-sector Defined Benefit Plans and Defined Contribution Plans, 1975-2018



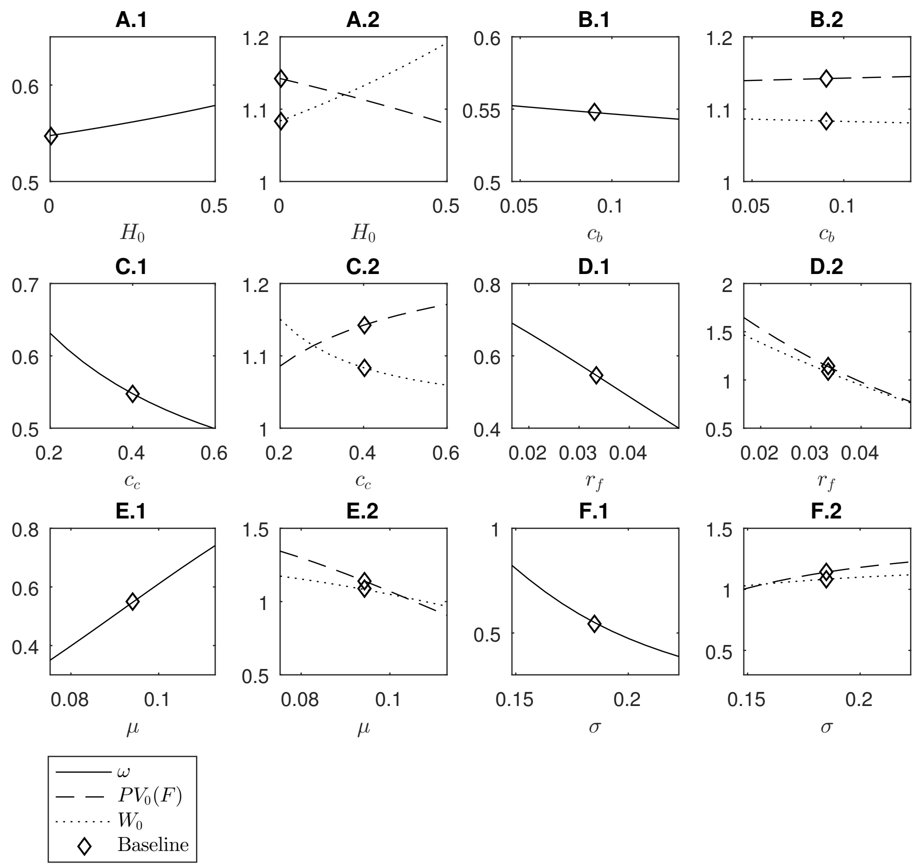
This figure plots the total assets of private-sector defined benefit plans and defined contribution plans, from 1975 to 2018. The data are from Private Pension Plan Bulletin, Department of Labor.

Figure 3: Impact of Risk Sharing on Pension Investment and Pension Funding Decisions



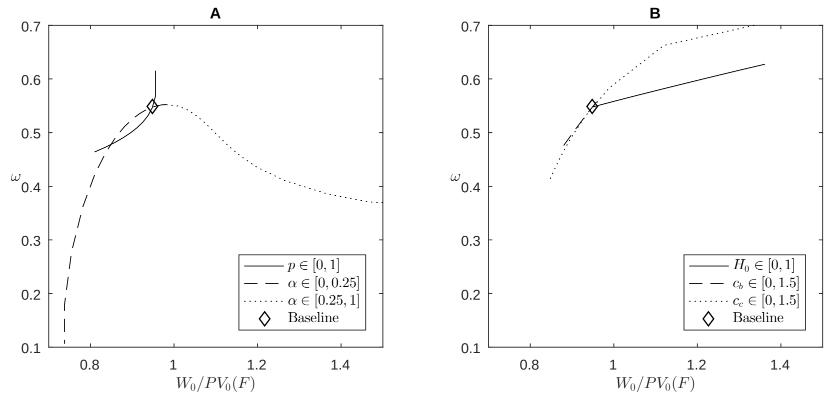
This figure illustrates the impact of bankruptcy probability and surplus sharing on pension investment and funding decisions. Panel A displays the pension decisions under six bankruptcy probabilities, which are calibrated to those for firms with credit ratings of AAA, AA, A, BBB, BB, B, and F (sure bankruptcy) respectively. Panel B displays pension decisions under varying pension surplus sharing parameter α which is the employee's share of pension surplus. In each panel, the first plot is on the portfolio weight of the risky asset, w. The second plot is on the present value of pension benefit $PV_0(F)$ and the initial pension contribution W_0.

Figure 4: Impact of Additional Model Parameters on Pension Investment and Pension Funding Decisions



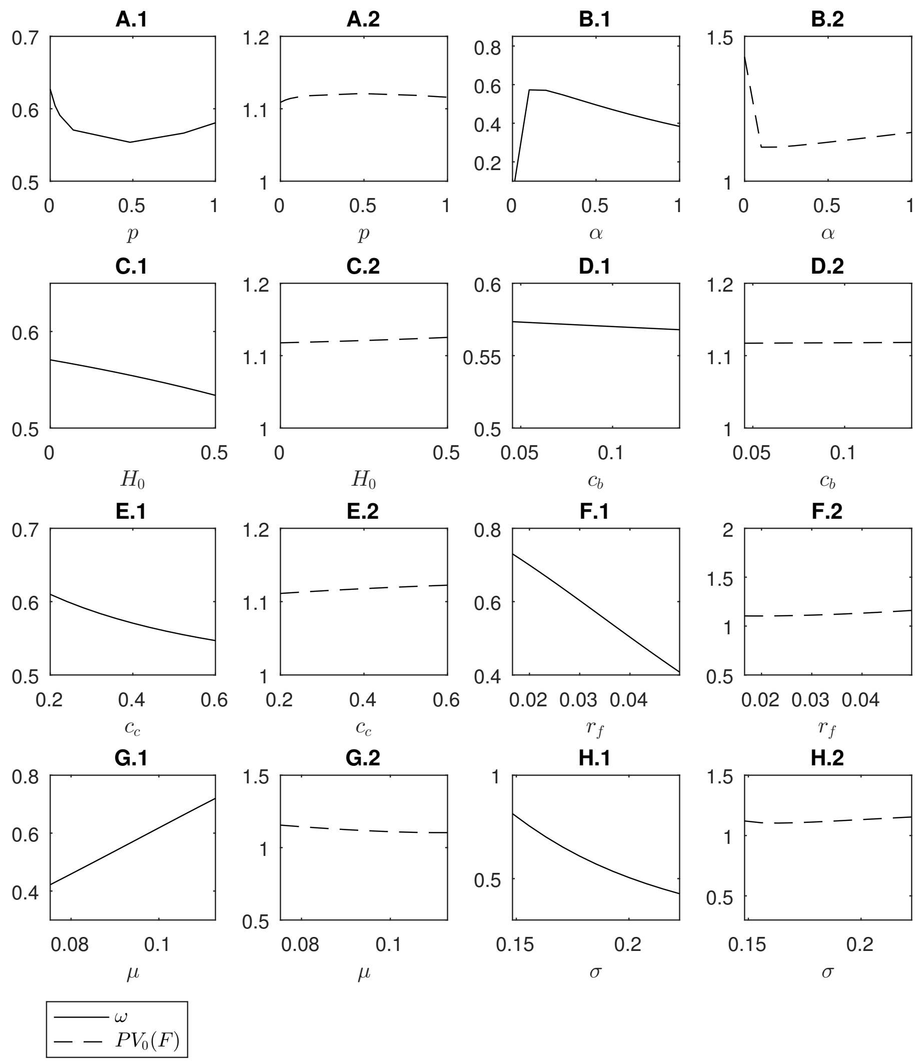
This figure illustrates the impact of an additional set of model parameters on pension investment and funding decision. Panels A to F display the impact of the initial cash available H_0, the coefficients for the linear (c_b) and quadratic (c_c) of financing cost, the risk-free rate r_f, the risk premium $E(r_m) - r_f$, and the volatility of the risky asset σ, respectively. In each panel, the first plot is on the portfolio weight of the risky asset, w. The second plot is on the present value of promised pension benefit $PV_0(F)$ and the initial pension contribution W_0.

Figure 5: Relation Between Optimal Investment and Optimal Funding Ratio



This figure illustrates the endogenous relation between the optimal pension investment (w) and the optimal funding ratio ($W_0/PV_0(F)$) under various parameter values. The left panel shows the relation under various values of bankruptcy probability (p) and surplus sharing (α). The right panel shows the relation under varying values of initial free cash (H_0), the linear (c_b) and quadratic (c_c) financing cost coefficients.

Figure 6: Decisions by Initially Fully-funded Pensions



This figure illustrates pension decisions under various parameter values when the pension is required to have full funding initially. Panel A displays the pension decisions with various bankruptcy probabilities p. Panel B displays the pension decisions under different pension surplus sharing (α). In both panels the first plot displays the portfolio weight on the risky asset (w) and the second plot displays pension benefit ($PV_0(F)$). In Panel C, Plots C.1 to C.6 display the portfolio weight w for different values of the initial cash (H_0), the linear component of financing cost (c_b), the quadratic component of financing cost (c_c), risk free rate (r_f), the expected return of the risky asset (μ), and the volatility of the risky asset (σ), respectively.

A Online Appendix

I.1 Nash Bargaining with Reversion Tax

When the pension investment results in a surplus, the firm and the employees need to come to agreement on how to share the surplus. We examine the surplus sharing problem within a stylized Nonsymmetric Nash bargaining game setup (**?**). We denote the bargaining power of the employee by γ and according $1-\gamma$ for the firm. The standard Nash bargaining solution (**?**) is a special case of $\gamma = 1/2$. Note that the firm pays a tax rate of t for the surplus it keeps.

While the firm has means to convert surplus to its assets, we assume that firms sharing α of the pension surplus S with its employee gains a reputation capital of $f(\alpha) \times S$, with $f(0) = 0$ and increasing in α. A firm that is willing to share large surplus to its employee may be viewed as internalizing its employee's welfare and therefore may benefit from easy future recruiting and enjoy higher workplace morale.

Consistent with the notation in the rest of the paper, we denote the employee share of the surplus by α. The Kalai bargaining solution maximize the following:

$$\max(\alpha S)^{\gamma} \times \left(\left[f(\alpha) + (1-t)(1-\alpha)\right] S\right)^{1-\gamma} \tag{A1}$$

Equivalently,

$$\max \gamma \ln \alpha + (1-\gamma) \ln \left(f(\alpha) + (1-t)(1-\alpha)\right)$$

The first order condition is:

$$\frac{\gamma}{\alpha} + \frac{(1-\gamma)\left(f'(\alpha) - (1-t)\right)}{f(\alpha) + (1-t)(1-\alpha)} = 0$$

We consider a simple linear function specification of the firm's reputation capital, $f = \phi\alpha$. Clearly, if $\phi \geq (1-t)$, we have the corner solution of $\alpha = 1$. If $0 < \phi < (1-t)$, we have:

$$\alpha = \gamma + \frac{\gamma\phi}{1-t-\phi}.$$

The first term above is increasing in γ, the second term is increasing in γ, ϕ, and t, because its numerator is increasing and its denominator is decreasing in the respective variables. We collect the observations in the following proposition:

Proposition 2. *When the pension generates a surplus of S, the firm and the employee reach the Kalai bargaining solution, of αS goes to the employee, $(1-t)(1-\alpha)S$ goes to the firm, and consequently $t(1-\alpha)S$ paid as penalty tax, by maximizing the objectives in Equation (I.1), where parameter γ represents the employee's bargaining power and the firm's reputation capital $f(\alpha) = \phi\alpha$. When $\phi \geq (1-t)$, the bargaining reaches the corner solution* *of* $\alpha = 1$. *With* $0 < \phi < (1-t)$, *we have*

$$\alpha = \gamma + \frac{\gamma\phi}{1-t-\phi}.$$

and

$$\frac{\partial\alpha}{\partial\gamma} > 0, \frac{\partial\alpha}{\partial\phi} > 0, \textit{and } \frac{\partial\alpha}{\partial t} > 0.$$

From the proposition, the employee's share, α, is increasing in her bargaining power (γ), increasing in the firm's reputation concern on employee welfare (ϕ), and increasing in the penalty tax rate (t) of converting the surplus to the firm assets. Consequently, the employee's share of the pension investment risk, denoted by Δ in the main text, is increasing in γ, ϕ, and t. However, the relations between those parameters and the resulting pension investment decision (W) are not monotonic, because, as demonstrated by the inverted U-shape shown in Panel B.1 of Figure 3 in the paper, W increases in α when α is small and decreases when α is large. The pension's initial investment, W_0, is monotonically decreasing in the three model parameters because, as shown in Panel B.2 of Figure 3, it is decreasing in α.

I.2 No Surplus Sharing in Bankruptcy

For robustness of our baseline results, we repeat the comparative analysis in Section IV.2 with no surplus sharing (i.e., $\alpha = 0$) when the firm faces bankruptcy (i.e., $D = 1$). The time-T pension payment to the employee, S', is then:

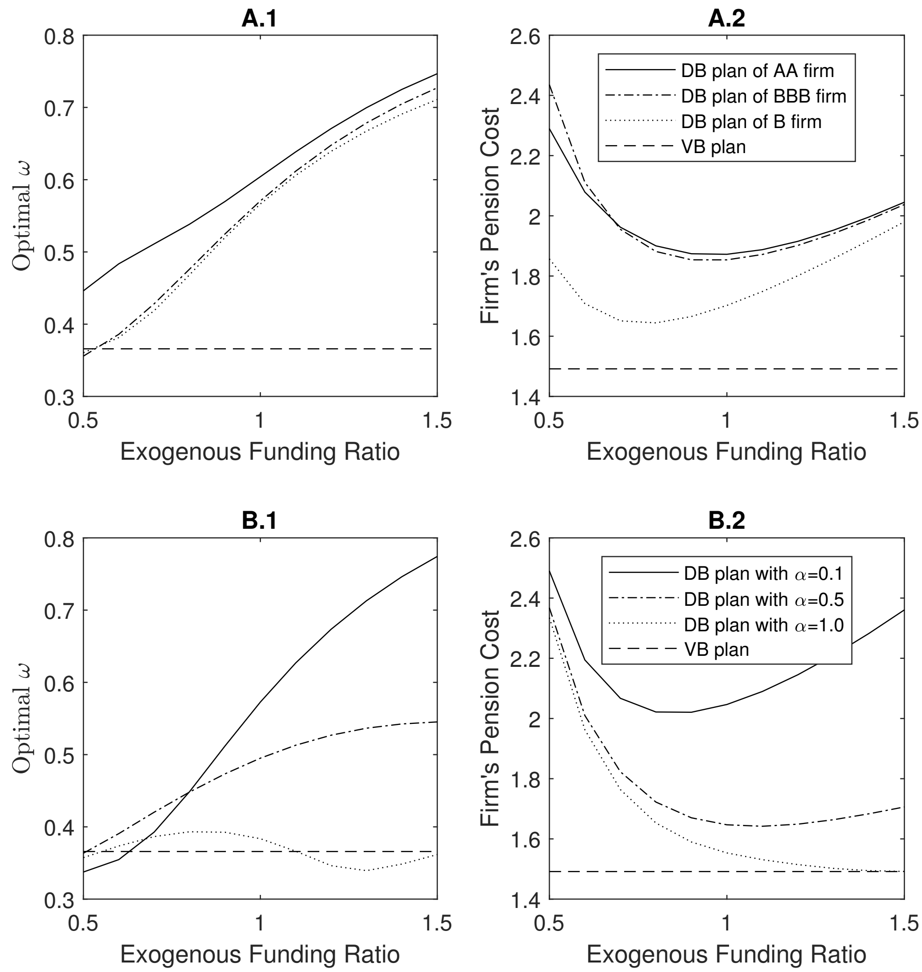
$$S' = (1-D)F + D\min(W_T, F) + \alpha(1-D)\max(W_T - F, 0)$$

Figure A1 shows similar patterns as Figure 3, with a few noticeable differences. The impact of bankruptcy risk on investment risk-taking, as shown in A.1 of both figures, remains the same for moderate bankruptcy risks. However, for high-bankruptcy risk firms, the risky allocation in pension investment slowly increases with the probability of bankruptcy. B.2 illustrates that when surplus sharing is not executed in the event of bankruptcy, the firm always underfunds the pension plan at the beginning regardless how much it commits to share the surplus with the employee if no bankruptcy occurs.

I.3 Additional Analysis of Surplus Sharing and Pension Cost

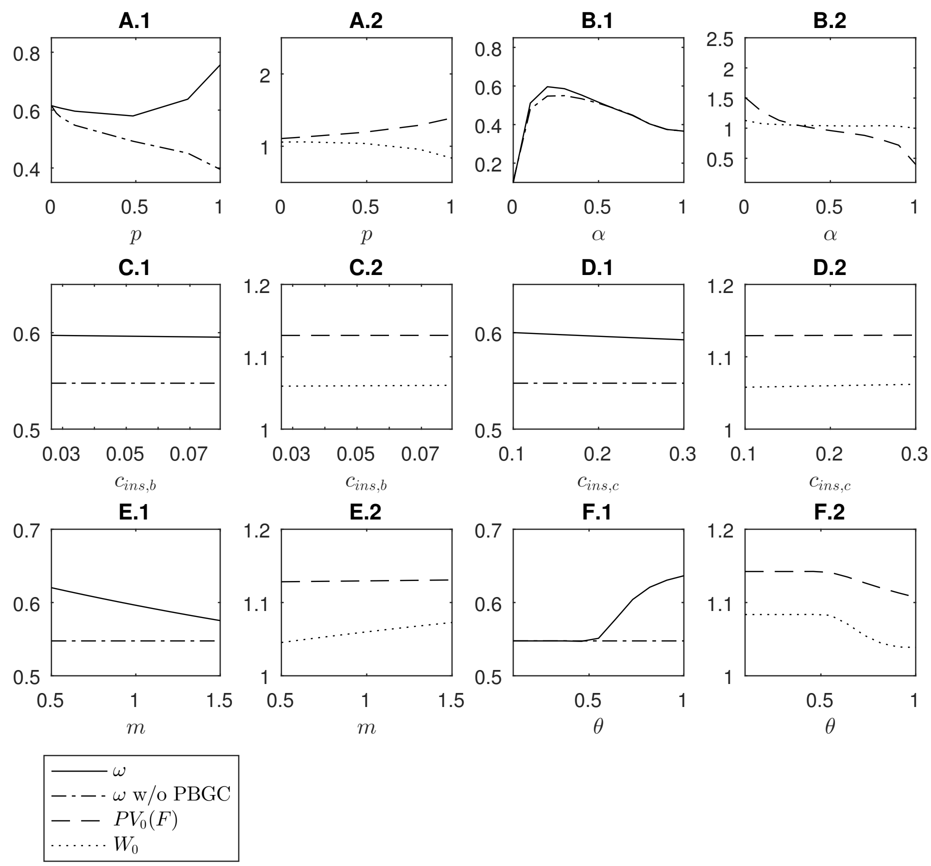
In the main body of the paper, we consider the value of surplus sharing parameter α as exogenously given. In this section, we investigate the optimal α that minimizes the firm's pension cost and meets the employee's participation constraint. We proceed the investigation by plotting different values of surplus sharing parameter with the corresponding pension costs, calibrated with the baseline values of other parameters. As shown in Figure A2, the firm's pension cost decreases monotonically with α for both DB plans, the traditional one characterized by the baseline model, and the one with zero (0) bankruptcy risk. The pension costs of both DB plans converge to the VB plan when the employee receives 100% of the surplus.

Figure 7: Investment and Funding Cost: DB vs. VB Plans



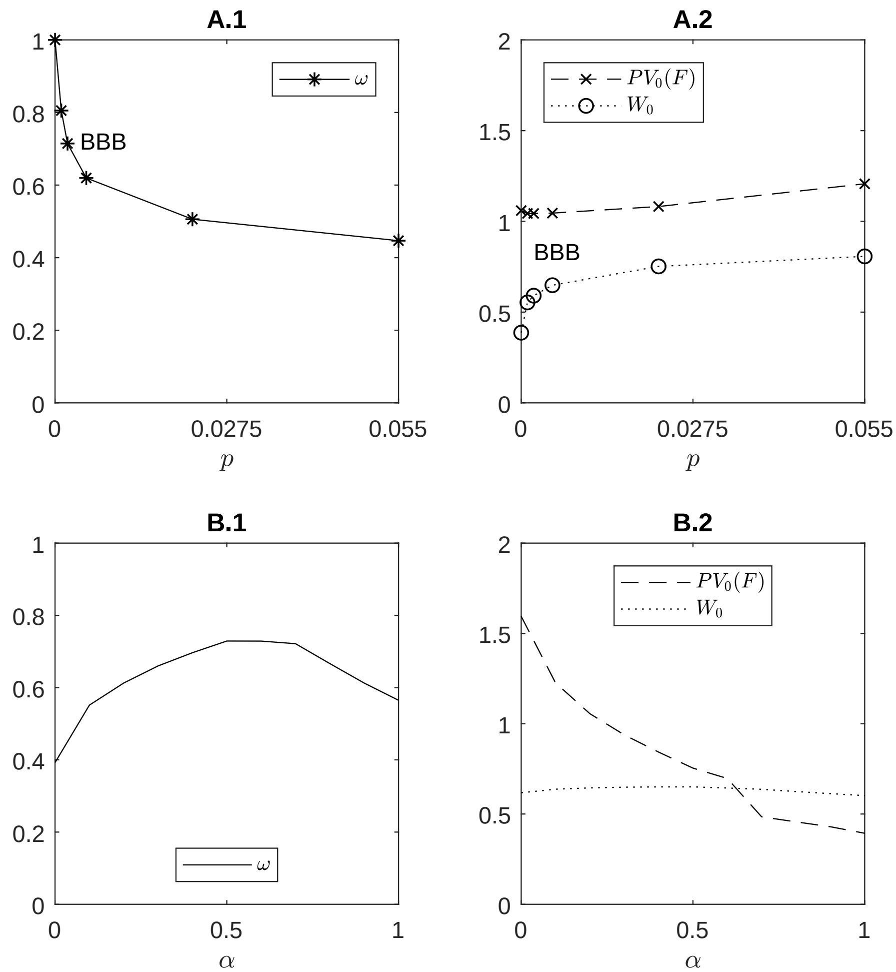
This figure illustrates pension portfolio weight (w) and expected funding cost under DB and VB plans. The pension funding cost of the DB plan is the firm's cash outflow at time 0 plus the present value of expected cash outflow at time T ($-CF_0 - \frac{1}{R_F}E_0^Q(CF_T)$). The funding cost for the VB plan is the initial contribution plus financing cost $W_0 + C(h_0)$. We keep employee's reservation utility the same in all cases, and vary the funding ratio exogenously. In Panel A, we plot pension decisions for the VB plan and three DB plans where the employee's share of pension surplus is 20% and the bankruptcy probabilities are calibrated to those of AA-rated firms, BBB-rated firms, and B-rated firms. In Panel B, we plot the pension decisions of the VB plan and three DB plans where the employee keeps 10%, 50%, and 100% of pension surplus. In each panel, the first plot is on pension portfolio weight and the second plot is on the expected pension funding cost.

Figure 8: Pension Investment and Funding Decisions with Pension Insurance



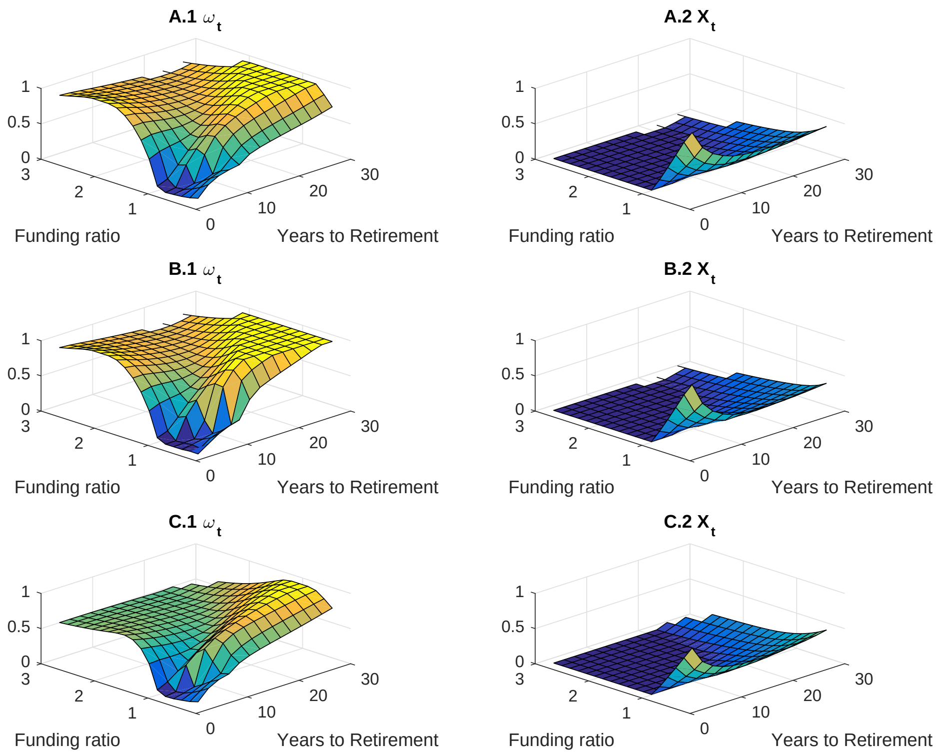
This figure illustrates pension decisions under PBGC pension insurance coverage and with various parameter values. Panels A to F display pension decisions under different values of bankruptcy probability p, surplus sharing α, the coefficients for the linear and quadratic components of PBGC funding cost $c_{ins,b}$ and $c_{ins,c}$, the pension insurance pricing factor m, and the pension insurance coverage θ, respectively. In each panel, the first plot is for the pension portfolio weight w, and the second plot is for the present value of pension benefit $PV_0(F)$ and initial pension contribution W_0.

Figure 9: Initial Investment and Funding Decisions in the Dynamic Model



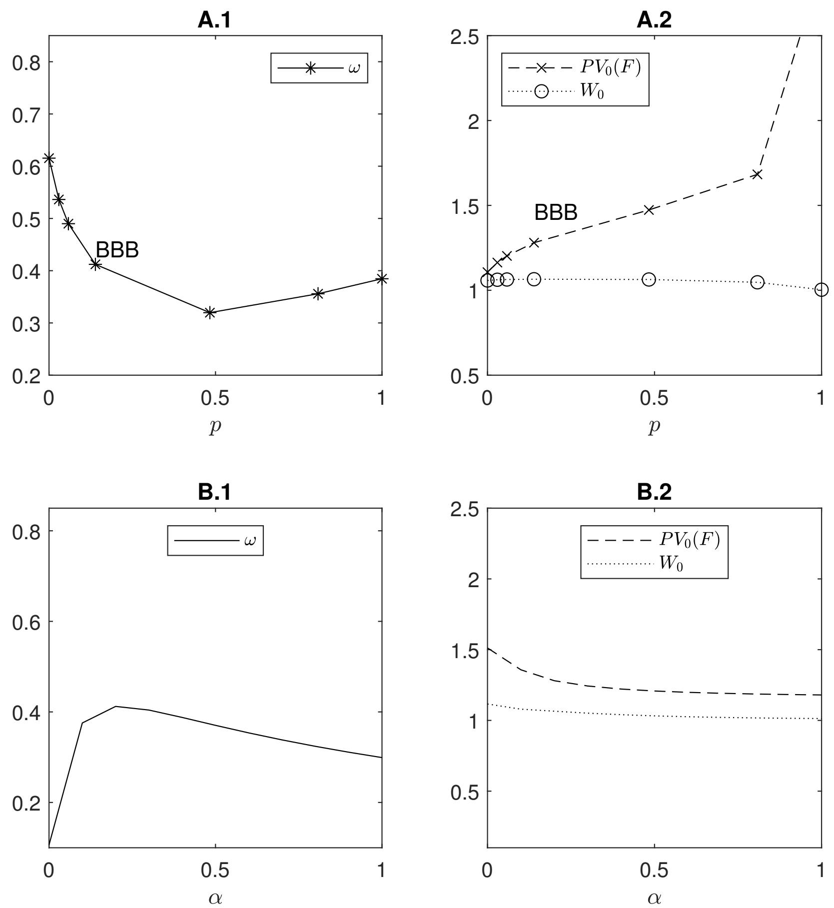
This figure illustrates the initial pension decisions under different values of the bankruptcy probability p (in Panel A) and surplus sharing parameter α (in Panel B), for the dynamic model. In each panel, the first plot is for the pension's time-0 portfolio weight (ω_0), and the second plot is for the present value of pension benefit $PV_0(F)$ and time-0 pension funding W_0.

Figure 10: Optimal Investment and Pension Funding Decisions Over Time



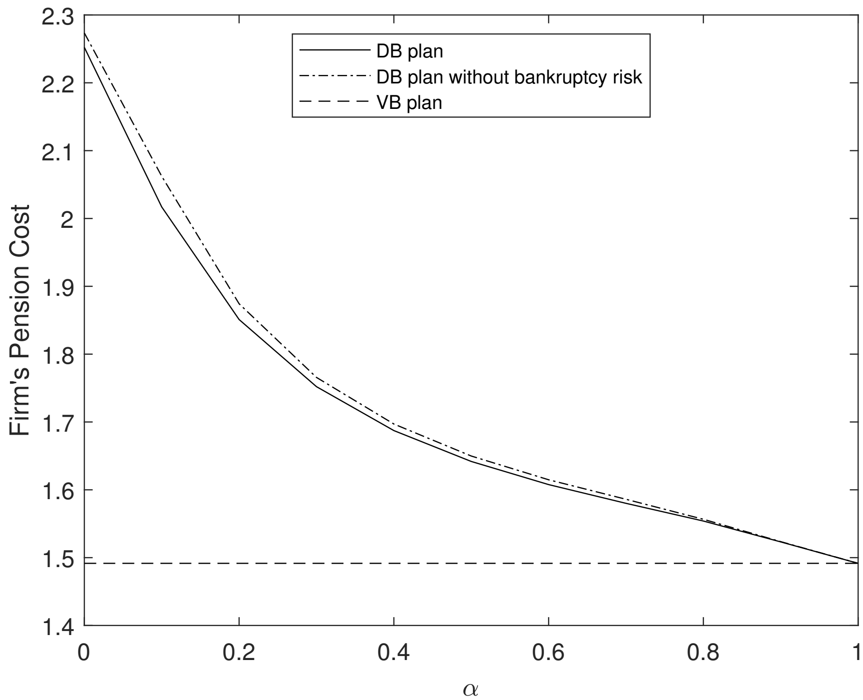
This figure illustrates pension investment (ω_t) and pension contribution (X_t) decisions across time and states (funding ratio). Panel A shows the optimal investments and contributions under the baseline parameters, where the bankruptcy probability is calibrated to that of a BBB-rated firm and the employee keeps 20% of pension surplus. Panel B shows the optimal investments and contributions under a lower probability of bankruptcy (for an AA-rated firm). Panel C shows the optimal investments and contributions when the employee keeps a higher (α=40%) share of the pension surplus.

Figure A1: Impact of Risk Sharing on Pension Investment and Pension Funding Decisions: with No Surplus Sharing in Bankruptcy



This figure illustrates the impact of bankruptcy probability and surplus sharing on pension investment and funding decisions. Compared to the baseline case, we assume that no surplus sharing ($\alpha = 0$) happens when the firm files bankruptcy. Panel A displays the pension decisions under six bankruptcy probabilities, which are calibrated to those for firms with credit ratings of AAA, AA, A, BBB, BB, B, and F (sure bankruptcy) respectively. Panel B displays pension decisions under varying pension surplus sharing parameter α which is the employee's share of pension surplus. In each panel, the first plot is on the portfolio weight of the risky asset, w. The second plot is on the present value of pension benefit $PV_0(F)$ and the initial pension contribution W_0.

Figure A2: Impact of Surplus Sharing on Pension Cost



This figure illustrates the impact of surplus sharing α on the firm's pension cost. The solid line represents the traditional DB plan, as defined in the baseline model. The dot-dash line represents the DB plan with zero (0) bankruptcy risk, that is $p = 0$. The dash line represents the VB plan defined and discussed in Section IV.5